ELECTRICITY

Larder

PRINTED BY
SPOTTISWOODE AND CO., NEW-STREET SQUARE
LONDON

ELECTRICITY

FOR

PUBLIC SCHOOLS AND COLLEGES

BY

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AUTHOR OF 'A SCHOOL COURSE IN HEAT,' IN USE AT RUGBY, CLIFTON, CHELTENHAM, BEDFORD, BIRMINGHAM, KING'S COLLEGE LONDON, AND IN OTHER SCHOOLS AND COLLEGES.

SECOND EDITION

LONDON
LONGMANS, GREEN, AND CO.
AND NEW YORK : 15 EAST 16th STREET
1889

PREFACE

TO

THE SECOND EDITION.

IN this edition many misprints have been corrected, several sections have been revised and improved, and some additions made on the subject of modern measuring instruments.

In answer to certain criticisms the author would add, firstly, that the book is not intended to be a practical companion to the laboratory, but rather to indicate the amount of theory that it is advisable to master before entering upon technical work ; secondly, that it is assumed that the student can see and examine the instruments referred to, the descriptions being intended merely to enable the learner to understand at once the principle of each instrument that comes under his notice.

The writer wishes to acknowledge his obligations to Dr. A. H. Fison for help most kindly given with respect to corrections and additions for this edition.

W. L.

August 1888.

PREFACE.

In this Course the writer aims at giving a sound though elementary knowledge of the modern science of Electricity. With a view to rendering the book suitable for use in public schools, it has been thought better to assume no more mathematical knowledge than is usually possessed by the higher boys in a classical school.

For reasons of space, no attempt has been made to treat the history of the subject at all completely. Names and dates may be occasionally mentioned; but for information on this head the reader can consult works in which the historical sequence of discovery has been carefully traced.

In that part of the subject in which an elementary knowledge of *Chemistry* is demanded, the student is referred to the elementary text books of Roscoe and of others, in which he can easily read up to the desired standard.

But with respect to such knowledge of *systems of units* and of *mechanical principles* as is essential for the proper understanding of the present Course, the writer conceives that the case is different. Believing it to be far from easy for the student to collect from various sources just the kind of mechanical knowledge required for the present purpose, the writer had originally included in the Course all the mechanics needed. He has, however, since been obliged, for reasons of space, to curtail; and for the most part to indicate merely the exact amount of knowledge required.

With regard to *diagrams*, it may be remarked that there have

to be considered two classes of drawings. There are the simple diagrams illustrative of principles, and there are the complete figures representative of actual pieces of apparatus. After some deliberation the writer decided to draw the former specially for the Course; but the latter have, for obvious reasons, been freely borrowed from other works, wherever, through the courtesy of the publishers, this was permitted. For such use of diagrams the writer is chiefly indebted to Messrs. Longmans, Green, & Co.; and also to Messrs. Elliott, to the publishers of 'Nature,' and to others.

At the end of the book will be found questions and numerical examples; these are intended both to serve as an exercise, and also to some extent to supplement the teaching of the text. Some few of the numerical exercises have, with the kind permission of the publishers, been borrowed.

It would be impossible for the writer to acknowledge in detail all that is owed to other books, papers, and lectures. But he would here express his thanks to Professor Tait, Professor D. E. Hughes, and Professor Crookes, for most kindly answering questions and giving information on certain points. Also to Mr. Lupton, of Harrow, for kindly giving permission to use some tables given in his 'Numerical Tables.'

But especially would he acknowledge his obligations to his friends Professor W. H. Heaton, of Nottingham, who most kindly looked over the MSS. and the proofs of the first ten chapters; and Professor W. N. Stocker, of Cooper's Hill, who, with like kindness, looked over the MSS. of the remaining fifteen chapters. To both of these the writer is indebted for many valuable suggestions and criticisms. Also to his friend and colleague, Mr. A. S. Davis, who undertook the work of looking over the proofs of the last fifteen chapters.

CHELTENHAM COLLEGE: *June* 1887.

REFERENCES.

The writer here gives some references to works which the student ay consult if he desires to go deeper into the subject. He has ought that it may be useful to some if he gives also references various matters discussed in 'Nature,' volumes of this paper ing found in most libraries that are even partly scientific.

The list that follows is, however, very far from being complete.

I. Magnetism.

Lloyd's Magnetism. [Longmans, Green & Co.]
Airy's Magnetism. [Macmillan.]
Admiralty Manual of Scientific Enquiry. [Murray.]
Elementary Manual for the Deviation of the Compass, by F. J. Evans.
Theory of Electricity, by L. Cumming. [Macmillan.]
Electricity and Magnetism, by Mascart and Joubert; translated by E. Atkinson. [De la Rue & Co.]

II. Theory of Potential.

Elementary work, not employing the notation of the calculus, Theory of Electricity, by L. Cumming. [See under I.]
Treatise on Statics, vol. ii., by G. M. Minchin. [Clarendon Press.]
Mascart and Joubert. [See under I.]
Elementary Treatise on Electricity, by Clerk Maxwell. [Clarendon Press.]
Electricity; the large work by Clerk Maxwell. [Macmillan.]

III. Electrolysis.

Faraday's Experimental Researches.
Stewart and Gee's Elementary Practical Physics, vol. ii. p. 72, gives a list of references. [Macmillan.]
Sprague's Electricity [Spon & Co.] gives useful practical details.
There is to be a discussion on Electrolysis at the B. A. meeting for 1887. Probably therefore much useful information will be given in the report of this discussion.

IV. Electric Testing, Telegraphy, &c.

Handbook of Telegraphy, by Culley. [Longmans.]
Stewart and Gee's book. [See under III.]
Practical Physics, by Shaw and Glazebrook. [Longmans.]
Electric Testing, by Kempe. [Spon & Co.]
Telegraphy, by Preece and Sivewright. [Longmans.]
Practical Electricity, by W. E. Ayrton. [Cassell.]

V. Phenomena in High and Low Vacua.

A Physical Treatise on Electricity and Magnetism, by Gordon [Sampson Low], gives description and figures of experiments by Crookes, Spottiswoode, De la Rue, and others.

We may refer also to 'Nature,' vols. xx. pp. 174, 199, 228, 250, 438; xxii. pp. 101, 125, 149, 153, 174, 196; xxiii. pp. 442, 461; xxiv. pp. 66, 89, 130, 255, 346, 569; xxv. pp. 539, 594; xxvii. p. 434; xxviii. p. 381; xxx. p. 230.

VI. Dynamos and Motors; Electric Lighting, &c.

Electric Transmission of Energy, by Kapp. [Whittaker.]

Dynamo-Electric Machinery, by S. P. Thompson. [Spon.]

Modern Applications of Electricity, by Hospitalier, translated by Julius Maier. [Kegan Paul.]

Arc and Glow Lamps, by Julius Maier. [Whittaker.]

VII. General.

Clerk Maxwell's book. [See under II.]

Die Lehre von der Elektricität, von G. Wiedemann. [Friedrich Vieweg und Sohn, in Braunschweig.]

Cours de Physique, par M. J. Jamin. [Gauthier-Villars, Paris.]

Œuvres de Verdet. [Imprimerie Nationale, Paris.]

Mascart and Joubert's book. [See under I.]

VIII. Miscellaneous Papers in 'Nature.'

Hydrodynamical Analogies to Electric Phenomena, vol. xxiv. p. 362; xxv. p. 271; xxvi. p. 134.

Electricity, Medical Use of, vol. xxv. p. 521; xxviii. p. 463.

Electric discharge by red-hot balls, and towards a flame, vol. xxv. 475, 523.

Electric Light and Horticulture, vol. xxiv. p. 567.

Aurora Borealis, &c., vol. xxii. pp. 33, 146; xxiv. p. 613; xxv. pp. 53, 319, 368; xxvi. p. 571; xxvii. pp. 31, 389, 443; xxviii. pp. 60, 107, 128; xxix. p. 409; xxx. p. 80; xxxi. p. 372; xxxii. pp. 275, 348; xxxv. 433.

Electric Current Meters, vol. xxviii. p. 162; xxx. p. 220; xxxiv. p. 508.

'*Contact-Electricity*' *of Metals*, vol. xxxv. p. 142.

Magnetism; Hughes' Researches in, vol. xxviii. pp. 159, 183; xxix. p. 459.

Earth's Magnetism, vol. xxv. p. 66.

Magnetisation, Change in Dimensions of Magnetised Iron, &c., vol. xxii. p. 543; xxxii. p. 45; xxxiii. p. 597.

Saturation of Cores of Electromagnets, vol. xxxiv. p. 159.

Magnetic Field produced by rotation of an Insulator, vol. xxii. p. 89.

Floating Magnets, vol. xvii. p. 487.

Diamagnetism, vol. xxxiii. pp. 484, 512.

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SOME ABBREVIATIONS USED.

Magnetic pole-strength	μ
Magnetic moment	m and μl
Field-strength	H and I
Work and Energy	W
Electric Quantity	Q
Electrostatic Capacity	K
Specific-inductive-capacity	σ
Electric and Magnetic density	ρ
Potential	V
Difference of Potential	ΔV
Gold-leaf electroscope	g.l.e.
Current-strength	C
Resistance	R and r
Equivalent-resistance	R′
Battery-resistance	B
Galvanometer-resistance	G
Shunt-resistance	S
Electromotive Force	E.M.F., E, and e
Joule's equivalent [This is the factor which reduces *calories* to *ergs*; and $= 4{\cdot}175 \times 10^7$, or 42×10^6 nearly.]	J

Note.—In cases where the above symbols have other meanings, the context will obviate ambiguity. Thus *V* may sometimes mean *velocity*; *H* may mean *heat*; and ρ may mean *specific resistance*.

ELECTRICITY.

CHAPTER I.

GENERAL PHENOMENA OF MAGNETISM.

§ 1. **Introductory.**—The subjects of *magnetism* and *electricity* are in reality not two, but one; all magnetic, electro-magnetic, and voltaic phenomena—to use terms with which most of our readers will have some acquaintance—belong to one great branch of science for which we have not yet one comprehensive name. Perhaps it might be more logical were we to give at once a general survey of all the above-named classes of phenomena before proceeding to a more detailed discussion of each. But, as in this course no previous knowledge has been assumed, it has been thought better to avoid any chance there might be of confusing the mind of the student by the presentation of a multitude of strange facts, and hence we shall first discuss the main phenomena of *magnetism*. A study of these will serve as a training to the beginner, and he will incidentally become acquainted with many facts and conceptions that will prove to be of great value in the study of the other branches of our science.

§ 2. **First Phenomena observed.**—Accustomed as most of us are to the use of a 'magnet,' there is still something very startling in the simple elementary experiment with a 'magnet' or a piece of lodestone. When a piece of lodestone (magnetic oxide of iron, Fe_3O_4, magnetised by the influence of the earth) is held above a piece of iron or steel of not too great weight, the piece of iron or steel will move up to the lodestone, against the force due to gravity. And, more remarkable still, the lodestone can convert pieces of hard steel into permanent 'magnets,' it being

n

merely necessary to make a convenient bar of steel and to stroke it repeatedly from end to end with the lodestone, taking care that the strokes are alike in all respects. We shall henceforth employ such steel magnets instead of lodestone.

§ 3. **Polarity.**—If we examine our magnetised bars of steel we shall find that, though the lodestone passed along every portion of them equally, the ends always possess far greater magnetic powers than do the middle portions. Further than this, we find that in each magnet these two ends are different. If we magnetise a bar A B, and another A′ B′, in such a way that A and A′, B and B′ are respectively corresponding ends (and we can do this by stroking the one bar from A to B, the other from A′ to B′ with the same portion of one piece of lodestone), then we find that A repels A′, B repels B′, but that A attracts B′ and A′ attracts B. We find also that if such bars be suspended they will take up a definite position with respect to the earth, setting themselves in a direction called *magnetic north and south*, the similarly treated ends A and A′, B and B′, lying in similar directions.

We may say that the *ends* of the bar have acquired remarkable properties, these properties being, in a sense, of opposite natures at the two ends respectively. It is usual to call the ends where the magnetic properties are displayed *poles*, and the bar is said to have acquired *magnetic polarity*.

We may here point out a possible source of error. We have seen that like poles repel, and unlike poles attract, one another; and further that all needles or bars, that are magnetised and suspended, turn so as to point in a northerly and southerly direction. That end of the bar that points towards the geographical north pole of the earth is usually called a *north pole*, or *north-seeking* pole; the other end is called a *south pole*, or *south-seeking* pole. But from what we have seen it is evident that the *north-seeking pole of the bar must be of opposite polarity to the 'north pole' of the earth towards which it turns;* and so with the south-seeking pole of the bar.

It is better, therefore, to speak of the 'north pole' of the earth, but the 'north-seeking' pole of a magnetised bar.

§ 4. **Constitution of a long thin Magnet.**—Much light is thrown upon the nature of this *Polarity* by the following simple experiment:

A strongly magnetised knitting-needle is taken, and the

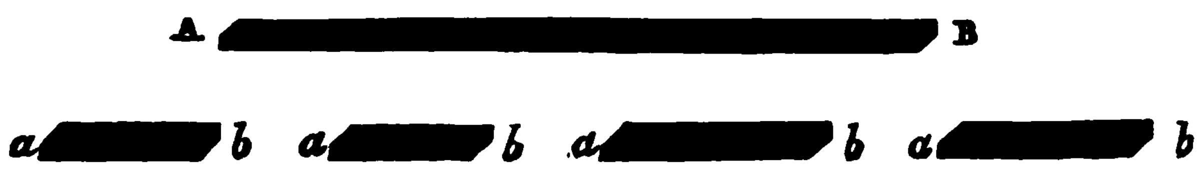
polarity of the ends, and neutrality of the middle, tested by use of filings and compass-needle.

It is then broken in half.

We now find each half a complete magnetic needle; two opposite poles having apparently started into existence in the previously neutral middle.

Each half is then again broken, and so on.

The original needle, and the condition when it is broken, are shown in the accompanying figure.



It would seem that all these intermediate poles were existing in the original needle, but that they neutralised each other as far as any external action went.

As far as experiment goes there is nothing against the supposition that if we had a magnetic bar very thin, in fact, a single row of molecules of steel, we might continue this process of breaking up until we should find each molecule a complete little magnet with poles at its ends (if we may use the term *ends* with respect to a molecule which may be spherical), and a neutral region at the middle.

Experiments.—(i.) Magnetising a bar of steel with lodestone, we find that iron filings will cluster chiefly at the ends.

(ii.) Take a series of unmagnetised knitting-needles; testing them by seeing that either end acts the same on the '*north*' end of an ordinary compass-needle.

Lay them down side by side, and magnetise them successively with a piece of lodestone or with a steel magnet; marking with gummed paper the ends that must, by the process of magnetisation, be similar.

It will then be found that (*a*) similar poles repel, dissimilar attract, one another; (*b*) the suspended needles will all set themselves with the marked ends turned in the same direction.

§ 5. **Molecular theory of Magnetism.**—The above experiment suggested, very early, that probably magnetism is *molecular*. The view adopted was that when a long thin needle was magnetised, its molecular constitution was somewhat as represented in the accompanying figure. N S is the magnetic needle, supposed to

consist of a single line of molecules. Each of these molecules is a small magnet, and they are arranged so that north-seeking poles are opposed to south-seeking poles all along: there being left at the one end a north-seeking pole, and at the other end a south-seeking pole, uncompensated.

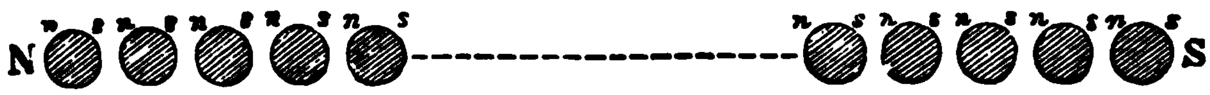


FIG. i.

It is assumed, as justified by such experiments as that alluded to in § 4, that the south-seeking pole of one molecule can neutralise, as far as external manifestation of magnetic properties goes, the opposed north-seeking pole of another molecule, while the north-seeking and south-seeking poles of the same molecule, though very close to one another, do not so neutralise one another.

The state of neutrality was formerly considered to be a 'higgledy-piggledy' arrangement, or rather absence of arrangement, producing on the whole external neutrality.

Thus the end of a magnetised bar would present to external bodies a whole set of molecular north-seeking or south-seeking poles, while the end of an unmagnetised bar would present a mixed surface of north-seeking and south-seeking poles whose total external action would be nil.

Professor D. E. Hughes has taken up the molecular theory, and has reduced it to order, much extending it. In fact, he has done so much that the chief questions left unsolved are the fundamental ones: 'What is the polarity of a molecule? and, Why do similar poles repel and dissimilar attract one another?' The question of *diamagnetism* also needs some further research.

The main drift of Professor Hughes's theory, each portion of which he has supported by experiment, is somewhat as follows:

(i.) Each molecule (possibly each atom) of every substance possesses to a fixed and unalterable degree a property called polarity. The *poles* of the molecule are probably fixed in that molecule, and can be made to lie in a changed direction only by rotation of the molecule.

(ii.) The opposite *poles* of consecutive molecules do, when the molecules are so rotated that these 'poles' lie one over against the other, neutralise each other with respect to external action.

(iii.) *In neutrality* there is always some symmetrical arrangement by which the molecules neutralise one another with respect to any external action ; either by the molecules being paired off with opposite poles together, or by a whole line of molecules neutralising each other's poles in one long chain, or by some equivalent arrangement.

One of the great features of Hughes's theory is the *symmetry* of arrangement that is considered to exist under all circumstances. The accompanying figures indicate two possible cases of neutrality.

Since the molecule, whatever its real form, is practically for magnetic purposes a short line with poles at the ends, we have here represented the molecules as lines ; indicating thereby their magnetic, though not their actual shape.

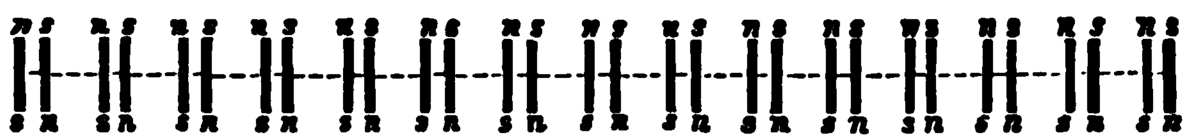


FIG. ii.

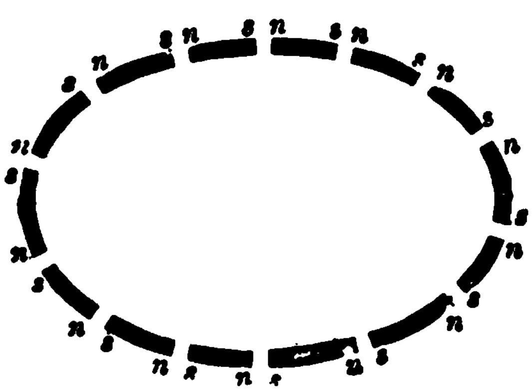


FIG. iii.

(iv.) When the bar is magnetised to the greatest possible degree, the arrangement will be such that there is a chain of molecules in which the north-seeking pole of the molecule at the one end of a chain, and the south-seeking pole of the molecule at the other end of the same chain, will be left unneutralised ; while between these there is complete neutralisation. And, further, these chains must be arranged side by side so that the free north-seeking poles are all at one end of the bar and the free south-seeking poles at the other.

In the above two cases this might be effected in fig. ii. by a rotation of the molecules so as to form one line as in fig. i., while in fig. iii. it would be only necessary to break the chain at any point.

Thus there is a limit to possible strength of magnetism. The utmost that can be done is to arrange the molecules in the

most advantageous position. The strength of magnetism will then depend on the ultimate properties of the molecules.

(v.) Usually, or perhaps in all cases, the setting of the molecules so as to give evident magnetism is not completed. The molecules are *partly* rotated away from their position of what we may call 'short-circuiting' or neutralisation, and we get all intermediate cases between perfect neutrality and perfect magnetisation. A piece of iron when magnetised to its fullest extent is said to be *saturated.*

(vi.) Arrangements of the molecules, in which they do not pair off, or in any other way neutralise one another, and so give zero external action, are unstable. In fact, if only there be given free movement to the molecules, so that they can act on each other without hindrance, they will always 'short-circuit' with one another and give us zero external magnetic action.

(vii.) The rearrangement of the molecules so as to give external magnetism is resisted both by their action on each other and by the mechanical difficulties in the way of rotation of the molecules. This *mechanical rigidity*, that opposes the 'magnetisation' of a bar and also opposes the molecular tendency to a return to the neutral, short-circuited, condition, is called by the inexact name of *coercive force.* Any action (as heating, hammering, and the like) that gives freer play to the molecules will diminish the *coercive force* of the bar. In general, therefore, it is useful to hammer a bar while it is being magnetised, but bad to do so when it is no longer under the magnetising influence.

(viii.) The degree of molecular rigidity depends upon the chemical nature of the metal and upon its temper. Thus, some specimens of 'soft iron' will hardly retain any trace of magnetism when the magnetising influence has been removed, while some specimens of 'hard steel' will remain unaltered for a long time.

(ix.) *Temporary and Residual Magnetism.*—In general, if we magnetise a bar, it will readily lose part of its magnetism, but will retain the remainder unaltered for a long time. It is supposed that the molecules can, without much difficulty, rotate part of the way back to their original positions of neutrality ; but that at a certain point their freedom of movement is limited by the 'molecular rigidity' (or coercive force) spoken of above.

It is found that when iron or steel is raised to a white heat, it is not attracted by a magnet; in fact, it ceases to be magnetic. This phenomenon has not been satisfactorily explained by any theory; and, until further investigation has been made, we cannot say whether it makes for or against the above molecular theory.

Experiments to illustrate molecular theory.—(i.) We can magnetise a tube of steel filings, and then render it neutral by shaking it up. As the filings are not free to traverse the tube, but merely turn about where they are, this illustrates the case of neutrality by the pairing off of the molecules.

(ii.) Magnetise a strip of watch-spring, and test it.

If this watch-spring be now made into a complete circuit, we get almost complete external neutrality; while the evident magnetism is again restored when we break the circuit.

(iii.) A poker may be magnetised by being held along the lines of the earth's magnetism (*see* Chapter III. § 8) and in that position hammered.

(iv.) The magnetism of a steel knitting-needle may be reversed if it is held with south-seeking end downwards along the lines of earth's magnetism, and in that position kept at a bright red heat for some time; being then allowed to cool in the same position.

(v.) It was shown by Joule that a bar when magnetised increases in length. This might well be the case if each pair of molecules (*see* fig. ii. above) does in fact rotate from a parallel, to an end-on, position.

(vi.) A tube is filled with water in which is suspended magnetic oxide of iron in a finely divided condition; the whole being nearly opaque to light.

If the tube be magnetised in the direction of its length, it is found that it is now less opaque to light in this direction. This would tend to show that the particles of the magnetic oxide have arranged themselves end-on in the direction of magnetisation.

§ 6. **Induction, General Phenomena.**—The matter of induction will be clearer to the learner when he has learnt something of 'Fields of Force,' in Chapter II.

At present we shall only describe the actual facts observed.

When a piece of iron is placed near the pole of a magnet, the end of the iron that is nearest the pole acquires a polarity opposite to that of the said pole; while the end of the iron that is furthest away acquires a polarity similar to that of the magnet-pole.

This action of a magnet, in making iron near it also magnetic, is called 'Induction.'

It is more powerful as the 'inducing' magnet is more powerful, and as the iron is nearer to the magnet.

It is stronger and less permanent as the 'coercive force' of the iron is small (or the molecular freedom great), weaker and more permanent as the 'coercive force' is greater.

Experiments in Induction.—(i.) NS is a powerful magnet; and ns, $n's'$, $n''s''$, &c., are pieces of very soft iron arranged near NS as shown. The letters of the accompanying figure indicate the observed polarities of the pieces of soft iron. One may test these polarities either by means of a small compass-needle applied to the ends of the iron pieces, or in some such way as that indicated in the next experiment.

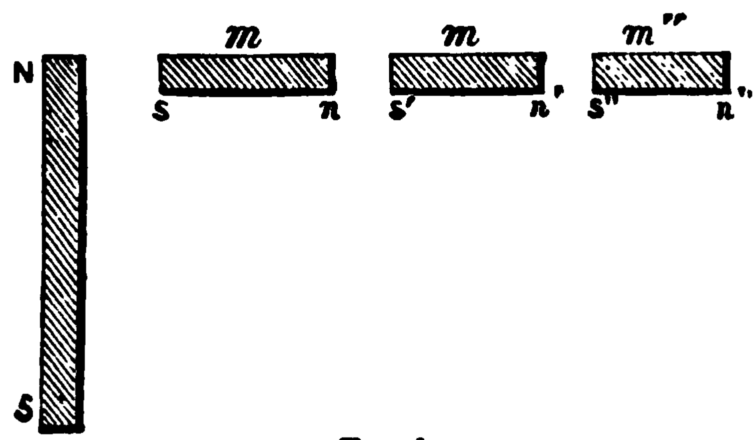


FIG. i.

(ii.) The writer believes that the following experiment is due to Professor Guthrie.

NS is a powerful permanent magnet and $\nu\sigma$ is a bar of soft iron hung above NS, parallel and close to it. Then $\nu\sigma$ will be acted upon inductively, and will temporarily become a magnet, its poles being at ν and σ.

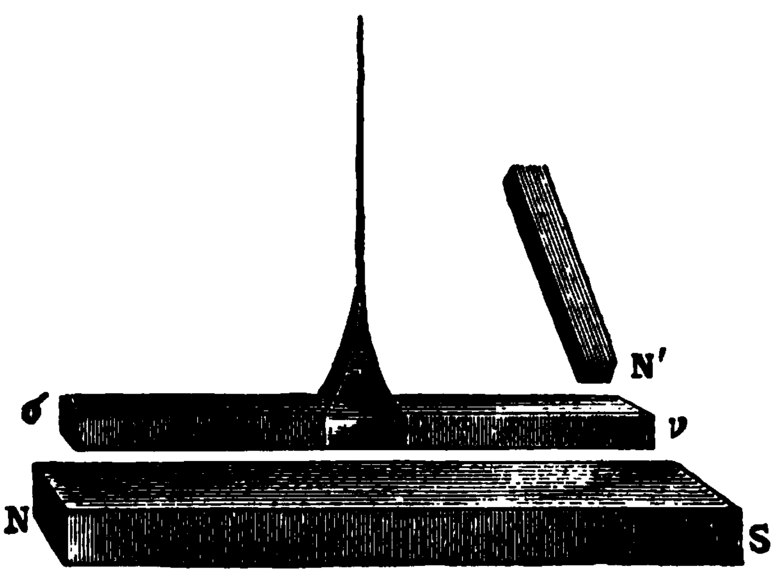


FIG. ii.

If a north-seeking pole N′ be approached to the end ν which lies above the south-seeking pole S, it is found that N′ repels ν. Hence ν is north-seeking, or S has induced at ν a polarity opposed to its own. This is, in fact, indicated by the simple fact that $\nu\sigma$ will, if displaced, return to the position represented in the figure.

We may say generally that induction takes place through any solid, liquid, or gas; it making no perceptible difference what substance is interposed between the magnet and the iron, unless we are making very delicate observations indeed.

There is, however, one very important exception. A sheet of perfectly soft iron, or iron with absolutely zero 'coercive force,' would act as a perfect screen; and no induction would take place on a piece of iron or steel suspended in a closed vessel made of such soft iron. To a greater or less degree, according to its 'softness,' all iron and steel will act as a screen.

We have spoken hitherto of *iron* and *steel* only. There are,

however, other bodies, notably *cobalt*, *nickel*, and *manganese*, which possess to a certain degree magnetic properties. More will be said of these in Chapter XX.; and it will there be pointed out how feebly magnetic are these bodies when compared with iron or steel.

§ 7. **Use of Keepers.**—When a bar is magnetised, there are at the one end a set of molecules with their north-seeking poles 'free,' and at the other end a corresponding set with their south-seeking poles 'free.' This condition of things is, as we have said before, unstable; and there is in a magnetised bar a tendency for the molecules to pair off again in such a way as to give neutrality of the nature indicated in § 5, fig. ii. This tendency is resisted by molecular rigidity, or coercive force; but a loss of magnetisation is inevitable if the bar be subjected to molecular disturbance of any kind.

But if we can, by means of soft iron pieces applied to the magnet, make the molecular circuit complete, we shall obtain a

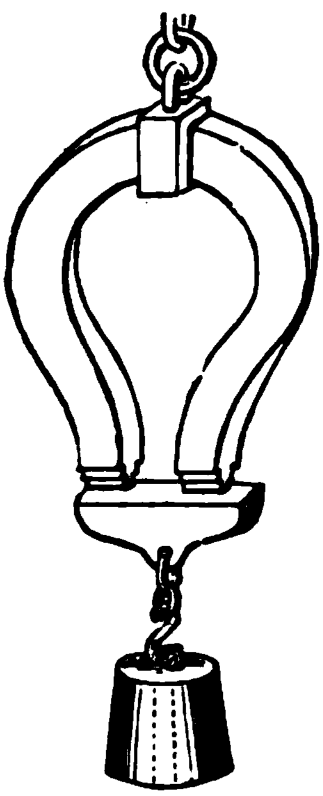


FIG. i.

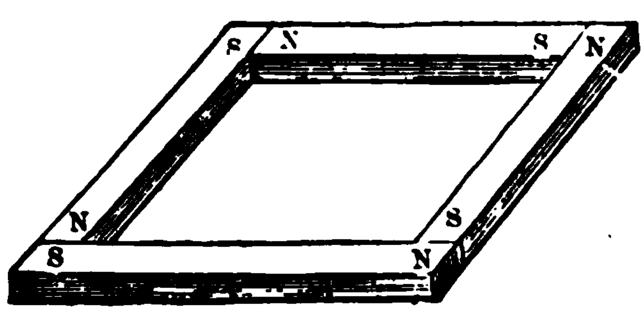


FIG. ii.

stable molecular arrangement similar to that of § 5, fig. iii. The whole system will give but little signs of external magnetism until the soft iron pieces are removed. The accompanying figures show how a horseshoe magnet, or a pair of bar magnets, may be provided with soft iron *keepers*.

The letters indicate the polarities temporarily induced in these

keepers. The molecular arrangements, similar to that of § 5, fig. iii., can readily be imagined.

§ 8. **Methods of Magnetisation.**—In making a magnet we must have several points in view.

(i.) The material must be one having great molecular rigidity or coercive force; so that the rearrangement of molecules, to which is due the external or evident magnetism, may be retained. Hence we use *hard steel.*

(ii.) During the process of magnetisation the molecules should be given as free play as possible. Hence, hammering, twisting, and other mechanical aids to molecular freedom may be employed if convenient. It is found that repetitions of the magnetising process, even when each action first undoes the work of the preceding action before redoing it, give better final results than a single application of the process.

(iii.) We must apply as powerful a magnetising influence as we can. The more powerful the 'field' (*see* Chapter II.) in which we place our bar, the more will the molecules be rotated from their 'short-circuited' position into the position giving most powerful external magnetism.

I. *Single Touch.*—Here a magnet is simply drawn from end to end of the bar in the position shown in the figure. This 'stroking' process is repeated many times.

Here we can imagine the molecules rotating, as the inducing magnet pole moves down the bar. They will always, *e.g.*, turn their south poles towards a north inducing magnet pole. Hence we should predict that finally the end last touched by a north inducing pole will be left of south polarity, while that first touched will be left of north polarity. Each time that the 'stroke' is repeated it is clear that the molecular arrangement due to the preceding 'stroke' will be undone, or at least seriously disturbed. It seems, therefore, at first sight somewhat strange that there is so much gained by repetition. Probably the explanation is that this repetition gives greater freedom of movement to the molecules; and hence the final stroke has greater effect than had the first.

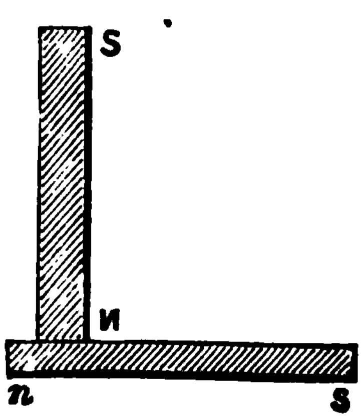


FIG. i.

II. *Double Touch.*—Here two magnets are used; they are

placed, N pole to S pole, in an inclined position, as shown. We begin the stroke in the middle of the bar, and we move the magnets up to one end, back to the other end, and then up to the middle again; this process being repeated many times.

The ends of the bar may also be placed on the opposite poles of two magnets. This increases the effect.

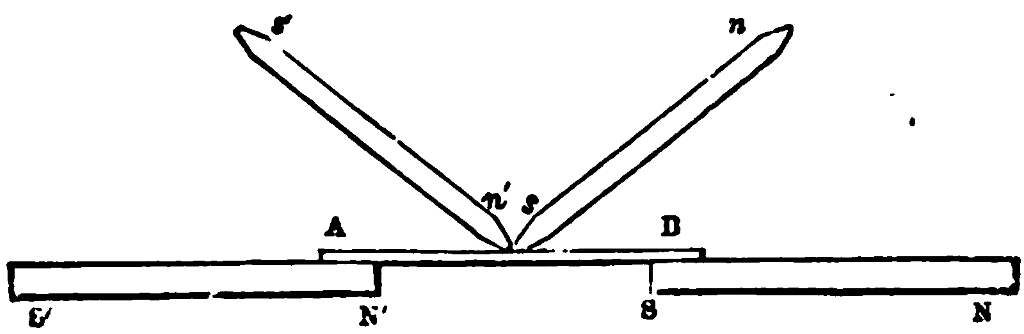


FIG. ii.

III. *Separate Touch.*—Or again the two inducing magnets may be drawn away from each other to the two ends respectively, and brought back to the middle by being lifted in a wide curve through the air; and this process repeated.

IV. *By an Electric Current*—But by far the most powerful method is magnetisation by means of an electric current. Of this we shall say much hereafter in Chapter XX.

CHAPTER II.

MECHANICAL AND MAGNETIC UNITS.

§ 1. **Introductory.**—For the study of magnetism and electricity it is necessary that the student should have a knowledge of the principles of elementary mechanics, and a clear understanding of what is meant by *a system of units.* He will then be in a position to understand in their exact sense the terms employed in the discussion of magnetic and electrical measurements. For reasons of space it is impossible to give here any satisfactory account of the mechanical principles involved in our present subject. We shall merely indicate briefly the nature and extent of the knowledge required.

§ 2. **Fundamental and Derived Units.**—The three fundamental units to which all other mechanical quantities can be ultimately referred, are the units of *length*, *time*, and *mass.* We shall in what follows employ the *centimètre* as the unit of length, the *second* as the unit of time, and the *gramme* as the unit of mass. The system of units built up upon this foundation is called the 'centimètre-gramme-second system,' or more usually the 'C.G.S. system.'

Examples of *derived units* will occur in §§ 3 and 4 and later on. Very simple cases of derived units are the *square centimètre* as the unit of *area*, and the *cubic centimètre* as the unit of *volume.*

The exact way in which any derived unit involves the fundamental units constitutes what is called the *dimensions* of the quantity measured by that derived unit. For the purpose of exhibiting the dimensions of any derived unit it is convenient to represent the three fundamentals by the symbols [L], [M], and [T] respectively. Thus the C.G.S. unit of volume has *dimensions* represented by $[L]^3$.

§ 3. **Velocity and Acceleration.**—In the same system the unit of velocity will be the velocity of *one centimètre per second.* If we represent this unit by the symbol [V], we may express the dimensions of *velocity* by the relation $[V] = \frac{[L]}{[T]}$.

The *rate at which velocity changes* is called *acceleration* ; and in our system *unit acceleration* will be the adding of unit velocity during unit time. Hence the C.G.S. unit of acceleration is *one centimètre per second, per second.*

We may express the 'dimensions' of acceleration by the relation—

$$[A] = \frac{[V]}{[T]} = \frac{[L]}{[T]} \div [T] = \frac{[L]}{[T]^2}.$$

§ 4. **Force.**—The C.G.S. unit of force is called the *dyne.* It is that which acting on unit mass for unit time gives to it unit velocity.

Thus we may express symbolically the 'dimensions' of *force* by the relation—

$$[F] = [M] \times [A] = \frac{[M] \times [V]}{[T]} = \frac{[M] \times [L]}{[T]^2}.$$

The student will have learned, in his study of mechanics, to distinguish carefully between *mass* and *weight.* In virtue of gravitation, a mass of one gramme is urged downwards with a certain force ; this force being called the weight of one gramme. Near the earth's surface this force is such that acting for one second it will give to the gramme mass a velocity of about 981 centimètres per second. Hence, near the earth's surface, the weight of our unit mass is a force measured by about 981 dynes. A *gramme* then is a *mass* ; and we take it for our unit of mass. But the *weight* of a gramme is a *force*, and is very far indeed from being unit force in our system.

Note.—In the English system of units we take the *foot*, *second*, and *pouna* as our three fundamental units.

The unit of force in this system is called the *poundal*, and is that which acting for one second on one pound mass will give it a velocity of one foot per second.

§ 5. **Parallelogram of Forces, &c.**—We shall make frequent use of the principle commonly called the 'Parallelogram of Forces.'

It will be assumed that the student is thoroughly conversant with the formulæ for the composition of two component forces into one resultant, and for the converse resolution of a single force into two components, in the case in which the directions of the two components are at right angles to one another.

The case of 'oblique resolution' will not occur.

§ 6. **Moments and Couples.**—Let us suppose that we have a body (such, *e.g.*, as a magnetic needle) capable of rotation about an axis. If one or more forces act on the body we shall in general have, as a result, rotation of the body about the axis.

In our present subject the only cases usually occurring are very simple; the forces acting in one plane which is perpendicular to the axis. We shall have to consider two cases.

(i.) *A single force acting.*—Let a single force of F *dynes* act upon the body, and let the perpendicular between the axis and the direction of the force be *p centimètres.* Then the 'turning power' or *moment* of the force about the axis is, as we know, measured by $F \times p$. This is the *moment* urging the body to rotate about the axis; and there is also a force of F *dynes* tending to move the axis bodily in the direction of the force.

If, for example, the body be a horizontal bar, suspended from its middle point by a thread, and we push one end of the bar, we shall find that the bar tends to rotate and also to move bodily in the direction of our push.

(ii.) *A couple acting.*—Now consider the case where two equal, parallel, and opposite forces act upon the body, each force being F *dynes*, and the distance between them being *p centimètres.* The *moment* of this couple is, as we know, measured by $F \times p$. In this case we have simply a tendency to rotation, and no pressure on the axis.

When the product $F \times p$ is numerically equal to unity—(F being measured in *dynes* and *p* in *centimètres*)—then we have what we call *unit moment* in the C.G.S. system.

§ 7. **Unit Magnetic Pole.**—Returning now to magnets and magnetic poles, we observe that similar poles of various pairs of magnets repel one another with forces of various magnitudes. Thus we naturally say that poles may be of different *strengths.*

We should then endeavour to express the 'strengths' of mag-

netic poles in terms of some convenient unit-pole; and further we should choose our unit strength of pole such as to give the simplest relation to *force of repulsion* expressed in *dynes*, and *distance apart* expressed in *centimètres.*

We have seen that a single pole never occurs alone. But in an indefinitely long straight thin magnet, perfectly magnetised, we have the poles exactly at the ends and indefinitely far apart; or, in other words, we can easily conceive of, and approximately obtain, a pole isolated from another pole. We can then in our definitions speak of 'a pole.' We choose to call the north-seeking pole +, and the south-seeking pole −.

It is natural to define *unit-pole as that which at a distance of* 1 *cm. from a similar pole repels it with a force of* 1 *dyne.*

So a pole of two units would be defined as that which at a distance of 1 cm. from unit-pole repelled it with 2 *dynes*; or at a distance of 1 cm. from a similar pole repelled it with $2 \times 2 = 4$ *dynes.*

We shall designate the numerical value of the strength of a pole by μ.

§ 8. **Magnetic Fields, and Unit Field.**—Any region where a pole (*e.g.* a + unit-pole, which we may conveniently employ as a test-pole) would be urged by magnetic force is called a *magnetic field.*

In so far as such force is due to any particular magnetic body, we speak of the *field due to that body.*

The direction in which our + unit-pole at any point is urged is called the *line of force* at that point.

If a + unit-pole is urged with 1 *dyne*, the field at that point is defined to have *unit strength.*

If with 5 *dynes*, then the field has a strength 5.

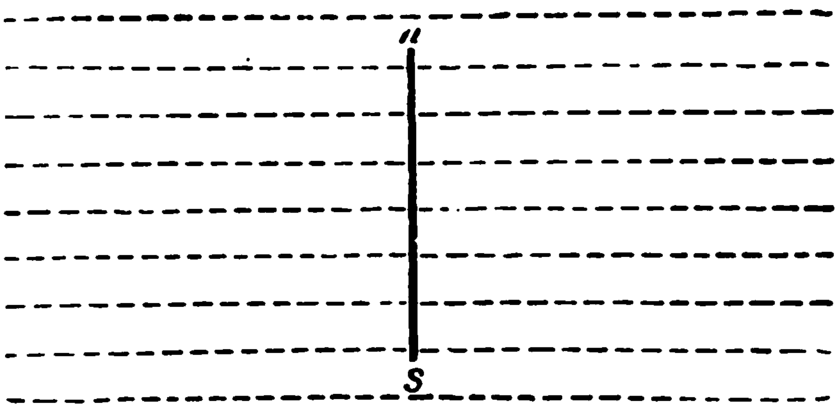
In general we shall designate the numerical value of the strength of a field by the letters H or I.

Thus a pole of strength μ in a field of strength H is urged by a force of $\mu \times \mathrm{H}$ *dynes* If a field is *uniform* it means that the strength is the same at all places in it, the lines of force being moreover, as a necessary consequence, parallel to one another; as will be seen in Chapter X. § 13.

§ 9. **Magnetic Moment of a Needle.**—Let us consider a magnetic needle in a uniform field. Since the poles, *n* and *s*, are equal and opposite, and since the lines of force are in a uniform

field parallel, it follows that the forces acting on the poles will be equal and opposite; or that a magnetic needle in a uniform field is acted on by a pure couple.

We have seen that each force will be μ H *dynes*. Hence if the needle lie at right angles to the lines of force, and if its length be l cm., the couple acting on the needle will be measured by Hμl units of couple.



The part μl depends on the needle alone; and into whatever field we introduce the needle, so long as it remains molecularly unaltered, this μl remains constant.

We give to this quantity μl the name of the *magnetic moment of the needle.* We call it m. When $\mu l = 1$, both μ and l being measured in C.G.S. units as given in §§ 2 and 7, then the needle is said to be of *unit moment.*

A needle of unit moment in unit field, placed perpendicularly to the lines of force, will be acted on with unit couple.

It will be useful for the student to prove for himself that when a needle of pole-strength μ, and length l, is placed in a field H so as to make an angle θ with the lines of force, then the couple acting on the needle is

$$H \mu l \sin \theta, \text{ or } H m \sin \theta.$$

If we make up a cube of needles by first laying similar needles side by side so as to form a layer, and then piling layer on layer, we can see that the following statement will be true; *the magnetic moment of a uniformly magnetised bar is proportional to the volume of the bar.*

The student should puzzle this out by the 'building up' method just suggested.

§ 10. **Magnetic Moment of Practical, not Ideal, Magnets.**— We have considered our magnet as consisting of two poles occupying points, separated by a distance called 'the length of the needle.' But the actual magnet may be considered to consist of an indefinite number of such pairs of poles, the poles of each pair being of equal strength and of opposite sign; thus we may

have at the two ends the poles μ_1 and $-\mu_1$ corresponding to one another and separated by a length l_1; then μ_2 and $-\mu_2$ with distance l_2 between, and so on. The poles μ_1 and $-\mu_1$ &c. near the ends will be the strongest; in fact, the strength of the pairs of poles ought, in a good magnet, to fall off very rapidly as we leave the ends and approach the centre.

So we have the moments $\mu_1 l_1$, $\mu_2 l_2$, &c. And the magnetic moment of the whole needle will be the sum of these, or will be

$$m = m_1 + m_2 + m_3 + \text{\&c.}$$

where the first terms, $m_1 + m_2 +$ &c. are the largest.

The complexity of this result is due to the difference between our ideal magnetisation and the imperfect manner in which we must needs perform it. The magnetisation is not effected uniformly all over and all through the bar; and hence there is *not* the complete internal neutralisation, with evident magnetism only at the ends, which would result from uniform magnetisation.

To sum up all these terms, $m_1 + m_2 +$ &c., about each of which too little would be accurately known, were impracticable.

We have, however, other methods of finding m. We may (*e.g.*) suspend the needle in a uniform field of known strength H, and measure by mechanical means the moment (in terms of unit couple or unit moment) required to keep the needle perpendicular to the lines of force. Let this moment be found to be S.

Then we have

$$\text{S} = \text{H}\, m \ (\textit{see}\ \S\ 9).$$

$$\text{whence} \quad m = \frac{\text{S}}{\text{H}}.$$

In fact, we measure the magnetic moment m *as a whole* by observing the action of a known field upon it.

§ 11. **Magnetic Curves.**—Consider a bar magnet N S, and a point P in its field, and suppose a + unit pole to be placed there.

This will be urged *from* N in the line N P; urged *to* S (by a weaker force if it be nearer to N than to S) in the line S P.

If we draw the parallelogram of forces we shall get a resultant force P R lying (in the case given in figure) nearer N P than S P in direction.

As the pole P moves, the component and resultant forces change continuously. *And if P is always allowed to follow the*

direction of the force urging it, then it will trace out a curve of force, called, in the subject of magnetism, a *magnetic curve.*

The curve is such that at any point in it the tangent gives the direction of the force at this point in the field. Now if we put into the field a needle so small that we can consider its two ends to be practically in the same part of the field, then the needle will be acted upon by a couple, and it will turn until it lies in a straight line with the lines of force. In no other position but this could there be equilibrium for the needle.

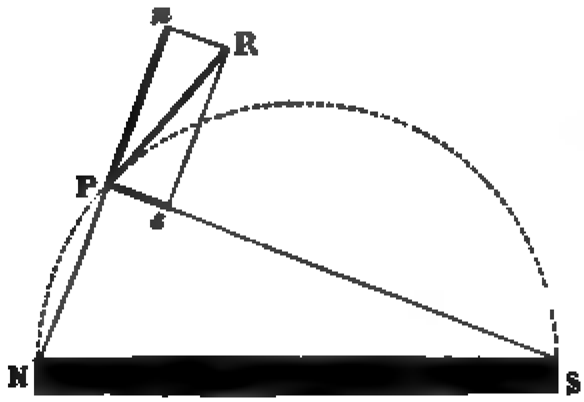


FIG. i.

Hence, a 'small needle' will always point out the direction of the lines of force at the place it occupies.

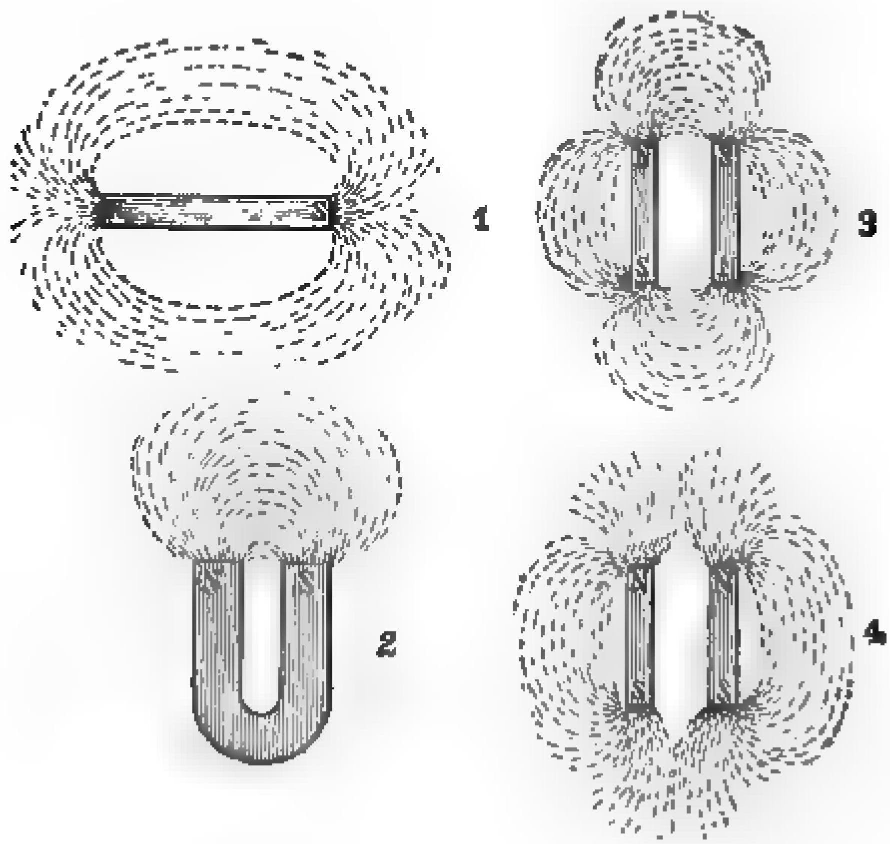


FIG. ii.

Now iron filings do, by induction, become small magnetic needles when they are placed in a magnetic field. Hence, if we

place a magnet under a sheet of glass and scatter over the glass iron filings, tapping the glass to allow of arrangement, we shall find traced out for us many of these lines of force that lie in the plane of the glass.

The accompanying figures show us the general aspect of the curves thus obtained.

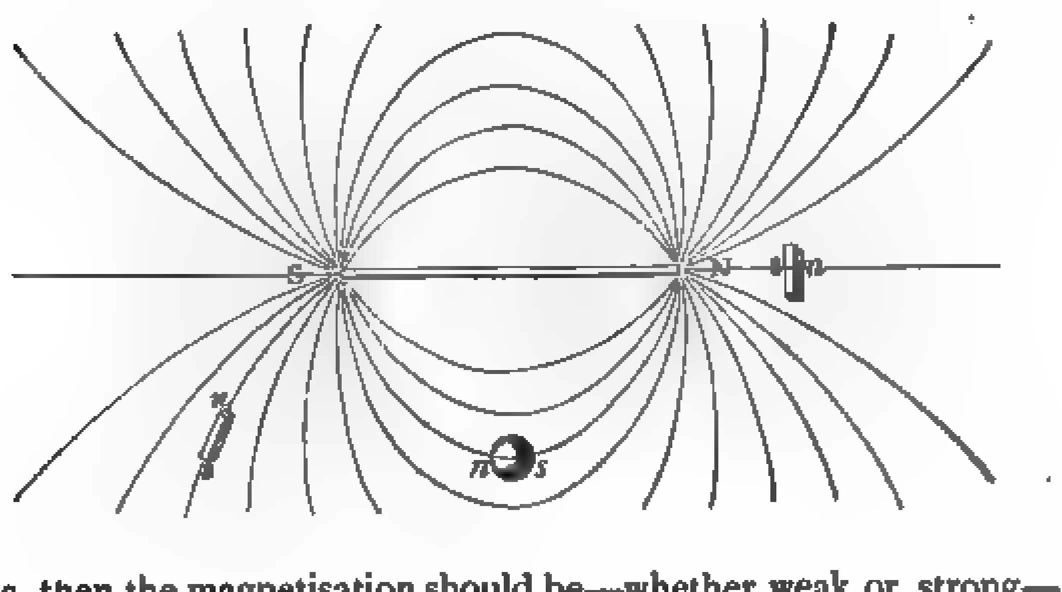
In fig. ii. we have respectively the cases of a bar-magnet; a horseshoe-magnet; two bar-magnets with unlike poles opposed; and the same with similar poles opposed.

§ 12. **Magnetic Induction takes place along the Lines of Force.**—It is along these lines of force that we must consider our molecules, regarded as small magnetic needles, urged to direct themselves.

Hence, if these molecules were perfectly free to move, any mass of iron would by the rearrangement of its molecules become magnetised along the lines of force of the inducing field.

But as the molecules are subject internally to mechanical constraint as well as to each other's action, the resulting magnetisation is determined by the combined influences, internal and external.

If the bar or needle lies so that its greatest length is along the lines of force, and if the form of the bar is symmetrical about this



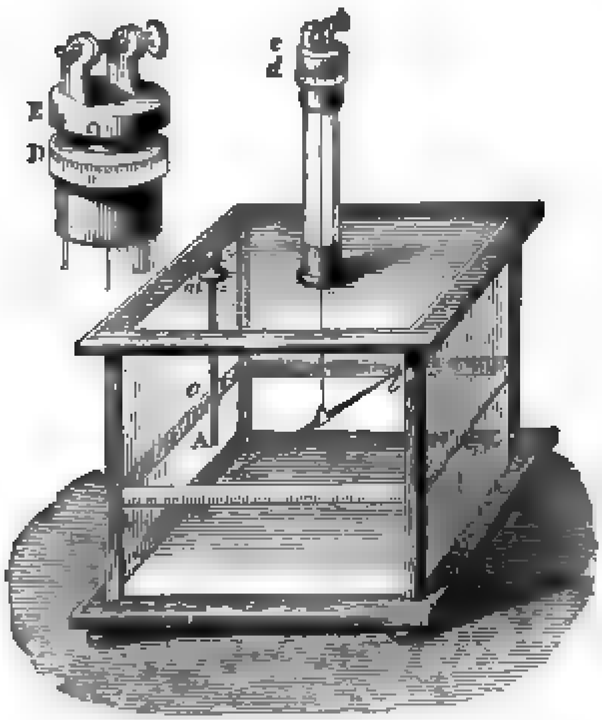
axis, then the magnetisation should be—whether weak or strong—along the lines of force.

But if the bar be unsymmetrically situated with respect to the lines of force, the resulting magnetisation may be quite unsymmetrical and be oblique to the lines of force.

CHAPTER III.

MAGNETIC MEASUREMENTS. THE EARTH'S MAGNETISM.

§ 1. **Coulomb's Torsion Balance.**—In considering the subject of magnetic measurements we shall first describe the *torsion balance.* This instrument deserves notice as the earliest by which exact magnetic and electrostatic measurements were obtained, and some study of it will be instructive. In practice it has, however, been now superseded by other instruments.



The figure represents one form of the instrument. A rectangular or cylindrical glass case is provided, either with a graduated scale round the sides, as here shown, or, what is better, with a plane mirror at the bottom, on which is marked a circle graduated in degrees.

In what follows we shall assume that the latter method of graduation has been adopted, and that the centre of this graduated circle is called O. Above the mirror is suspended (when the instrument is used in magnetism) a magnetic needle $a\,b$, in such a way that its axis of suspension is immediately above the centre O of the graduated circle.

By looking down from above we can tell over what degree the needle is lying. By moving the eye until the needle and its image coincide, we avoid errors due to the fact that the apparent position of the needle over the scale varies according to our point of view.

The needle is suspended by a fine wire or thread of glass, so hung that its prolongation would pass through O, the centre of the graduated circle.

This thread is fixed at its upper end to a brass piece *e d* (called a *torsion head*) that caps the glass tube shown. This piece is so constructed that the thread can be twisted without being displaced laterally, and the angle of twist can be measured.

A sight of the instrument will make clear how this is contrived.

Now to twist a wire or thread of glass without displacing it laterally requires a simple couple.

It can be proved experimentally that as long as the thread or wire is not permanently altered by a twist, the couple required to twist it is directly proportional to the angle of twist.

Or if we twist the wire through 10°, 20°, 30°, 170°, &c., we are exerting couples proportional to 10, 20, 30, 170, &c. We cannot *measure* the couples in absolute C.G.S. units unless we know more about the length, radius, and material of the thread. But we can, without this knowledge, *compare* couples. If the top of the wire be twisted through $\theta°$ one way, and the bottom through $\alpha°$ the other way, then the total angle of twist will be $\theta° + \alpha°$.

§ 2. **Use of Torsion Balance 'at constant angle.'**—We can use the torsion balance to compare the strength of magnetic fields and of magnetic poles; or even to *measure* them in C.G.S. units if we are acquainted with the 'constants' of the instrument we are using.

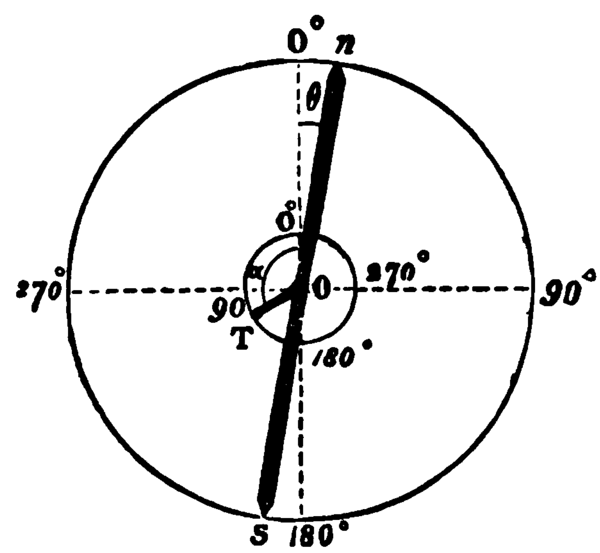


FIG. i.

In the accompanying diagram we are supposed to be looking down upon the whole instrument. The small circle represents the graduated circle of the torsion head; and, in the case given, the torsion wire has been twisted either through an angle $\alpha°$, or through $(n \times 360° + \alpha°)$, where n is some whole number.

The large circle is the graduated circle on the mirror; and, in the case given, the needle has been deflected through an angle θ from zero.

The opposite directions of twist of the needle and of the

torsion head respectively, that must exist where we are dealing with repulsions, are indicated by the opposite directions of graduation. In this figure the needle $n s$ answers to the needle $a b$ of § 1. Supposing now that we wish to compare the strength of two magnetic poles.

We start with needle and torsion index both at 0°. There is a hole in the glass top to the instrument in such a position that we can lower the one or the other pole of the magnets we are comparing into the place occupied by the pole of the needle $n s$ when this lies over 0°. The needle is swung to one side, and the first pole, which we will call μ_1, lowered. As we always take care to lower a pole similar to that pole of the needle whose place at 0° it takes, we have the needle deflected until the moment of the force of repulsion about the axis of suspension of the needle just balances the moment of the couple due to the twisting of the wire.

As a rule we shall find it advisable to turn the torsion head, and so to twist the wire, until the needle is forced back to some angle $\theta°$ of deflexion from zero, this angle being much less than the original angle of deflexion. One reason for this is that if we cause θ to be small, we may neglect the restoring couple due to the earth's field.

Let us suppose that the torsion head has been turned through $(n_1 \times 360 + a_1)°$. Then the total twist on the wire will be $(n_1 \times 360 + a_1 + \theta)°$; and the couple that it exerts will be measured by $k\ (n_1 \times 360 + a_1 + \theta)$, where k is some constant that depends upon the nature and dimensions of the wire.

Now this couple is balanced by the moment about O due to the action of the pole μ_1 upon the needle $n s$; and this moment will be measured by the product $\mu_1 h$. In this product μ_1 represents the pole strength of the first magnet; while h is some quantity depending upon the magnetic moment of the needle $n s$ and upon the angle θ of deflexion, and so will be constant while θ is constant and while $n s$ remains unaltered.

Since the two are in equilibrium, we have

$$\mu_1 h = k\ (n_1 \times 360 + a_1 + \theta)$$

$$\text{or} \quad \mu_1 = \frac{k}{h}(n_1 \times 360 + a_1 + \theta).$$

Repeating the process with the second magnet pole μ_2, and turning the torsion head through such a number of degrees that θ remains constant, we have

$$\mu_2 = \frac{k}{u}(n_2 \times 360 + \alpha_2 + \theta).$$

Finally, by division we obtain the result that

$$\frac{\mu_1}{\mu_2} = \frac{n_1 \times 360 + a_1 + \theta}{n_2 \times 360 + a_2 + \theta}.$$

Notes.—(i.) The reader should notice that we here compare primarily the two *moments* acting upon the needle *n s* in the two cases respectively.

But if the magnetic moment of *n s*, and also the angle θ, remain unaltered, it follows that we are really comparing the two field-strengths due to the poles μ_1 and μ_2 respectively.

And finally, since these field-strengths must be proportional to the pole-strengths μ_1 and μ_2 respectively when the conditions of distance and situation remain constant, it follows that we are comparing μ_1 and μ_2, as was desired.

(ii.) We assume that the magnetic moment of the needle *n s* is constant. As pointed out at the end of § 3, this is not true absolutely; and under certain conditions the error, due to the alteration in this magnetic moment, may be great.

This method may also be employed to observe the distribution of evident magnetism along a bar-magnet; since we may, without otherwise altering the process, lower the bar-magnet so that the different portions of it may be successively opposite to the pole *n* of the needle.

It will be found that the evident magnetism is distributed along a good bar-magnet somewhat as here represented by the

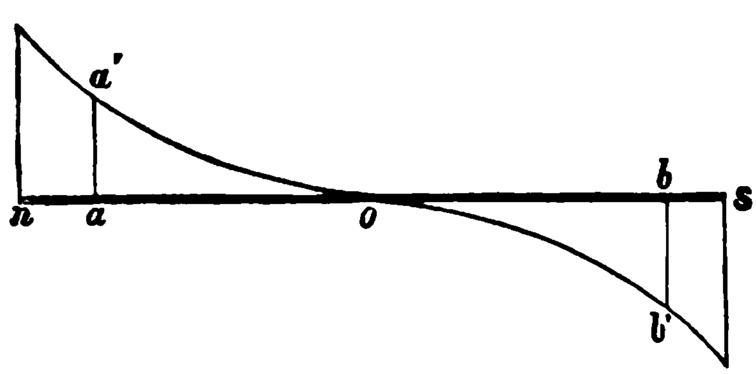


FIG. ii.

diagram. In this the height of any ordinate such as *a a'* represents the strength of polarity at the point *a* in the magnet.

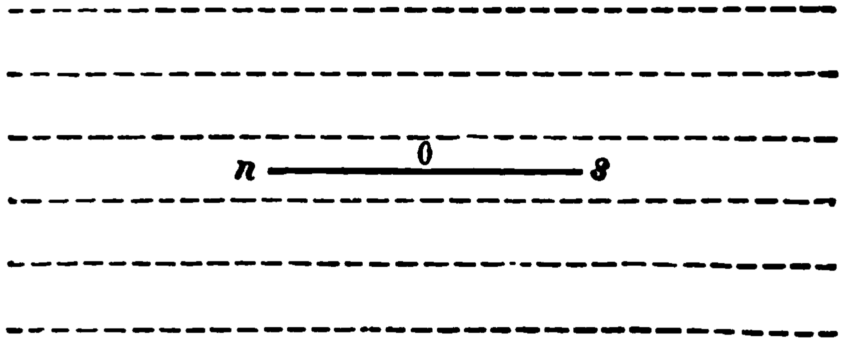
If we find the centres of gravity of the two *areas* included between the axis of the bar-magnet, the ordinates at its two ends, and the curve; and if through these centres of gravity we draw lines perpendicular to the axis of the magnet, meeting it at P and P′, these points P and P′ may be called the *principal poles* of the bar. If we consider any external point that is so remote from the bar that straight lines drawn from it to the extremities of the latter are practically parallel, then the action at this point will be the same as if the polarities of the bar were concentrated at the two points P and P′ respectively. But for points that are relatively near to the magnet, the action is not so simple.

In using the torsion balance it must not be forgotten that the needle *n s* is under the influence of the earth's field. This obliges us to place the instrument in such a position that the needle, when under the earth's action only, stands at the zero mark, or 0°.

When the needle is displaced through an angle θ the earth's field gives a restoring couple that is proportional to $\sin\theta$; and this is added to the torsion couple.

If θ be made small, we may often neglect this couple in comparison with the large couple due to torsion; since the smaller θ is, the smaller is the earth's couple, and the larger is the torsion couple. Or we may find experimentally what angle of torsion is required to displace the needle through θ° under the earth's action only. If the angle of torsion required is β°, then we must add β to the angle through which the torsion head was turned in the experiment.

§ 3. **Method of Oscillations.**—The figure represents a uniform magnetic field of strength H, and a needle *n s* of magnetic moment *m* placed in this field.



If disturbed from this position of rest, the needle will oscillate more or less quickly; the number of oscillations per second depending upon the field-strength H, the magnetic moment *m*, and mass and shape of the needle.

Now it can be shown that in all such cases of oscillation—(whether of magnetic needles or of pendula or of elastic rods)—

the time of a single oscillation is independent of the angle of oscillation, provided that this is very small. If this angle do not exceed a few degrees we may consider that this law is true. Hence we can speak of 'the rate of oscillation,' without specifying the extent of oscillation in degrees. In the case we are considering it can be shown by *mechanics* that for the same needle—

The product H m *is proportional to the square of the number of oscillations per second.*—This gives us a means of comparing, or of measuring, two field-strengths; for we may oscillate in the two respectively a needle of constant magnetic moment m, counting the number of oscillations per *second.*—We may also compare two pole-strengths; since, at constant distance, the field strengths will be proportional to these.

In this method we must reckon in the earth's action; since we cannot here make it relatively so small as to be negligible.

We so arrange matters that the lines of force of the field that we are considering act in the same direction as those due to the earth; *i.e.* so that the two fields, acting upon the needle, are simply added.

Let the needle *under the earth's action only*, or in the field h, oscillate O_0 times per *second*; *under the earth's action + the first field*, or in the field $h + H_1$, let the number be O_1; and *under the earth's action + the second field*, or in the field $h + H_2$, let the number be O_2.

Then by the mechanical law quoted above we have

$$\begin{cases} m\,h = k\,.\,O_0^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . & \text{(i.)} \\ m\,(h + H_1) = k\,.\,O_1^2 \quad . \quad . \quad . & \text{(ii.)} \\ m\,(h + H_2) = k\,.\,O_2^2 \quad . \quad . \quad . & \text{(iii.)} \end{cases}$$

where k is some constant involving the mass and dimensions of the needle.

$$\begin{cases} \text{Subtracting (i.) and (ii.) we have } m\,H_1 = k\,(O_1^2 - O_0^2) \\ \text{and subtracting (i.) and (iii.) we have } m\,H_2 = k\,(O_2^2 - O_0^2). \end{cases}$$

From this we obtain by division that

$$\frac{H_1}{H_2} = \frac{O_1^2 - O_0^2}{O_2^2 - O_0^2}.$$

This gives us also the ratio of the two pole-strengths, if the distance be constant.

Experiment.—Such experiments are best made with a short massive needle; the oscillations being counted for a sufficient length of time to give accurately the number per *second.*

Notes.—(i.) The needle must be so short that there is no practical difference in the field-strengths at its two ends.

(ii.) The reader must notice that we assume that m is constant in fields of different strengths. This is unfortunately not true; as m will, as a rule, change when the field-strength changes. But the error is not great provided that the fields are so weak that the magnetism of the needle is found not to be permanently affected.

Comparison of the magnetic moments of two needles.—This method enables us to compare the magnetic moments of two needles. For if we oscillate them in the same field H, *e.g.* that due to the earth, we have

$$\begin{cases} m\,\mathrm{H} = k\,.\,\mathrm{O}_1^{\,2} \\ m'\,\mathrm{H} = k'.\,\mathrm{O}_2^{\,2} \end{cases}$$

for the two respectively. From this it follows that

$$\frac{m}{m'} = \frac{k\,.\,\mathrm{O}_1^{\,2}}{k'.\,\mathrm{O}_2^{\,2}}.$$

Here, the constants k and k' depend upon the masses and dimensions of the two needles respectively; and may be readily calculated, provided that the needles are of simple forms. If they are identical in mass and in dimensions, we have simply that

$$\frac{m}{m'} = \frac{\mathrm{O}_1^{\,2}}{\mathrm{O}_2^{\,2}}.$$

§ 4. **Laws of Magnetism.**—There are two fundamental laws in magnetism.

I. *Like poles repel, unlike poles attract, one another.*

This simple observed fact needs no comment.

II. *The force between two poles varies inversely as the square of the distance between them.*

It may also be stated that *the force between two poles μ and μ' is proportional to the product $\mu \times \mu'$.* But this is hardly a separate law; since the magnitude of μ and μ' would be found by measuring forces of repulsion; and hence we should be reasoning in a circle were we to give this as a third independent law.

The second law is very important, and requires experimental proof.

§ 5. **Proof of Law II. by Torsion Balance.**—Here we keep the same pole acting on the needle-pole n, but vary the distance. (*See* § 2, fig. i.) Now if we only keep the angle θ small enough, the following statements will be approximately true.

(i.) When the needle is deflected through $\theta°$, $\frac{1}{2}\theta°$, $\frac{1}{3}\theta°$, &c., the *arm* of the repulsive moment acting upon $n\,s$ remains constant, the force only changing.

(ii.) If the pole n be at a certain distance from the repelling pole when the deflexion is $\theta°$, then at $\frac{1}{2}\theta°$ it is at half this distance, at $\frac{1}{3}\theta°$ it is at one-third of this distance, and so on.

The method is simple enough.

Let the needle be deflected through $\theta°$, and let the torsion angle be $(n_1 \times 360 + a_1)°$. Then the total angle of torsion is $(n_1 \times 360 + a_1 + \theta)°$.

Now let us force the needle up to $\frac{1}{2}\theta°$ by a larger angle of torsion $(n_2 \times 360 + a_2)°$. Then the total angle of torsion is $(n_2 \times 360 + a_2 + \frac{1}{2}\theta)°$. (*See* §§ 1 and 2.)

Hence

$$\frac{\text{Force at distance } \theta}{\text{Force at distance } \frac{1}{2}\theta} = \frac{n_1 \times 360 + a_1 + \theta}{n_2 \times 360 + a_2 + \frac{1}{2}\theta}$$

If Law II. be true we shall find that this fraction comes out to be $\frac{1}{4}$.

So again for an angle of deflexion $\frac{1}{3}\theta°$, we should find that the ratio comes out to be $\frac{1}{9}$; and so on.

We here suppose that the earth's action has been allowed for, or is relatively insignificant.

§ 6. **Proof of Law II. by Method of Oscillation.**—Using a 'short' needle we can count the oscillations at distances 10 cm., 20 cm., 30 cm., &c., from a given pole. The magnet whose pole we use should here—as in all cases where we wish to consider the action of one pole only—be so long that for all these distances the other pole has a negligible action on the vibrating needle.

Eliminating the earth's action as before, we get the ratio of the field-strengths at 10 cm., 20 cm., 30 cm., &c. If the law be true, then we should have

$$H_{10} : H_{20} : H_{30} : \text{\&c.} = 1 : \tfrac{1}{4} : \tfrac{1}{9} : \text{\&c.}$$

§ 7. **Measurements as affected by Induction.**—The same needle will give quite different results under the same external conditions if its own magnetism alters.

Hence, in all the above, where a needle is used for two or more measurements, and where it is assumed that there is no change in its magnetic moment, we must take care that the fields are not strong enough to alter its magnetism permanently.

This can be tested by vibrating it under the earth's action only before and after the series of experiments, counting the oscillations per *second.*

Even then there will be error due to the temporary alteration of the magnetic moment.

§ 8. **Earth's Magnetism. General Ideas.**—We have spoken of the 'earth's action,' implying that the earth acts as a magnet.

It does so act; but from the nature of the causes of its magnetic action we cannot give a simple account of the position of its poles or distribution of its magnetism.

The lines of force viewed as a whole (*see* Chapter II. § 11) would be seen to be wavy; and it would probably be seen that they converged to more than one north and south pole.

But it will give some idea if we say that the earth's action is nearly that which there would be if the whole earth were neutral, and if there were buried at the centre a powerful magnet about half as long as the earth's axis, whose position lay about 20° from the earth's axis, and varied from year to year and century to century.

This would give us lines of force lying horizontal over the equatorial regions, dipping more and more as we go north or south, and plunging perpendicularly into the earth at the points cut by the prolongation of the axis of our buried magnet.

Any magnetic needle used will be so very small with respect to the earth, that the lines of force will be with respect to it practically parallel, or the earth's field practically uniform. In fact, we could not detect any difference between the earth's field in different parts of the same room.

The earth's action therefore on a needle will be a pure couple, and will simply direct it.

If it were not so we ought to find besides the couple a horizontal force, or a vertical force, or both.

But experiment shows that no such force exists.

Experiments.—(i.) A needle floating on a cork shows absence of any horizontal force, by not moving in any direction.

(ii.) A needle weighed before and after magnetisation shows absence of a vertical force by absence of change in weight.

Note.—In what follows we shall often speak of the *single force* acting on one pole of a magnet needle, instead of *the couple* that acts on the needle as a whole. The reader will see that this is for convenience, and does not in any way contradict the above.

§ 9. Compasses.

Definition of magnetic axis.—A B represents a magnetic needle, perfectly symmetrical; its ends A B and its point of suspension O lying in a straight line.

If A B be also symmetrically magnetised, then it will come to rest with the geometric axis A O B lying along the lines of force acting upon it.

But if the needle be unsymmetrically magnetised, then, in the position of rest, some other line as *n* O *s* will lie along the lines of force. This line through O, that lies along the lines of force when the needle is at rest, is called the *magnetic axis of the needle.*

It ought to coincide with the geometric axis A O B; and in a well-made lozenge-shaped needle this will be the case.

When an ordinary compass-needle comes to rest, the direction of its magnetic axis shows us the direction of the horizontal component of the earth's lines of force at the place where the compass is used. (For *horizontal component*, and *resolution*, see further on.)

This will be all that we can learn. Unless we have further information as to the *declination* (*see* § 11) at the place in question, we cannot tell the direction of the geographic north and south.

The *mariner's compass* differs from the ordinary compass in that the whole card turns on a pivot; there being several needles, parallel to each other, attached to the card underneath. There is a fixed mark on the side of the compass-box, and one reads off what point on the card lies against this mark; whereas in the other compass one reads off the point on the fixed card over which the needle has come to rest.

The compass-box is, moreover, provided with two sets of

pivots at right angles to each other, so that the card may remain horizontal in spite of the rolling of the ship.

§ 10. **Modification of Earth's Lines of Force by the Presence of Iron Masses.**—Any masses of iron or steel will modify the earth's field of force in their vicinity.

This disturbance of the field is of especial importance in ships, since there it may bear a large proportion to the whole field.

To render the magnetism of an iron ship as symmetrical as possible, it should lie along the lines of force while it is being subjected to the prolonged hammering during its construction.

§ 11. **The Earth's 'Magnetic Elements.'**—If we take any point on the earth's surface there will be through this point a line of force. This will lie more or less horizontally in equatorial regions and vertically in polar regions. In our latitude it will lie obliquely, dipping down towards the north. That vertical plane, through the point, that contains the line of force, is called *the plane of the magnetic meridian*; just as the vertical plane, that passes through the geographical north and south points, is called the plane of the geographical meridian.

The earth's field can be resolved into a vertical and a horizontal component in this plane, as we shall see further in § 13. It is the horizontal component that acts on the ordinary compass, so that the magnetic axis of the needle will come to rest in this line.

Hence, *the plane of the magnetic meridian* can also be defined as that vertical plane that contains the magnetic axis of a compass-needle at the place in question.

At any place we know all about the earth's field when we know the three particulars given below. These are called the *earth's three magnetic elements* at the place.

(i.) *The declination* is the angle between the planes of the magnetic and geographic meridian. It is called *west or east declination*, as the needle points to the west or east of the geographic north respectively.

(ii.) *The inclination* is the angle between the direction of the lines of force and the horizontal plane. It is also called the *dip*.

(iii.) *The intensity* is the field-strength measured in C.G.S. units, as explained in Chapter II. § 8.

Besides the 'elements' there are other terms that need explanation.

Variations are changes taking place (from hour to hour, day to day, year to year, and age to age) in any of the elements. Most of these changes, and probably all, are periodic.

Magnetic storms are 'irregular' disturbances, or those which as yet we cannot predict. The most powerful are those accompanying marked and sudden alterations in the sun, such as a sudden appearance or disappearance of sun-spots.

Magnetic equator is an irregular line round the earth's equatorial regions at all points along which there is no inclination, the lines of force being horizontal.

Iso-clinic lines answer to the parallels of latitude. They are irregular lines, only very roughly parallel to the magnetic equator, connecting points of equal inclinations.

Iso-gonic lines answer to meridians of longitude. They also are wavy and irregular. They converge towards points where the lines of force are vertical; points which would be called 'the earth's magnetic north and south poles,' only that there appear to be more than one of each, and each of these is perhaps not a point but an area.

§ 12. **Measurement of Declination.**—The finding of the angle between the planes of the magnetic and geographic meridian is effected by means of an instrument called a *declinometer*; this being provided with means by which we can find both meridians.

Near the base of the instrument is a compass-box with a very carefully graduated card, over the centre of which is suspended a compass-needle. This compass-needle is of lozenge shape, and has at each end a fine cross scratched. It is suspended in a sort of stirrup, so that it can be turned over if required. Thus, we can use it with either surface uppermost.

On a vertical axis, coinciding with the axis of suspension of the needle, and passing through the centre of the graduated card, turns a telescope. This telescope is provided with an object-glass so made that both distant and near objects can be viewed without altering the focus of the instrument. It is capable of being inclined to the horizontal plane as well as of turning about a vertical axis.

There is a fixed horizontal graduated circle and an index movable with the telescope, to tell us through what angle we turn the telescope about its vertical axis.

The general method, without detail, is as follows.

With the telescope observe some distant object whose geographical bearing is known, *e.g.* some star not far from the horizon. Read off the position of the index on the graduated circle; and, from knowledge of the bearing of the object, note down at what division on the graduated circle the index would stand if the telescope were directed to the geographical north. Next set the telescope parallel to the magnetic axis of the needle,

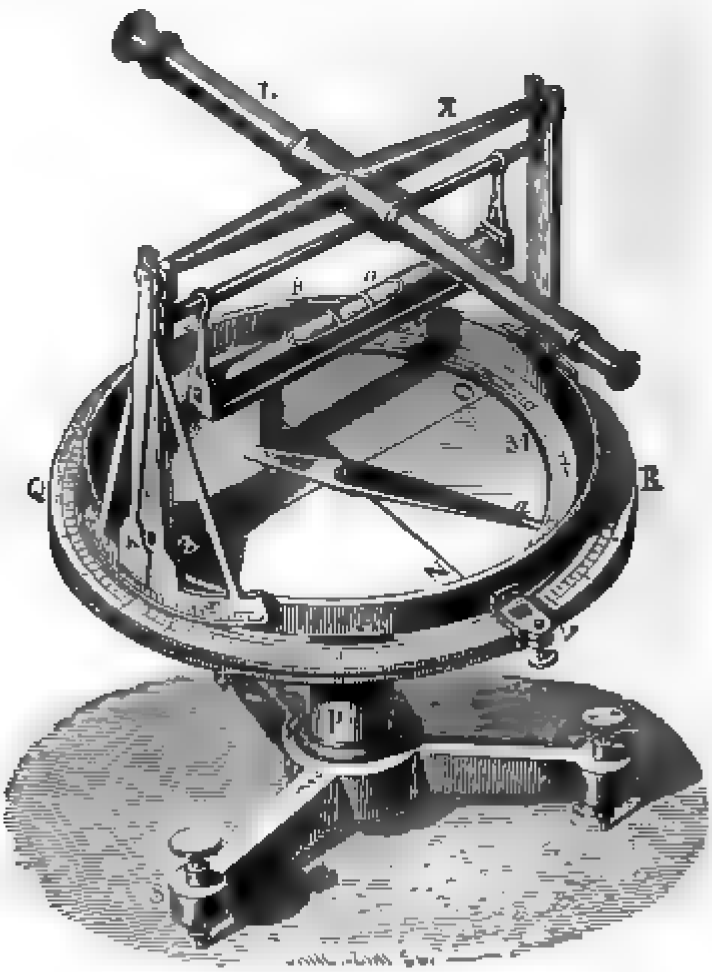


Fig 1.

which, if the instrument be perfectly made, will be done by turning the telescope round till the end of the needle is viewed. Read the position of the index. The difference between this reading and the last will be the angle between the magnetic axis of the needle and the geographical north and south line, or it will be the *declination*.

Corrections.—(i.) If the vertical axis about which the telescope turns does not pass through the point of suspension of the needle, then the

telescope, when the whole instrument is viewed from above, would be seen to lie off to one side of the needle. The consequence of this would be that when the telescope was directed so as to point at one end of the needle, it would be inclined to the needle and not parallel to it.

In fig. ii. (which represents this state of things as viewed directly from above, or is a horizontal projection of the needle and optical axis of the telescope) O and O′ are projections of the vertical axes of rotation of needle and telescope respectively. When we view one end *n* of the needle, the direction O′ *n* of the telescope is not parallel to the axis *n s* of the needle. But if we first view *n*, then view *s*, and bisect the angle between O′ *n* and O′ *s* produced back through O′, then the direction O′ *n*′ is that parallel to *n s*.

Thus we first regard the geographical north along O′N; then, by taking two readings and striking an average, we find the line O′ *n*′ parallel to *s n*; and so arrive at the declination N O′ *n*′. We will call this angle δ.

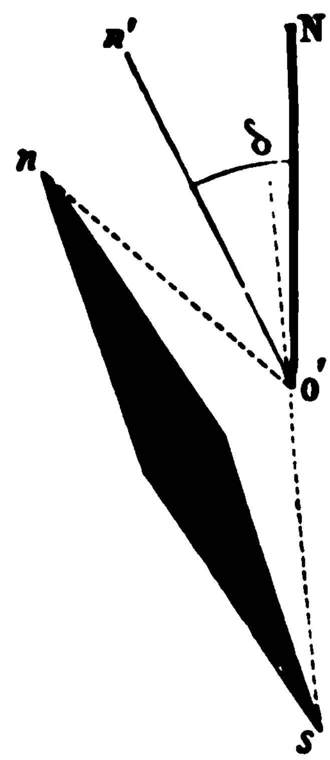


FIG. ii.

(ii.) In the above, *s n* is the axis of figure (or geometric axis) of the needle. It may not be the true magnetic axis.

If it be not we proceed as follows.

If it only be remembered that it is the magnetic axis of the needle that preserves a constant direction, it will be seen that if the needle be

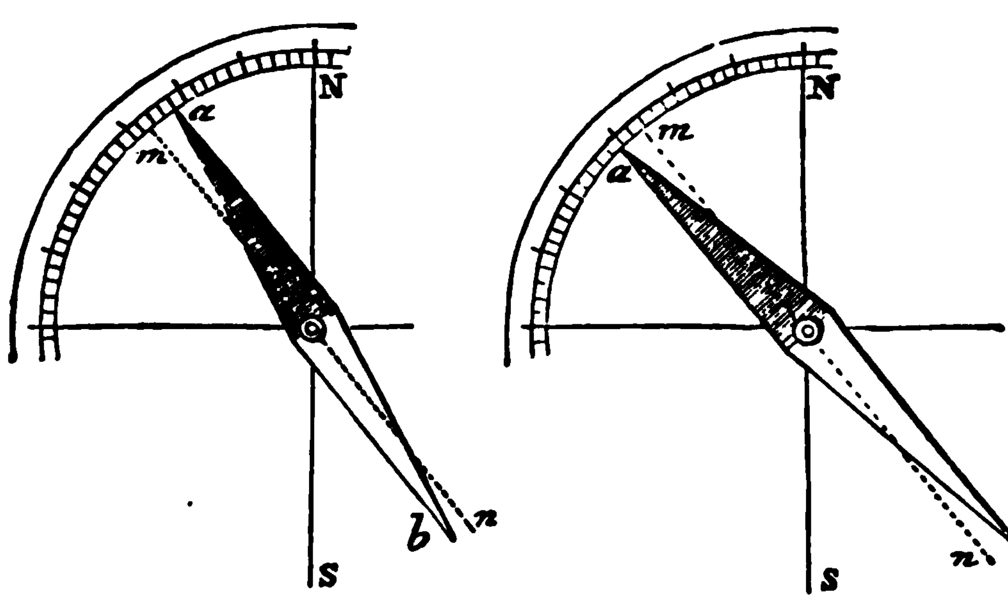


FIG. iii.

turned over in its stirrup, it will now lie as much to the one side of magnetic north and south as it before did to the other.

Hence, the average position of the needle will give us the true direction *n m* of its magnetic axis (*see* fig. iii.)

So that if the process given in correction (i.) be applied over again with the needle turned over in its stirrup, then the average value of δ, as obtained from these *four* readings, will be the true declination.

[There are also other corrections not given here.]

§ 13. **Resolution of Earth's total Field into two, or three, Components.**—Let us first suppose that we are viewing the plane of the magnetic meridian at a place, from a position off to the west of this place.

In the diagram, V O N represents the plane of the magnetic meridian; O R represents in direction and magnitude the earth's total field; which, as we know, lies in this plane. We can resolve this into two components; O H, or H, horizontal; O V, or V, vertical. In the case of an ordinary compass-needle, which can only move in a horizontal plane, the vertical component V is lost, and only H acts on the needle to direct it.

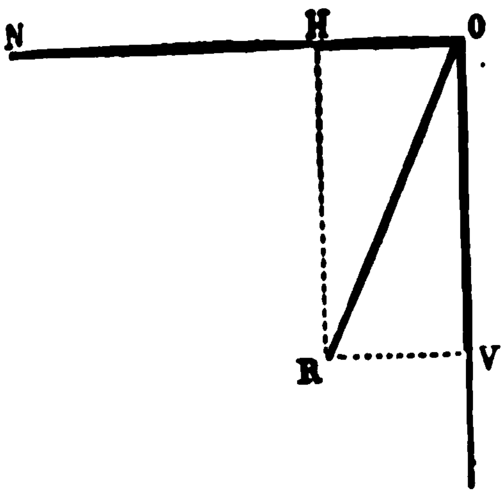


Fig. 1.

But let us consider a needle swinging in a vertical plane, and supported on a horizontal axis. The figure in § 14 shows such a needle. We can turn the instrument round so that the vertical plane in which the needle is free to swing is in any position; from one in which it coincides with the plane of the magnetic meridian, to that in which it lies magnetic west and east.

In the first position the whole force R acts to direct the needle; and the needle, if properly balanced, is directed along the lines of force.

In the second position mentioned, only the vertical component V can act on the needle to direct it; the horizontal component being directly resisted by the axis.

In intermediate positions, in which its plane of swing makes an angle θ with the plane of the magnetic meridian, we have the whole of V acting, and part of H.

If R is the whole force, and α the angle of dip, then we have

$$\begin{cases} H = R \cos \alpha. \\ V = R \sin \alpha. \end{cases}$$

Now in the next figure it is supposed that we are looking down from above.

P O P′ is the projection of the vertical plane in which the needle swings; O N is the magnetic north and south line; E O W

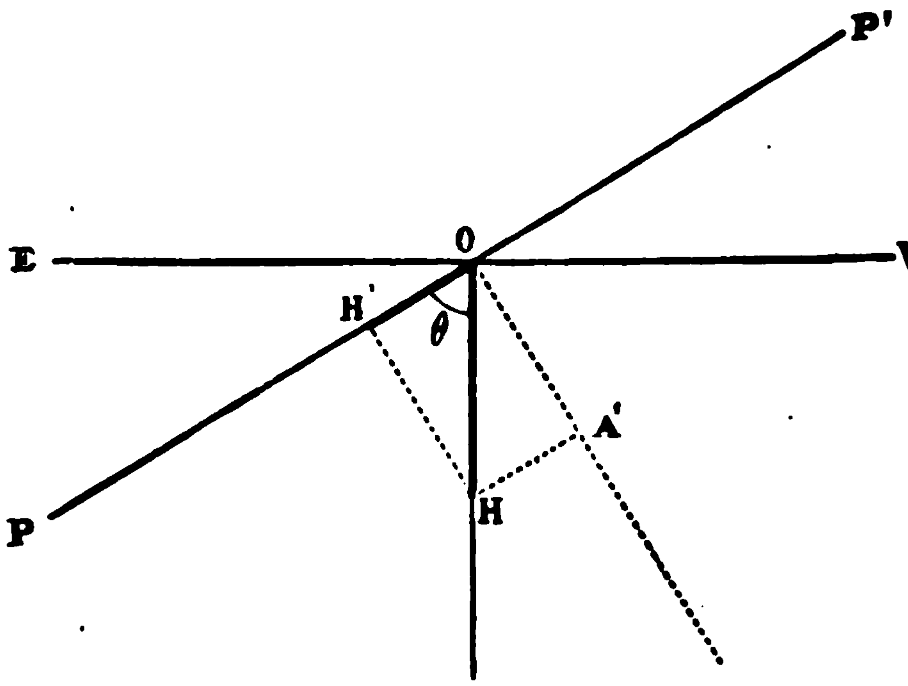


FIG. ii.

is the magnetic east and west line; O H is seen in its full length; O V has sunk into the point O; and O R is not given, since it would appear in a foreshortened condition. The angle P O N is θ.

Resolving H into two components, of which the one O H′ acts upon the needle, while the other O A′ is directly resisted by the pivots, we have acting upon the dip-needle in this position

$$\begin{cases} \text{O H}', \text{ or } \mathrm{H} \cdot \cos\theta \\ \quad \text{or } \mathrm{R} \cdot \cos\alpha \cdot \cos\theta \quad \ldots \quad \textit{horizontally.} \\ \text{O V, or } \mathrm{R} \sin\alpha \quad \ldots\ldots\ldots \quad \textit{vertically.} \end{cases}$$

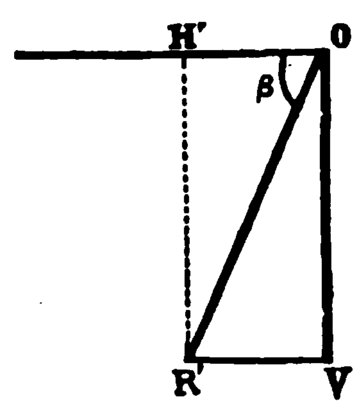


FIG. iii.

In our third figure we give a side view of the vertical plane in which we suppose our dip-needle to be swinging; this vertical plane being at an angle θ from the plane of the magnetic meridian.

$$\begin{cases} \text{Horizontal component} = \mathrm{H}' = \mathrm{R}\cos\alpha \cdot \cos\theta. \\ \text{Vertical component} = \mathrm{V} = \mathrm{R}\sin\alpha. \\ (\text{Total force})^2 = (\mathrm{R}')^2 = \mathrm{R}^2\cos^2\alpha \cdot \cos^2 . \theta + \mathrm{R}^2 . \sin^2\alpha \\ \qquad \tan\beta = \dfrac{\sin\alpha}{\cos\alpha \cdot \cos\theta}. \end{cases}$$

Here β is the angle made by the needle with the horizontal plane, and R′ is the resultant field acting upon the needle in the given position of its plane of swing.

§ 14. **To find the Inclination, or Dip.**—We will first give a *general* description of the method of finding the dip; and later will give more detail.

We will suppose that a steel needle has been accurately balanced on a horizontal axis, so that it swings in a vertical plane. While unmagnetised it will, if truly balanced, take up any position in this plane indifferently.

Now suppose that this needle is accurately magnetised so that its magnetic axis coincides with its axis of figure. The needle will now set in the direction of the resultant field R′ of the last section; this direction depending upon the angle θ which its plane of swing makes with the plane of the magnetic meridian, as well as upon the angle of dip a at the locality in question. If θ be *zero*, then the needle takes up the direction of dip, or it makes with the horizontal plane the angle a; for in this case R′ is the same as R.

If θ be 90°, or if the plane of swing stand magnetic east and west, the needle stands in a vertical position; since in this case we have only the vertical component V acting upon it. Hence we can find the angle a of dip, if we have a vertical graduated circle fixed near to the needle. We first obtain the direction of magnetic east and west, by turning the plane of swing round until the needle stands vertically. We then turn this plane through 90°, so that it may coincide with the plane of the magnetic meridian. The angle now made by the needle with the horizontal will be the required angle a of dip.

Corrections.—In practice, however, it is necessary to make several corrections owing to unavoidable imperfections in the instruments.

(i.) *For error in the magnetic axis of the needle.*—We wish to find the position in which the magnetic axis of the needle is vertical, since it is this that gives us true magnetic east and west. Hence, we must turn the plane until the needle is vertical; must then take the needle out of its pivots and replace it in a reversed position; must, if necessary, turn the plane slightly until the needle is again vertical; and finally, must give to the plane the mean of these two positions. This will be the true magnetic east and west.

(ii.) *Error of balance.*—It is almost impossible to balance the

needle so truly that, when unmagnetised, it will take up any position indifferently. Such a defect can be corrected for by repeating the above processes with the needle reversed in magnetism.

Thus we have in all *four* measurements to make.

[There are also other corrections not given here.]

'*Goolden's and Casella's Dip-circle.*'—The accompanying figure represents a very convenient form of apparatus by means of which the dip can, for teaching purposes, be found with sufficient accuracy.

Full instructions as to its use are supplied with the instrument, and need not be given here.

For ordinary purposes we can assume that error (ii.) has been corrected by the makers. If we follow carefully the instructions given, it will be found that they amount simply to—

(i.) Showing us how to set the plane of swing, and its axis, in a vertical position.

(ii.) Showing us how to correct for error (i.) given earlier; the error *viz.* respecting the magnetic axis of the needle.

There is a very simple device by means of which we can turn the plane of swing through 180° or 90° from *any* initial position.

§ 15. **Measurement of the Earth's Magnetic Elements.**—We have described in §§ 12 and 14 how two out of the three magnetic elements (*see* § 11) at any place can be measured. It remains only to show how the intensity R of the earth's field can be determined.

Now by § 13 it is clear that, if we have measured the inclination α, we can find R by measuring the horizontal component H of the earth's total field. For we have the simple relation that $H = R \cos . \alpha$. This is convenient; since, for practical reasons, it is easier to measure the strength of a horizontal field than of one inclined at any angle to the horizontal.

The *general* method of finding H is somewhat as follows.

A steel bar is made, of simple rectangular shape; such that its

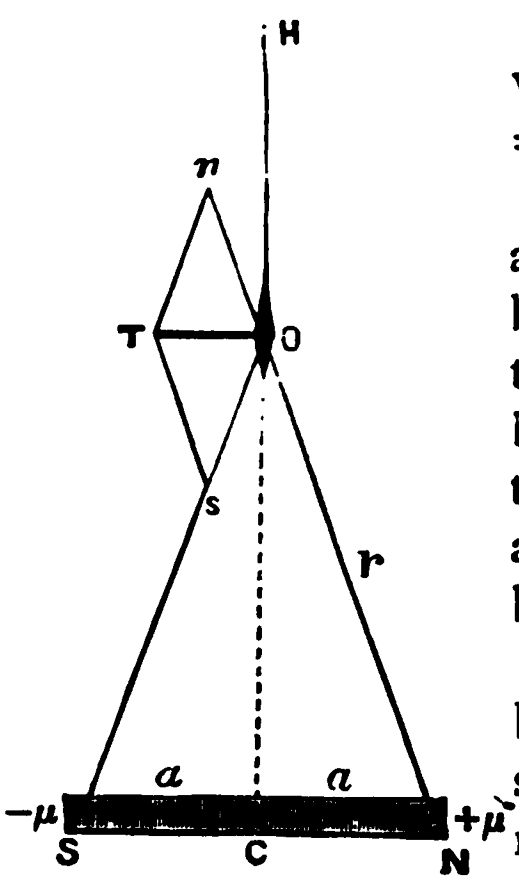
mechanical properties, as depending upon its mass and its dimensions, can be readily calculated. This is very carefully magnetised, and is then suspended by a thread of fibre of negligible torsional elasticity, so that there is complete freedom of vibration in a magnetic field.

It is then vibrated in a horizontal plane under the influence of the earth's action only. As explained in § 3, this enables us to determine, in absolute C.G.S. units if we so desire, the value of the product H m; where H is the horizontal component of the earth's field, and m is the magnetic moment of the bar. But we cannot in this way separate H from m.

Next we find the value of the quantity $\frac{H}{m}$. To effect this, we compare the actions of the earth's field and of the magnetic bar respectively upon a small magnetic needle; as it were setting the field H against a field that can be shown to be proportional to m. We shall in § 16 describe more exactly how this is done.

We thus have

$$\begin{cases} \text{(i.)} & H\,m = a \text{ (where } a \text{ is known).} \\ \text{(ii.)} & \dfrac{H}{m} = b \text{ (where } b \text{ is known).} \end{cases}$$



Hence, by multiplying, we obtain the value of H; and so also of R, since R $=$ H . sec α.

§ 16. **The 'Method of Deflexions.'**—If a small needle O of magnetic moment m' be under the horizontal component H of the earth's field only, it will lie along the lines of this field; and if it be deflected to an angle $\delta°$ from this direction, it will be acted upon by a restoring couple measured by H . m' . sin δ (*see* Chapter II. § 12).

Now if the needle be thus deflected by a field T, due to a magnetic bar N S so placed that its lines of force run at right angles to the earth's lines, the deflecting couple due to this magnetic bar will be measured by T . m' . cos δ. When there is equilibrium it must be that these couples are equal; and thus . . . . . .

$$\frac{\mathrm{T}}{\mathrm{H}} = \tan \delta.$$

(Compare with the theory of the 'Tangent galvanometer,' in Chapter XVII. § 5.)

We thus can find the ratio of T to H, without any knowledge of the magnetic moment m' of our small needle. For this moment m' 'cancels out' when the ratio between the two fields is considered. When the bar and small needle are suitably situated with respect to one another, it can be shown that the field T is proportional to the magnetic moment m of the bar.

In what follows we shall investigate the relations that exist between the angle δ of deflexion, the two fields H and T, the magnetic moment m of the bar, and the distance of this latter from the small needle when these two are suitably arranged. And we shall show how we may experimentally determine the value of $\frac{\mathrm{H}}{m}$; a result of which the importance is indicated in § 15.

'Total action' of a magnetic bar upon a small magnetic needle; and the determination of $\frac{\mathrm{H}}{m}$.—In the figure the small needle is placed at O; the word 'small' having the usual relative meaning, and implying that we may consider its poles to be in a field that does not differ sensibly from the field at its centre O. The line O H represents in direction and in magnitude the earth's horizontal field H. The magnetic bar N S is so placed that the line H O passes through its centre C, and is perpendicular to its axis. The pole-strength of N S is μ, and its length is $2a$, so that its magnetic moment m is $2a\mu$. The distances N O and S O from the centre of the small needle to the poles of the magnetic bar are each measured by r.

If we mark out on S O and on N O produced, the equal distances O s and O n representing the two components of the field due to the two poles respectively, then the diagonal O T of the completed parallelogram will represent on the same scale the resultant field-strength T at the point O due to the magnetic bar; the lines of force thus being perpendicular to the earth's lines.

The small needle will take up a position lying along the resultant of O T and O H, so that if the angle of deflexion from the magnetic meridian be δ, then we have the relation $\tan \delta = \frac{\mathrm{T}}{\mathrm{H}}$.

Now the lines O n and O s each represent the components of the

field due to one pole, *i.e.* represent $\frac{\mu}{r^2}$. And by similar figures we have the proportion $\frac{OT}{On} = \frac{2a}{r}$; or $T = \frac{2a}{r} \cdot \frac{\mu}{r^2}$. Since we may write m for $2a\mu$, we have

$$\left\{\begin{aligned} T &= \frac{m}{r^3}, \\ \tan\delta &= \frac{T}{H} \end{aligned}\right. \text{ whence } \frac{H}{m} = \frac{\cot\delta}{r^3};$$

and these results give us all that we desired.

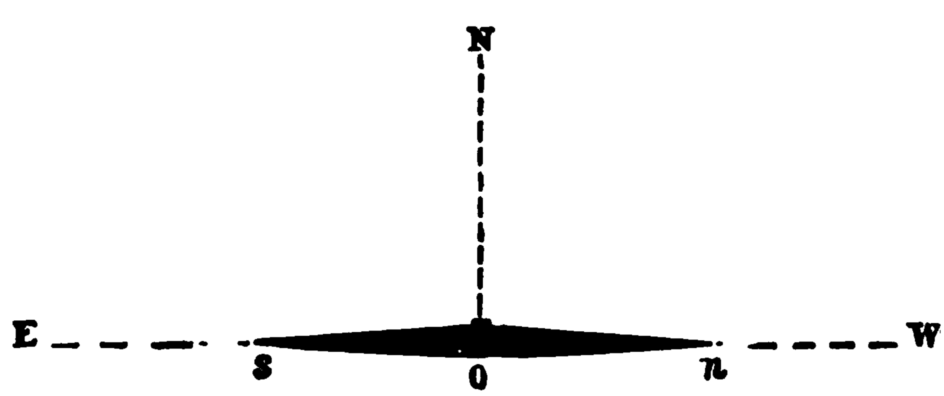
It is clear that the same method enables us to eliminate H and to find m, if we wish to do so.

And the latter part of the experiment, or the 'method of deflexion,' enables us to compare the magnetic moments of two magnetic bars such as N S, and that without any knowledge of their relative masses. We must, however, know approximately the positions of their 'poles,' otherwise we could not substitute a measured value for r.

§ 17. **Magnetometers. Changes in the Earth's Field.**—When a magnetic bar is so constructed and suspended that by means of it we can determine the absolute value of H (and hence of R) at any place, we have the essential part of that important instrument, the *Magnetometer*.

For the construction and use of this class of instrument the reader must consult more technical works. He will there see how a magnetometer can be also caused to record automatically, in a continuous manner, the daily and hourly changes in the earth's field.

We will here merely indicate in a general manner how such changes could be observed.



(i.) *Variations in* H.—Let O N represent the direction of the lines of force in the horizontal component of the earth's field, and let ns be a magnetic needle that has been deflected into a position lying due magnetic east and west, by means of torsion.

In this position the couple due to H is first balanced by the torsion couple.

Any change in H will evidently destroy the equilibrium, and the position of the needle will change. Very slight movements can be detected if we employ a beam of light reflected from a mirror attached to the needle. (*See* Chapter XVII. § 10.)

(ii.) *Variations in* V.—Any needle must be balanced *after* magnetisation, if it is to maintain a horizontal position. In such a position we then have on the one hand a gravity couple, if the centre of gravity is not directly under the pivot; and on the other hand a magnetic couple, due to the vertical component of the earth's field. These two couples are in equilibrium.

Any change in the vertical component V of the earth's field will disturb this equilibrium, and the consequent movements of the needle (in a vertical plane) can be observed by means of the reflected beam method.

CHAPTER IV.

THE SIMPLER PHENOMENA OF ELECTROSTATICS.

§ 1. **Introductory.**—We think it best to give the reader a general view of the phenomena of statical electricity before discussion of the theory of electricity as based on ideas of potential and fields of force.

So we propose, in this preliminary view, to use language that may not be very scientific, but at any rate will not be misleading to a learner previously warned.

The reader must carefully note in what follows that in electrostatics as in magnetism all the phenomena are *dual* in nature.

In *magnetism* there are two polarities that, together, neutralise each other's external effect; apart, they give a field of magnetic force; the two ends of all the lines of force terminating against matter of + and − polarity respectively.

So, in *electrostatics*, we shall find always two sorts of electrifications that, together, neutralise each other's external effect; apart, they give us a field of electric force, the two ends of all the lines of force terminating against + and − electrifications respectively.

In magnetism this polarity is molecular, and we can never isolate the one polarity. In electrostatics we shall see that the case is somewhat different; but yet every + electrification has corresponding to it a − electrification of just the magnitude to neutralise it.

§ 2. **Electrical Excitement.**—As early as 640 B.C. it was noticed that when certain bodies, such as amber, were rubbed, they became possessed of many peculiar properties. Thus they could 'attract' to them light bodies such as bits of paper or pith-balls; when held near the face they gave rise to a peculiar sensation, similar to that produced by coming into contact with cobwebs; and, under certain conditions, luminous phenomena could be observed.

Experiments.—(i.) We hang up a large pith-ball by means of a fine wire If now we bring near a glass rod rubbed with silk, or an ebonite rod rubbed with flannel, the pith-ball will be attracted to the rod. Many experiments of this sort can be tried.

(ii.) If we approach the excited rod to the face the peculiar sensation, referred to above, is at once felt.

(iii.) If we rub the rod in the dark, luminous effects will be at once noticed.

Note. —In order to succeed in the above experiments we should warm all the apparatus used, so as to insure its perfect dryness.

§ 3. **Dryness needed; not High Temperature.**—It was found that in order to make these experiments succeed it was better to warm the rubbers and the rubbed at a fire.

But a little further experiment will show that it is not high temperature, but the dryness of surface obtained, that we need.

Experiments.—(i.) If we heat the bodies in damp steam we fail to get a result, though the bodies are hot.

(ii.) If we dry the bodies by leaving them for some time under a glass case in presence of some desiccating body, the experiments will succeed though the bodies are cold.

§ 4. **Attraction and Repulsion Phenomena.**—We find that after a body has been in contact with an excited body, and if the conditions are such that it itself becomes and remains excited by contact, then it will be repelled by the body from which it received its excitement.

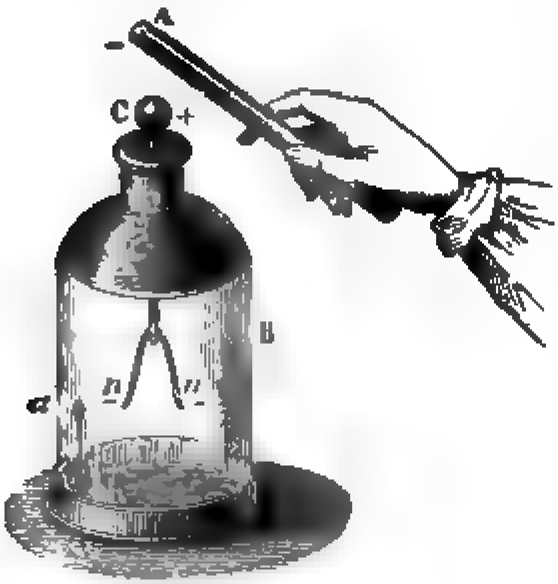
But if the excitement it has received be removed from it, the original attractive action will be again seen.

Experiments.—(i.) If we hang our pith-ball by a dry silk thread, we find that it becomes and remains excited after contact with the excited rod. In this condition it will be repelled by the rod. But if we hang it by a metallic wire, or if we keep touching with the hand, it will not retain its excitement, and it will always be attracted by the rod.

(ii.) The two leaves of a gold-leaf electroscope repel one another when charged.

Note on the Gold-leaf Electroscope.—This instrument is a very sensitive indicator of electrical excitement, and it may be constructed in various ways. In the ordinary form there are two strips of gold leaf, *n n*, suspended from a brass wire which passes through the neck of a glass vessel B. This wire terminates in a knob or plate C. Inside the vessel is usually placed some desiccating substance, such as *calcium chloride.* Two strips of tin-foil are often gummed on to the inside of the vessel so as to be opposite to the gold-

leaves; and these strips should be in metallic connection with the earth, or with the table upon which the instrument stands. The glass of the vessel B should be dry, and the upper part may be coated with shellac-varnish. The reasons for these details of construction will be understood later on.



Test for electrification.—We shall also understand by what follows later that the only certain test for electrification is the *divergence* of the leaves. If the leaves be charged and be divergent, it is found that the approach of a neutral (or unexcited) body to the knob C will cause a greater or less *convergence* of the leaves. Hence, if in any case we obtain such convergence of the leaves by the approach of a body, we should compare the amount of this convergence with that produced by the approach of the same, or a similar body when unexcited. It is only if the former convergence is markedly greater in amount than the latter that we can say that it indicated electrical excitement in the body approached.

§ 5. **Conductors and Non-Conductors.**—Such threads as do not allow the pith-ball, after excitement by contact with the excited rod, to remain excited, were said to *conduct* away what was formerly called 'the electric fluid.' Other such experiments were made, and it was found that certain bodies kept any electrical excitement isolated, while along others the excitement leaked away to the earth, or would pass to and excite other isolated bodies.

When a body showed the one or the other property to a marked degree, it was called a *non-conductor* or *conductor* respectively. In reality all bodies are to a greater or less degree conductors, as in the case of heat, but non-conducting properties are more strongly marked for electricity than for heat.

Experiments.—(i.) By experiments with the suspended pith-ball we find *dry silk*, *glass fibres*, *threads of sealing-wax*, &c. to be *non-conductors*; while *damp silk*, *cotton* and *wire* are *conductors*.

(ii.) Excite an ebonite rod, and throw over it threads or wires of various sorts in turn. In each case let the thread or wire, as it hangs from the excited rod, come into contact with the knob of a gold-leaf electroscope.

The 'conducting' threads will allow a charge to pass to the gold-leaves from the rod, and the leaves will remain permanently divergent.

The 'non-conducting' threads will not allow a charge to pass.

Note.—Table of some bodies remarkable for good or bad conduction respectively.

Conductors.	*Non-conductors.*
Metals.	Ebonite.
Well-burnt charcoal.	Resins, shellac, amber, &c.
Graphite.	Caoutchouc
Acids.	Dry gases.
Saline solutions.	Sulphur.
Moist bodies.	Glass.
Flame.	Silk.
Linen.	Wax.
Cotton.	Many dry bodies.

§ 6. **Electrics and Non-Electrics (so-called).**—If various bodies be held in the hand and rubbed with other bodies, some give signs of excitement and some do not.

This rough method of experiment gave rise to a division of all bodies into *electrics* (from which it was easy to obtain electric phenomena), and *non-electrics* (from which it was not).

This division was premature.

It was soon found that there was no reason for supposing but that all bodies rubbed together gave electric phenomena; at least, if they were not absolutely identical in material, surface, temperature, &c. But conducting bodies allow the electrification to pass away at once, while non-conducting do not. Hence *electrics* and *non-electrics* mean *non-conductors* and *conductors* respectively.

Experiment.—A brass rod held in the hand and rubbed with silk gives no signs of excitement. But, if fixed at the end of an insulating ebonite rod, it will now remain excited after the rubbing.

§ 7. **Two sorts of Electrification.**—It can be shown that there are two sorts of electrification such that *similarly electrified bodies repel one another, and dis-similarly electrified attract one another.*

The reader will of course be at once reminded of the two polarities in magnetism.

Experiments (*see* § 2, note).—(i.) Suspend a gilt pith-ball by dry silk thread. Having left it excited and repelled by an ebonite rod rubbed with flannel or catskin, bring up to it a glass rod rubbed with silk. It will be attracted.

Various experiments of this nature may be tried.

(ii.) Let a gold-leaf electroscope be charged. Then the approach of the excited ebonite will cause the leaves to diverge or converge according to the

nature of the charge of the leaves. Note which is the case, and then try the effect on the leaves of the approach of the excited glass rod, or of other excited bodies (*see* end of note to § 4).

(iii.) Make a simple wire frame and suspend it by dry silk, so that excited rods may easily be placed in it and hang insulated.

Excite various rods and try their action on each other.

Thus, if an ebonite rod rubbed with flannel be so hung, and another similarly rubbed be approached, they will repel each other. But if we approach a glass rod rubbed with silk, or another ebonite rod rubbed with silk on which has been spread some electric amalgam, we have attraction. (Such electric amalgam is usually an amalgam of mercury and tin; this is reduced to a powder, and is then made into a paste with lard.)

It is usual to call '+' that electrification that we get on smooth glass rubbed with silk; and '—' that which we get on ebonite rubbed with flannel.

Sometimes the former is called *vitreous*, and the latter *resinous*, electrification (or electricity).

§ 8. **The two sorts of Electrification are always produced together.**—Further experiment will show us that rubber and rubbed are always oppositely electrified; and that this simultaneous occurrence is the only thing that one can, without previous investigation, predict for certain when one body is rubbed with another.

Thus, ebonite may be made either +ly or —ly excited; but it is certain that in either case the rubber will be oppositely excited.

Experiments (*see* § 2, note).—(i.) Charge an electroscope, and then hold near it the rubber and the rubbed in turn. The leaves will always diverge more in the one case, and fall together somewhat in the other; showing the rubber and rubbed to be oppositely electrified.

(ii.) In the simple wire frame, mentioned in § 7, hang an excited rod. It will always be attracted by the rubber that has excited it, but repelled by another similarly excited rod.

The following pairs of substances can be tried as rubber and rubbed.

Ebonite and catskin, ebonite and amalgamated silk, smooth glass and silk, rough glass and flannel, sealing-wax and flannel, sealing-wax and gun-cotton, gun-cotton and cotton-wool, flannel and silk, two silk handkerchiefs of different makes, brown paper and india-rubber.

It is found that we can arrange bodies in a certain order, invariable under ordinary conditions, such that when two of the bodies are rubbed together, that which is higher on the list becomes +, and that which is lower becomes —.

Such a list is here given; it is taken from Ganot.

Catskin.	Metals.
Flannel.	Caoutchouc.
Ivory.	Sealing-wax.
Rock-crystal.	Resin.
Glass.	Sulphur.
Cotton.	Gutta-percha.
Silk.	Gun-cotton.
Wood.	

Note.—It is sometimes not easy to obtain signs of electrification on bodies, as *flannel*, *gun-cotton*, &c., which are not good insulators. In such cases we may fasten them to the ends of thin ebonite rods. Thus, if we so fasten a piece of gun-cotton and of cotton-wool to the ends of two ebonite rods, and then rub them together, we shall find them to be strongly excited with opposite electrifications; but if we hold the two in the hand and then rub them, we shall get little or no such signs.

§ 9. **Equal quantities of the opposite Electrifications are always produced simultaneously.**—This is a fact of great and fundamental significance; and is indissolubly bound up with the essentially dual nature of electrostatic phenomena.

In every case of electrical excitement there is a separation of a state of equilibrium into what may be expressively, though perhaps not very exactly, termed an *overbalance* and an *underbalance* respectively.

What '+ and − electrifications' mean we do not yet know. But one thing at least is certain; that the total electrifications produced in any way from a state of neutrality will, if collected together on one conductor, give zero electrification. Or, we never have any + or − electrification produced without an equal quantity of − or + electrification (respectively) being simultaneously produced. The sense in which the word 'equal' is used has just been explained.

Experiments.—(i.) The following experiment is well-known.

A flannel cap is made, so as to fit on to the end of an ebonite rod, and the whole is carefully discharged in a Bunsen's flame. Then, by means of a long and dry silk thread attached to the cap, this latter is twisted round upon the rod.

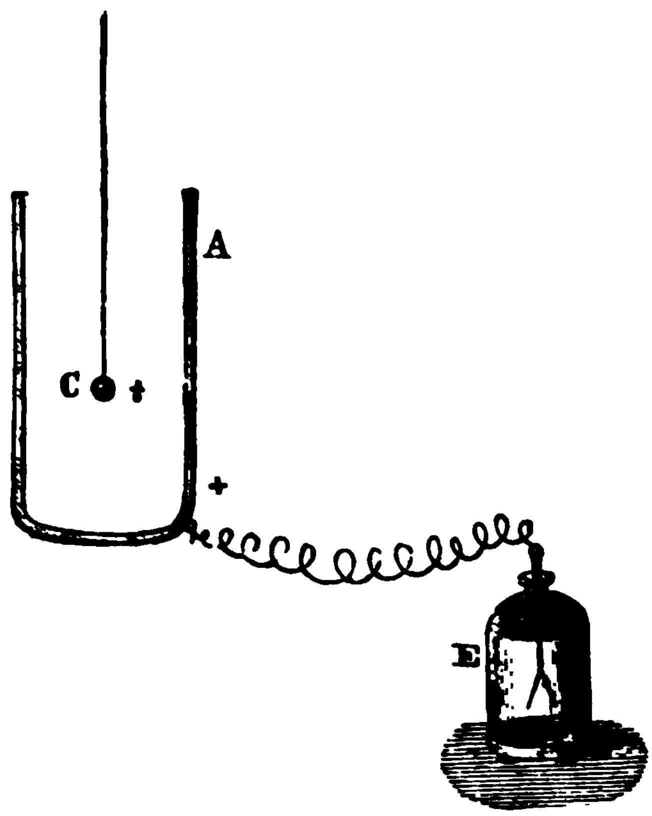
Still it will be found that the *whole* has no effect upon the leaves of an electroscope.

But if the cap be drawn off the rod, still insulated by means of the silk thread, it will be found that the two are oppositely excited.

Here then we have opposite electrifications produced, whose equality was shown by the fact that the *whole* system had zero action on the electroscope.

(ii.) In § 16 we shall see that when a charged conductor is lowered into a hollow vessel such as A, one that is nearly closed and is open only towards the comparatively remote ceiling, the effect upon this vessel will be the same as if it had received the entire charge of the body. This will be the case whether the charged body be a good or a bad conductor, whether it touch the vessel or hang insulated within it.

In the figure we have such an insulated vessel A, and with it is connected an electroscope E. Hence, in all cases of electrical excitement we may introduce the rubber and rubbed into the vessel A simultaneously. If this be done with the rod and cap of the last experiment, it will be found that they give together a zero action, while separately they cause the leaves to diverge equally with opposite electrification.



Notes.—(i.) *On discharging bodies.* All bodies may be readily discharged by passing them through the flame of a Bunsen's burner. The completeness of discharge should be tested by their having zero action on an electroscope.

(ii.) *On testing the charge for 'sign.'* When the leaves are divergent with any charge, a rod of ebonite excited with flannel may be approached. If the leaves diverge still more, their charge was similar to that of the rod, or negative; but if they fall together, their charge was positive. The reason for this will be explained later; that it is the case can easily be proved experimentally by the student.

§ 10. **The 'Fluid' Theories of Electricity.**—The mobility of electrification (or electricity), its ready passage along conductors, and certain other characteristics early suggested the name *electric fluid.* But the absence of weight, the attractions and repulsions of the two sorts of electrification, and other phenomena, such as the extraordinary speed of movement, arguing absence of mass, all show us that if we are to hold previous ideas associated with the word *fluid* we must regard the term 'electric fluid' merely as a rough analogy, if indeed it should be used at all.

Believing that too little is known as to the *nature* of electrification, and of the phenomena of electric strains and movements, to make any term at present in the language really a good one, we

shall, for lack of better terms, speak of '+ and − *electrifications* or *electricities*.'

The expressions '+' and '−' retain their algebraic meaning. Equal and opposite electrifications, when imparted to the same conductor, give *zero* electrification; while, if the quantities be unequal, we have left simply the balance of the one or the other as the case may be.

When it is assumed that there are two fluids possessing, in a sense that has been made clear, 'opposite' properties, we are said to be employing the *two-fluid theory* of electricity.

When it is assumed that there is but one fluid of which an excess in one place and a defect in another give rise to the phenomena of + and − electrical charges respectively, we are said to employ the *one-fluid theory*. The reader is again warned that in the present state of knowledge it is unadvisable to lay much stress upon either view.

§ 11. **The Three Laws of Electrostatics.**—With respect to the attractions and repulsions of electrified conductors, it is found that three main laws hold. These are as follows.

Law I. *Like electricities repel, and unlike attract, one another.*

This means that if two bodies are charged with electrifications of like or of unlike sign respectively, there will be observed between them 'repulsion' or 'attraction' respectively. (We here use the usual terms without asserting that they are accurate.)

§ 12. **Law II. The Force varies as $Q \times Q'$.**—This means that if we have two quantities of electricity, Q and Q' respectively, on two conductors, and if the conditions as to distance remain constant, then the force in *dynes* that exists between the conductors is proportional to $Q \times Q'$, being repulsive if the algebraic product be +, attractive if the product be −.

We must here suppose the bodies to be in the middle of a very 'large' room; and the conductors should be two 'small' spheres, so that we may consider the distance between Q and Q' to be the distance between the centres of these spheres.

Note.—The word *large* means very great compared with the distance between the spheres; the word *small* means very small compared with the same.

Measurement of Quantity; unit Quantity.—It was said that in *magnetism* (*see* Chapter III. § 4) we had no independent law

that *force varied as* $\mu \mu'$, since we measured the product $\mu \mu'$ by the force, and would be arguing in a circle if we made of this an independent law.

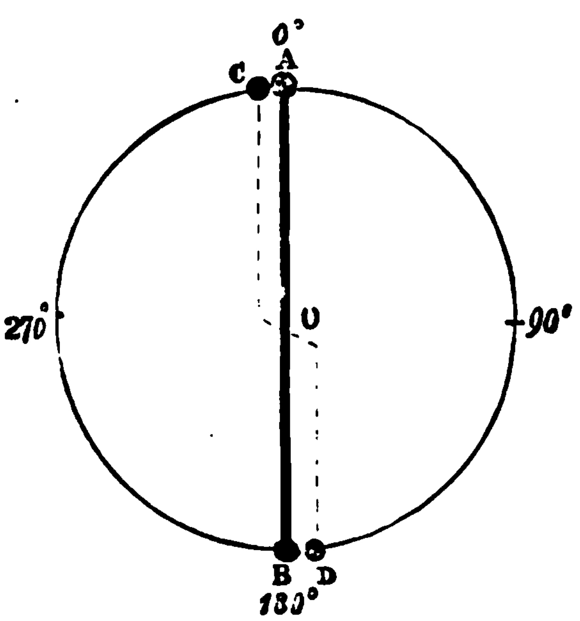
In electricity the case is somewhat different. We do indeed define *unit quantity as that which at unit distance from an exactly similar quantity repels it with unit force*, thus measuring our unit by force. But we can find multiples or submultiples of this unit without reference to force.

Thus we can empty a series of bodies, each carrying unit charge, into an insulated hollow body; the charge will go wholly to the outside surface, thus charging one and the same surface with any number of units desired.

Or again, we can charge a sphere with unit quantity, and then, hanging it in the middle of a large room, bring into contact with it one, two, three, &c., exactly similar spheres.

Theory and experiment will show that the unit is subdivided into $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., provided that the spheres are arranged with perfect symmetry.

(i.) *Proof of Law II. by torsion balance.*—Referring to the figure of the torsion balance we have to imagine only the following modifications.



Instead of the needle we have a light arm of straw, A B, carrying a small gilt sphere at each end.

When the straw arm is at zero each of these equal spheres rests against another equal sphere at C and D, these latter being connected with each other, but otherwise insulated. This arrangement of *two* spheres &c. is to insure its being a true couple that acts on the needle. A like arrangement would have been desirable in magnetism, but we could not have found two exactly equal poles to act on the two poles of the needle, whereas we can readily get equal quantities of electricity on the gilt balls. If C is now charged, the charge will be shared by all four balls alike if they have been made accurately equal spheres, and the arm A B will be deflected. We bring it back by torsion to some conveniently small angle, and measure the total angle of

torsion (*see* Chapter III. § 2), and so the torsion couple. We may suppose that we have a sphere S_1 charged, so large that the quantity on it is not appreciably altered by several times charging the torsion balance from it. Let us charge the balls of the torsion balance from the sphere S_1, and measure the torsion as above. Then discharge the whole.

Next make S_1 share its charge with an equal sphere S_2, and charge the balls again from either S_1 or S_2.

A little thought, and reference to the assumption as to the size of S_1 made above, will show us that each ball now has one-half the charge that it had before. We shall now find that for the same deflexion the torsion couple required to balance the electric couple is one-quarter what it was before ; that is, the force between the balls is one-quarter the former force.

Now since the quantity on each ball is one-half what it was, this result confirms the law that 'the force varies as Q × Q'.

§ 13. **Law III. The Force varies as** $\frac{1}{r^2}$.—The meaning of this law has been already explained in Chapter III.

In order to test this law by direct experiment it is clear that we must deal with charged spheres of very small size as compared with the distance between them. Otherwise, we could not assign a distinct meaning to the distance r between the two quantities of electricity.

All direct experiment is only approximate, and serves to confirm a law for which there is much indirect evidence.

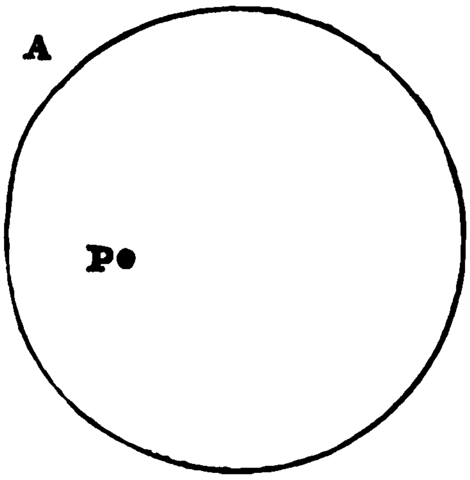
The law theoretically refers to quantities of electricity collected at *points*, just as do the corresponding laws in magnetism and in gravitation. This gives a definite meaning to r.

Assuming that our methods of experiment and of reasoning have proved the law, the integral calculus enables us to deal with the action on each other of conductors on which there is a known distribution.

(i.) *Torsion balance method.*—If the student will read Chapter III. § 5, and has learnt the modifications in the torsion balance required for electricity as given in the last section, he will need no further explanation of this method.

(ii.) *Indirect, but exact, proof.*—One of the easiest matters to

prove experimentally is that if a hollow vessel A B be charged there is no force due to this charge acting on a small charge P inside; or *there is no field of electrical force inside a hollow, charged, conductor.*



Faraday constructed a small insulated room. He went inside it with delicate electroscopic apparatus, and then the whole was highly charged. No electric field inside, due to this charge, could be detected.

Now it can be proved mathematically that no possible law of force between the small charge P and the different portions of electricity on the vessel A B, except the law of inverse squares, could account for this resultant *zero* force on P.

Formula expressing the three laws.—We can express all the three laws by the one formula

$$F = \frac{Q \times Q'}{r^2}.$$

Here we pay proper attention to the 'rule of signs' in the product $Q \times Q'$, and consider the force to be repulsive if the product is $+$, attractive if the product is $-$. The force F is in *dynes* if r is measured in *centimètres*, and if Q and Q' are measured in the *absolute units* given in § 12.

§ 14. **First Ideas as to Induction.**—As stated in § 1, we shall at present use the somewhat unscientific 'two-fluid' language in this first sketch of the main phenomena of electrostatics.

Instruments used.—The *gold-leaf electroscope* has already been described. The insulated conductors need no comment.

But the *proof-plane* needs some explanation. It consists of a small metal (or gilt paper) disc at the end of a thin insulating rod. The theory of its use involves the following considerations. We shall see that electric charges reside only on the surfaces of conductors. Consequently, if the small disc of the proof-plane be applied to a conductor so as for the time to form part of its surface, the electricity that was on the portion of the conductor covered by the disc will now be found on the disc. When the proof-plane is removed (care being taken to remove it so that every part of it leaves the conductor at one and the same

moment) the charge that it carries away can be taken as a sample both in sign, and in degree of concentration, of the charge existing on that part of the conductor to which the proof-plane was applied.

The main facts of induction will be understood from the experiments that follow, and the figures represent roughly to the eye the condition of things in each case.

Note.—Insulating stands.—A very good, but somewhat expensive, form of insulating stand is that due to Professor Clifton. It can be obtained at Powell's glass-works, or through electrical instrument makers. Professor S. P. Thompson has devised a simple and very effective form, that can be made in any laboratory. A description of it may be seen in *Nature*, vol. xxix. page 385.

Well-polished ebonite rods are also good, but in delicate experiments care must be taken to discharge them frequently in a Bunsen's flame. They may be cleaned with paraffin oil, this latter having been rendered anhydrous by means of a few lumps of sodium kept in the bottle.

Experiments.—(i.) A B is a large rod of ebonite excited with catskin and placed in a stand. C D is an insulated cylinder, previously discharged, placed

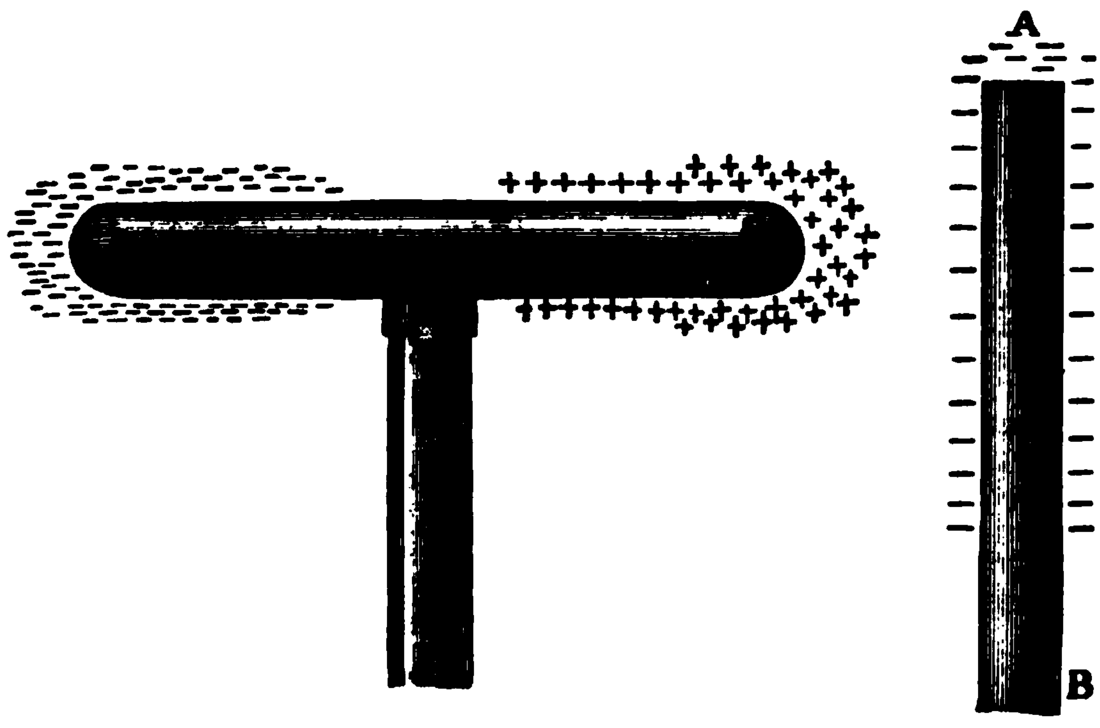


FIG. i.

near A B. It will be found that the state of equilibrium of C D is altered. Whereas it was totally uncharged, we now find on it a distribution somewhat as given below.

Applying the proof-plane to different parts on its surface in succession, and converging the charges so obtained to an electroscope, we can show that the end C opposite to the – charged A B is strongly + charged. The charge is feebler as we approach the centre of the cylinder; about the centre there will be a region where no charge is to be detected; while, as we approach

the end D, we find a stronger and stronger − charge. The proof-plane must be discharged in a Bunsen's flame between each two trials.

Note.—When the charge has been conveyed to the gold-leaf electroscope by the proof-plane, its *strength* can be roughly seen by observing the amount of divergence of the leaves, while its *sign* can be recognised by the approach of an excited ebonite rod.

(ii.) While the conductor C D is in the presence of the charged body A B, let us touch C D at any point. This will connect it with, and so make it form part of, the earth.

We shall find that *all* the 'repelled' − electricity 'goes to earth,' leaving the body charged +.

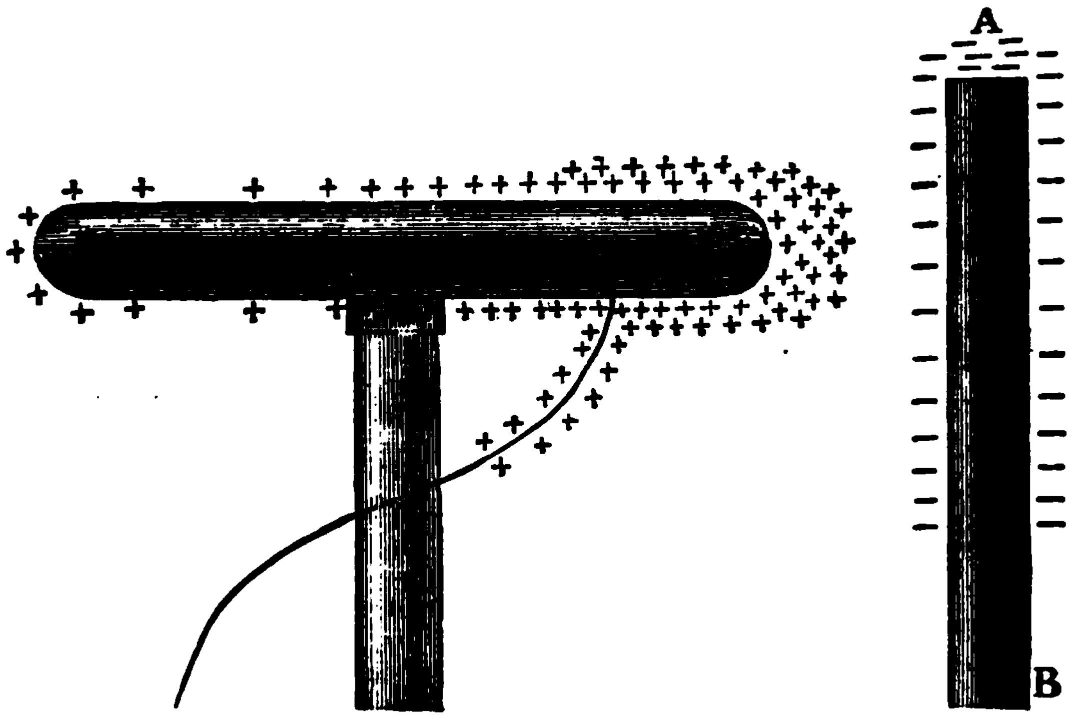


FIG. ii.

We should have expected this, since the portions of the earth near A B would probably, judging from experiment (i.), be + ly charged, while the − would go as far away from A B as possible. But still it seems at first a little surprising that the − charge should escape as readily if we touch the end C as if we touch the end D. This will be explained in Chapter V. § 6 (*d'*). Here we may only say that the presence of the + at the end C renders it as easy for the − to pass to earth by the route near A B as by the other route.

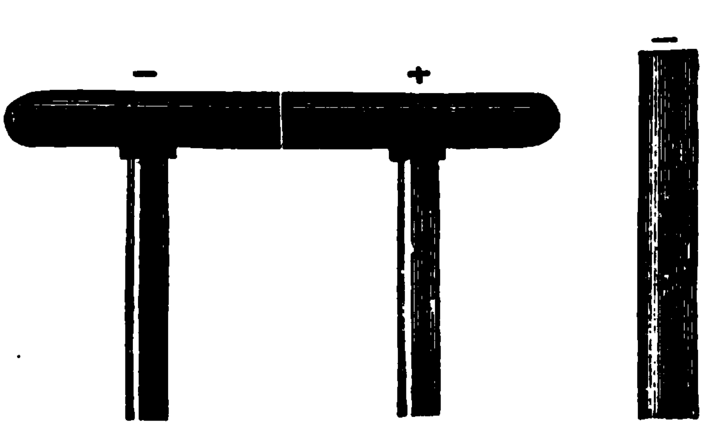


FIG. iii.

(iii.) *The divided cylinders.*—We may construct our cylinder of two halves separately insulated.

If we allow the whole, when forming one cylinder, to be acted upon inductively as in fig. i., and then separate the two halves, we shall find the half nearer A B to be + ly charged, the half further from A B to be − ly charged.

(iv.) *Charging the gold-leaf electroscope by induction.*—The second experi-

ment shows us how we may charge a conductor by induction; and this method is almost invariably employed when we desire to charge the gold-leaf electroscope.

An excited rod is held near the knob of the electroscope, so that the leaves diverge with a charge of a similar sign to that of the rod. We then discharge the electroscope of this 'repelled' electricity by touching the knob, so that the leaves again collapse. If we remove the hand and then withdraw the excited rod, the leaves will diverge with a charge whose sign is opposite to that of the inducing rod.

§ 15. **First Ideas as to Distribution.**—In considering the distribution of an electric charge on a conductor, we assume that we are dealing with *simple continuous conductors*, or those in which all the parts are connected by conducting matter. Such cases as that of an insulated ball hung inside a vessel will be considered later under the head of *condensers*. They are cases of *systems of conductors* and not of simple conductors.

We shall find the following general facts to hold, it being taken for granted that, unless the contrary is stated, the conductor in question is in the middle of a large room (*see* § 11, note).

(i.) The charge is on the surface of the body only. In the case of closed, or nearly closed, hollow conductors, the charge will in general be on the outside surface only.

(ii.) The distribution depends upon the shape of the body. The density of charge is greatest on all salient points; all projecting corners, or prominent regions of great curvature, having a far greater density than have the other regions.

We here give some facts with respect to the distribution on bodies of certain forms.

Note.—By *density* of charge we mean *quantity* of electricity *per unit area.* We shall say more concerning this in Chapter X.

On a *sphere* the distribution is uniform.

On a *ellipsoid of revolution* the densities of the charges at the extremities of the axes are proportional to the lengths of the axes.

On a *cube* Riess found that the density at the middle points of the edges is about two and a half times as great as at the middle of a face, while at the corners it is more than four times as great.

On a *circular disc* the density is greatest round the edge. As

we move from the edge the density falls off rapidly, and for a considerable region near the centre of either face of the disc the density is nearly uniform.

More exactly, Coulomb found the following distribution on a disc of 10 inches diameter.

At the centre . . .	the density was	1·000
1 inch from the centre .	„	1·001
2 inches „ . .	„	1·005
3 inches „ . .	„	1·170
4 inches „ . .	„	1·520
4·5 inches „ . .	„	2·07
5 inches, or at the edge .	„	2·90

On a *cylinder* of an elongated form the density is much greater at the ends than at the middle.

In the case of a cylinder of circular section whose diameter was 2 inches, length 30 inches, and whose ends were hemispherical, Coulomb obtained the following results.

At the middle . . .	the density was	1·00
2 inches from the ends .	„	1·25
1 inch from the ends .	„	1·80
At the ends . .	„	2·30

It is to be noticed that this is somewhat the form given usually to the 'prime-conductors' of electrical machines.

On a *cone* the density is greatest at the apex.

When the cone is of a very small angle, and the point is very sharp—a common needle is an extreme case of this kind—the density at the point becomes very great indeed. As a consequence of this, though why it is a consequence it is not so easy to explain as would at first sight appear, the air no longer serves to separate the charge residing on the point from the charge induced upon the walls or other surroundings. It is found that a stream of electrified air proceeds from the point towards the induced charge of opposite sign that exists on the walls, &c.; and the conductor is thus discharged. Thus a sharp projecting point will almost completely discharge any conductor to which it is fixed. In a similar manner, but by inductive action, an earth-connected pointed conductor will nearly discharge a charged body, towards which it is presented; for there will proceed from the point to the

body a stream of air charged with the electricity of opposite sign that is induced on the point by the charged body.

If there be a sharp point inside a closed or nearly closed vessel, there will be no charge upon it, and hence the above action will not occur. Let us, for example, take a tin vessel (such as a common pint measure), and let us fix inside it a needle, soldering it in an upright position to the bottom of the vessel. If the needle's point be well within the vessel, we may charge this latter, and shall find that the needle does not discharge it. But if the needle be long enough to rise out of the vessel, it will discharge it.

Experiments.—(i.) By the help of a proof-plane we can verify the above statements as to the general character of the distribution on insulated conductors of various shapes.

(ii.) Insulate a somewhat deep and narrow pail, and charge it. Examining it with the proof-plane we shall find that approximately all the charge is on the outside surface. (As the vessel is not quite closed there will be *some* charge on the inside surface, especially near the mouth of the vessel.)

This is approximately true even of a cylinder of wire gauze, or a butterfly-net. A well-known bit of apparatus is 'Faraday's net'; it was used to show the above.

(iii.) Connect a small insulated pail with the electroscope, having placed in the pail a quantity of brass chain to whose end is attached a silk thread. Now charge the pail, and note the extent of divergence of the leaves. Next raise a considerable length of chain into the air by means of the silk thread, leaving the lower end in the pail. The leaves will fall somewhat, showing the charge to be less concentrated than it was. Here we have increased the external surface of the whole conductor without changing its mass; and the result shows that the charge has spread.

§ 16. **Faraday's Ice-pail; illustrating the Laws of Distribution and of Induction.**—The experiments that follow illustrate to some extent the laws of distribution. But their chief value is that they throw much light on *Induction.*

We shall find that, among other things, the following facts are true.

(*a*) *Every charge induces on surrounding surfaces a charge equal to its own in magnitude, but of opposite sign.* If some of the surroundings be nearer to the charged body than are others, then the greater part of this induced charge will in general be on the former.

(*b*) A partly closed hollow conductor ceases to act as if

entirely closed when there is introduced into it (though not in contact with it) a conductor connected with the earth. The opening in the vessel now becomes of great importance; for we have introduced an earth-connected conductor. The whole surface of the vessel, inside as well as outside, is now opposite to part of the earth; and the charge will be distributed over the whole surface. Referring to the statement, which is in general true, that 'When a hollow conductor is charged, the charge resides upon the outside surface only,' we must in the present case consider that the only true *inside* is the interior of the mass of the metal. This case will be examined in experiment (vi.) below.

(*c*) The manner in which a charge distributes itself upon the outside surface of a hollow conductor is quite independent of the positions of charged or uncharged bodies situated entirely within the vessel. It depends solely upon the form of the conductor, and the situation and nature of surrounding bodies.

This is a particular case of the general fact that a conductor acts as a *screen* between two electrified systems (*see* Chapter X. § 17 (3)).

Apparatus used.—We use a tall narrow ice-pail placed on an insulating stand, and connected with an electroscope; also a brass ball, hung from a silk thread, to which a charge can be given from a small electrophorus (*see* § 17). The ball is kept insulated unless the contrary is stated.

Experiments.—(i.) We can show that the ball is totally discharged if allowed to touch the inside of the pail, while if allowed to touch the outside it only shares its charge with the pail. (*See* figure to § 9, experiment (ii.)).

(ii.) Charge the ball + and lower it insulated into the pail. The leaves diverge with a + charge repelled. Now let the ball touch the pail so as to be totally discharged. The leaves do not stir. This shows that the + charge repelled by induction equals in amount the inducing + charge on the ball. Hence also the induced – charge on the inside of the pail must be equal to the inducing + charge, since the pail *as a whole* has zero charge.

(iii.) While the ball hangs inside insulated, touch the pail outside with the finger. The leaves collapse, the repelled + charge having been removed. Now re-insulate the pail and remove the ball, when the leaves will diverge just as much as before, but with a – charge. This illustrates the above equality of inducing +, and induced –, charges respectively.

(iv.) Or again, as follows. Remove the repelled + charge, as in (iii.), and again insulate the pail. Then allow the ball to touch the pail. We get zero charge, showing that the inducing + and induced – have just cancelled each other.

can be shown by careful experiment with a proof-plane and a very ectrometer that (*a*) the − distribution, induced inside, moves about e the ball; (*b*) while the repelled + outside is independent of the the ball when this is once well inside the vessel.

harge the pail and lower into it the insulated ball uncharged. No the gold-leaves is observed. Now connect the ball with earth while ed from the pail. The leaves drop a little, showing that part of the s (as can be directly proved with a proof-plane) gone to what we rto called the *inside* of the pail, but what is now a part of the pail's posite to the earth. The charge on the pail induces on the walls, or, *and ball* (all of which are portions of the earth) a charge equal ite to its own, and a portion of this is now on the ball.

ortion can be isolated by again insulating the ball and removing it ail.

Ve may hang several pails, insulated from each other, one inside the

ments similar to the above may be tried. It will be a useful exercise lent to puzzle out for himself what will happen in each case, remem- two principles that—

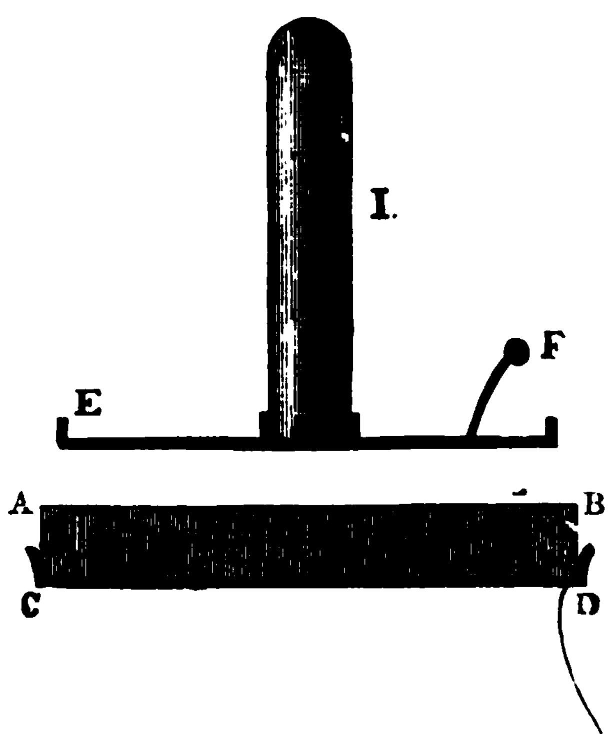
charge induces an equal and opposite charge on surroundings.

ch pail serves as a 'screen' (*see* Chapter X. § 17) between the charge r surface and any charge inside it.

. **Electrophorus.**—We shall here describe, in an elemen- ner, a simple contrivance for obtaining charges of elec- induction.

is a circular cake of resin, ite, or any of the good ' compounds that can be melting together various ; (*e.g.* the following sub- may be melted together in ortions by weight here given: hellac, 5 parts gum mastic, Venetian turpentine, and arine glue. This recipe is Professor Guthrie).

is fixed in a shallow pan C D, tinned iron or other metal.



s a circular disc of brass or tinned iron of smaller diameter cake, having well-rounded edges, and being provided nob F. This is fixed on to an insulating handle I.

se the instrument we proceed as follows.

(i.) The upper surface of A B is strongly excited by catskin, and acquires a strong — charge. This induces on surrounding objects an equal + charge. Now while E F and other bodies are remote from A B, nearly the whole of this induced + charge will be on the 'sole' C D; a repelled — charge passing away from the sole to earth.

The presence of C D helps to keep the upper surface of A B from getting gradually discharged.

(ii.) When E F is lowered on to the cake A B it is found that it behaves as if very close to A B but not in contact with it. In fact, the plate rests upon a few projections of the badly-conducting cake, and so is acted on inductively only.

(iii.) The repelled — charge can be removed from E F by touching the knob F; and there remains on E F a + charge opposite to the — charge on A B. In this position of things, the plate E F being so much nearer to the excited surface than is the 'sole' C D, we have on E F nearly all the induced + charge; the sole C D returning to a nearly neutral state by what may be described either as + electricity passing from C D to earth, or as — electricity returning to C D from the earth.

(iv.) The plate E F can now be moved by its insulating handle, and its + charge utilised. By the time that E F is again remote from A B, the 'sole' has returned to the condition in which it was in (i.).

The reader should draw for himself figures representing the condition of the charges, &c., in all these stages of the use of the electrophorus.

Experiment.—By the use of a substantial insulating stand, on which the electrophorus is placed *before* excitement, the changes in the condition of the sole, &c., as given above, can be easily demonstrated. A proof-plane of large size will carry off enough of the charges to affect inductively a previously charged electroscope. The fact that in (i.) — electricity would pass from the sole to earth, in (ii.) and (iii.) + electricity would do so, and in (iv.) again — electricity would pass away——all this can be shown well.

Of course we must put the sole to earth at the stages (i.), (iii.), and (iv.), after showing what nature of charge is ready to pass away.

§ 18. Frictional Electric Machines.

I. *The glass cylinder machine.*—The principle of frictional electrical machines is best understood from a description of one of the simpler forms. As seen in the figure the essential parts are

as follows. A glass cylinder rotates on a horizontal axis. This is pressed on one side by a rubber made of leather stuffed with horsehair and faced with silk; on the surface of the silk is spread a coating of 'electric amalgam' made plastic by mixing it in a

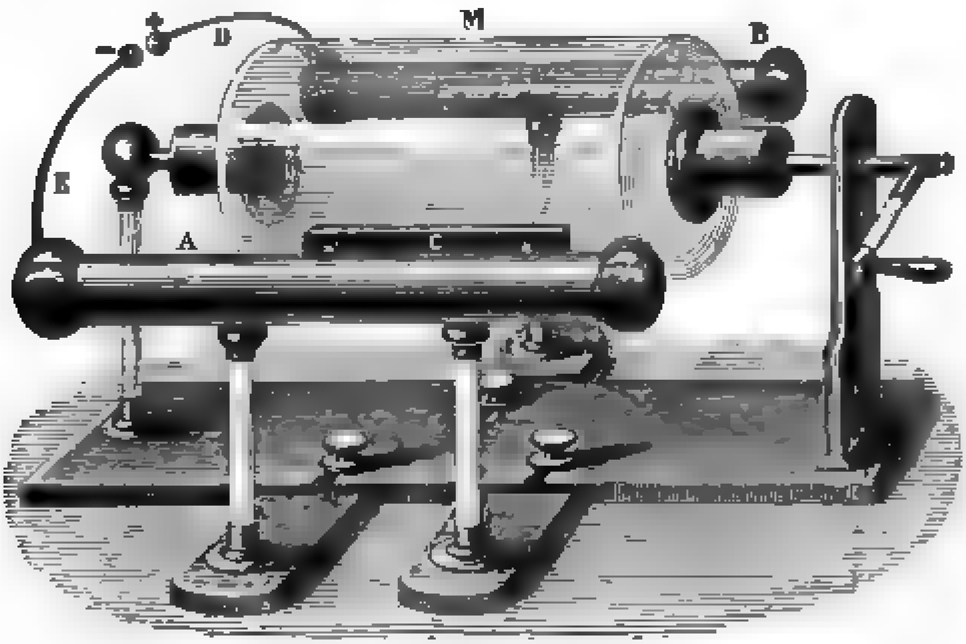


FIG 1.

powdered form with lard. A silk flap passes over the cylinder from the rubber, descending on the other side to nearly half-way down.

The rubber can be insulated when required. On the other side to the rubber, a large insulated conductor presents a row of fine points to the cylinder. These points nearly touch the glass, and are just below the lower edge of the silk flap.

Excepting for these points, the insulated conductor is well rounded off.

When we turn the cylinder, the surface of the glass acquires a + charge, while the rubber acquires an equal − charge.

The +ly excited glass, its charge kept from leakage by the flap of silk, next comes opposite to the conductor with its row of sharp points. On the conductor it acts inductively, repelling + and attracting − charge. This − electricity is discharged, by convection currents of charged air, from the points on to the glass; so that this latter passes on in its original unexcited condition. The + charge is left on the prime conductor, to be utilised.

Use of putting the rubber to earth.—We have seen that the

glass surface returns to the rubber in a neutral, or nearly neutral condition. But the rubber was left charged —. Hence, if the original separation of + or — were to go on, then after some turns the rubber would become highly — charged, and the tendency to part with some of its — charge to the glass would at some point in the process at last balance the tendency to excite the glass +ly. Things would come to a standstill.

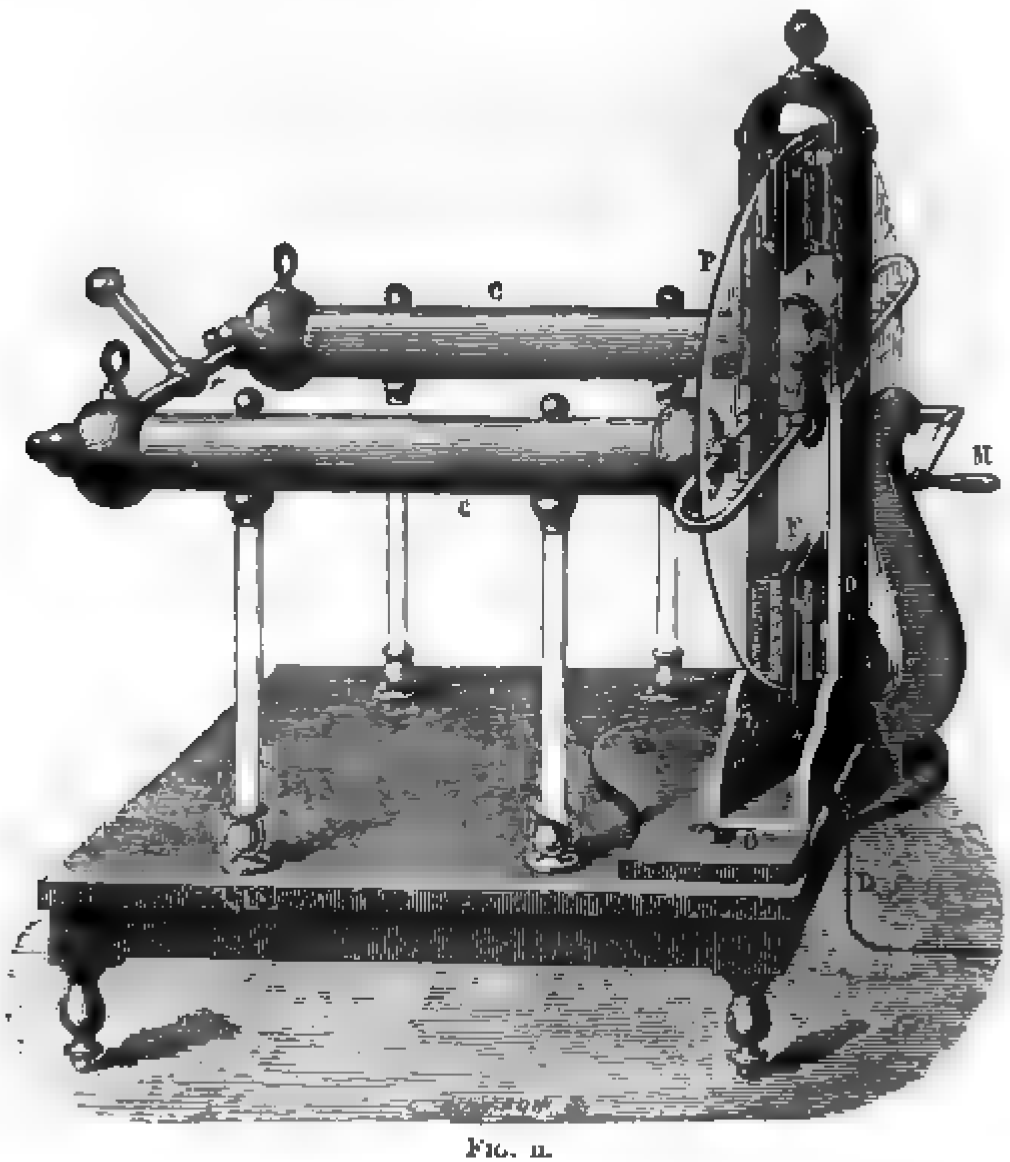


FIG. 11.

But if we put the rubber to earth, we shall get the initial condition of the rubber maintained by a steady passage of — electricity to earth. Later on we shall treat of this matter from a *potential* point of view.

So, if the prime conductor be put to earth, we can use the rubber as a source of — electricity. If the rubber and conducto

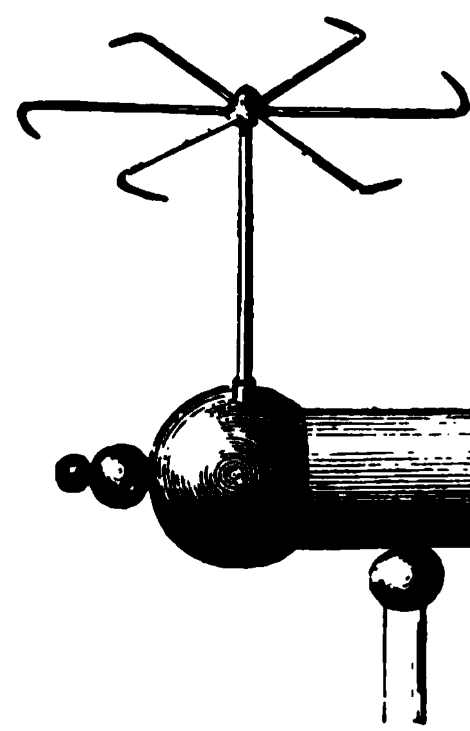
both be insulated, but they be *nearly* connected by conducting rods, when we turn the cylinder we shall see a series of sparks bridge over the gap left; the + charge of the conductor and the — charge of the rubber continually neutralising each other.

II. *The common plate machine.*—In this there is nothing essentially different from the cylinder machine. A glance at the figure will explain all.

There are generally two rubbers; and in this form of machine they cannot well be insulated if required; so the machine cannot be used as a source of both + and — electricity. Instead of glass, ebonite plates may be used, the rubbers being of amalgamated silk.

Note.—With ebonite two precautions must be taken; (*a*) never to heat the plate, only to warm it gently; (*b*) after use to clean the plate with paraffin oil from which all water has been removed by the introduction of a lump or two of sodium into the bottle.

III. *Winter's plate machine.*—In this the rubber and the points of the prime conductor are more widely separated; and the prime conductor can therefore acquire a higher level (or potential) of charge without discharge over the glass to the rubber. The rubber can be insulated or not, as required. A curious feature is an addition to the prime conductor in the shape of a large ring of brass enclosed in baked wood. This ring increases the 'capacity' (*see* Chapter V. § 4 &c.) of the prime conductor.



The machine gives very long sparks for its size, as compared with an ordinary plate machine.

§ 19. Miscellaneous Experiments with the Electrical Machine.

Experiments.—(i.) *Illustrating the action of points.*—Besides the simple experiment consisting in discharging the conductor either by presenting to it an earth-connected point, or by fixing the point on to it, experiments can be easily performed in which the rush of electrified air from the points is made use of. We may blow out a candle by presenting it to the point.

Or again, we may by means of the 'electric vane,' shown in the figure, make evident the repulsion between the charged particles of air and the points

whence they derive their charge. The air will stream one way, the van
in a contrary sense.

(ii.) *The insulating stool.*—A person standing on a stool supported
legs can be electrified by his placing the hand on the prime conducto
the machine is worked. He can then 'give sparks' to other persons, li
with sparks, &c. ; and his hair will, if fine in quality, show a tend
stand on end.

CHAPTER V.

INTRODUCTORY CHAPTER ON POTENTIAL.

(For further, see Chapter X.)

§ 1. **Quantity of Electrification.**—We have spoken of + and — electrification, or electricity, as something measurable and divisible, that is as a *quantity*. Formerly it was spoken of also as a fluid; a view that was fairly justified by certain observed phenomena, but not by others.

This conjunction of the words 'quantity' and 'fluid' leads us to consider in what respect there is, and in what respect there is not, a resemblance between electricity on the one hand, and such fluids as water and gases on the other.

(*a*) *Points of resemblance.*—Electricity can be measured as a quantity, as can water and gases.

In any system of vessels between which there is proper connection, there can be statical equilibrium of water only when all is at the same level, or of a gas only when all is at the same pressure. So on any electrical conductor there can be electrical equilibrium only when there is throughout the same *electrical level* or *electrical potential.*

In the former case the water or gas will move from places of higher level or of greater pressure to places of lower level or less pressure respectively, until the state of equilibrium is arrived at. So will there be a flow of + electricity from places of higher to places of lower potential, or of — electricity in the opposite direction, until a uniform *potential* is arrived at.

Water, and gases, and electricity, are alike unalterable in quantity; only in the case of electricity we must remember that it is the algebraic sum of any + and — quantities that is constant, equal and opposite charges giving a zero sum.

Note.—In the above we have neglected the compressibility of water, and the action of gravity on gases, as relatively insignificant.

(*b*) *Points of difference.*—Water and gases all possess *mass*; indeed, from a mechanical point of view they *are* merely masses endowed with certain mechanical properties, their chemical composition signifying nothing.

Electricity on the other hand has, most probably, no mass. There is, in the case of electricity, no gravitation-force, no mechanical inertia, no mechanical kinetic energy. In a word, electricity is not matter, while all fluids are.

If we are to call electricity 'a thing' at all, it must be justified by the precedent of calling *energy* 'a thing'; and we merely express by the term the unalterability of electricity as a quantity.

In the case of water and gases we have little or nothing answering to the duality of electricity; nothing like the attractions and repulsions between unlike and like charges respectively; nor is there anything like the variation of the intensities of these forces with the nature of the medium in which the charged bodies are situate (*see* Chapter X. § 25, end).

To return to the subject of our section.

Electricity is measured as a quantity; *and the unit is that quantity which at unit distance* (*i.e.* 1 *cm.*) *from an equal quantity repels it with unit force, or with one dyne; the medium between the two quantities being assumed to be air.*

§ 2. **Electrical Level, or Electrical Potential**.—In the consideration of the statical equilibrium of bodies of water, or the work to be got out of any water supply, we are concerned mainly with *level*, *differences of level*, and *quantity* of water.

We usually take 'the sea-level' as our arbitrary zero of level, and reckon the level of any body of water as so many feet + or —; the feet being measured vertically up from, or down from, this arbitrary zero respectively.

Now in electricity we have, roughly analogous to the above, the question of *electrical potential*, *difference of electrical potential*, and *quantity* of electricity.

We take as a zero of level (whether it is quite arbitrary or not we do not here discuss) the *electrical potential of the earth*; and we reckon potentials as + or — when above or below our *zero* respectively.

In gravity levels the test of *the same level* and *different levels* is

to see whether such a fluid as water will, or will not, flow from the one spot to the other when free to move. We call that the higher level from which the water flows. In electrical levels (or potentials) two points are at the *same potential* or *different potentials*, according as + electricity is not, or is, urged from the one to the other point when free to move. That is called the higher potential from which the + electricity is urged.

Thus in gravity a spot has a − level if water will run from the sea-level to it; in electricity a point is at − potential if + electricity will flow from the earth to it.

In the dual nature of electricity, and in the opposite movements of + and − electricity, we perceive the limitations of our rough analogy.

§ 3. **Measurement of Differences of Electrical Level by Work.**—In Chapter X. we shall discuss this matter of measurement of potential more fully. Here we wish only to show in what way the measurement is made.

The measurement of gravity levels may be made by means of a tape, caused to hang vertically by being weighted at the end; or by some equivalent method in which the action of gravitation gives us either a vertical line to measure along, or a horizontal plane from which the vertical can be deduced. In all such cases the measurement is made directly in feet.

But there is another way that is theoretically possible.

We shall in Chapter X. consider the meaning and measurement of *work*. Here we will only say that in lifting weights from a lower to a higher level we *do work*; that the work done is proportional to the weight raised, and also to the vertical height through which it is raised; and that it is independent of the route, whether direct or roundabout, by which the weight is raised from the lower to the higher level.

Now if we could in some way accurately register work done, then, by carrying some standard weight from one place to another, we could, by the amount of work done, estimate the difference of level of the two places. There has, however, been elaborated no exact method of this sort, because the former method is so very simple and direct.

In electricity, on the contrary, there is known no method analogous to the direct plumb-line measurement. The only

conceivable method of measuring the difference of electrical level between two points is by measuring the work done in moving a + unit of electricity from the one point to the other.

If we assume that the reader understands in what unit *work* is measured, we can say that we have now shown how to give an exact numerical meaning to that symbol 'V' which is used to express the potential of a point with respect to the earth as zero.

Notes.—(i.) We are said to do + work when we move against the lines of force, or expend energy; we are said to do − work when we move down the lines of force, or have energy expended on us.

(ii.) The reader will understand from the above section, that the electrical potential of a point in space (or of a body) is +, zero, or −, according as we do +, zero, or − work respectively in moving a + unit of electricity from the earth to the point in space (or body). The potential is higher as this work is greater.

The + unit of electricity was defined in § 1 of this chapter. Any work done in moving it from one place to another is due to the repulsive and attractive actions of other electrical quantities on this + unit. [As regards sign, see Chapter IV. § 7.]

§ 4. **Elementary Ideas on 'Capacity.'**—A certain quantity of water will fill a certain vessel to a definite level, this level depending on the quantity of water and on the dimensions of the vessel. The more the water, and the less the horizontal section of the vessel, the higher the level to which the latter will be filled.

In electricity, when we charge a conductor with a quantity Q of electricity, we raise the conductor to an electrical potential V. This potential will be greater or less according to a property of the conductor which we may naturally call, by analogy, its *electrical capacity*. We call the capacity greater or smaller according as the potential V, to which a given charge Q of electricity will raise it, is smaller or greater respectively. Now we have already indicated how Q and V are defined and measured. Since *capacity* (K) is a new term we may define it, and express it numerically, by the relation—

$$V = \frac{Q}{K}; \text{ or } K = \frac{Q}{V};$$

which is equivalent to saying *the capacity of a conductor is directly proportional to the quantity of electricity required to raise it to a*

certain potential, and inversely proportional to the potential to which it is raised by a certain quantity.

Of the *units* in which V and K are measured we shall treat more fully in Chapter X. We see, however, from the above definition of K, that—

A conductor has unit capacity when unit quantity raises it to unit potential, or to unit electrical level above the earth.

§ 5. **Lines of Force, and Equipotential Surfaces.**—We have in Chapter III. sufficiently explained what are meant by lines of force. The reader can easily, *mutatis mutandis*, define for himself lines of force in a gravitation field of force, or in an electrical field of force.

Thus in an electrical field a line of force is a line along which a particle charged with + electricity, and free from all other forces, would move under the influence of electrical forces; it being assumed that the movement is indefinitely slow, so that there is not any 'centrifugal' desertion of the line of force when this is curved.

It is along these lines of force that electrical level is measured; just as it is along the lines of gravitation force (*i.e.* vertical lines) that gravitation levels are measured with the plumb-line.

It is in movement of electricity along the lines of electrical force, as in movement of masses along the lines of gravitation force, that we *do work.*

Surfaces over which we can move our + unit of electricity without doing work are called *equipotential surfaces.*

In the case of gravitation these are clearly horizontal surfaces; a mass moved over these surfaces always cuts at right angles the lines of force of gravitation, and hence no work is done, or these horizontal surfaces are equipotential.

In electrostatics let us consider the simple case of a single isolated charged particle. Since a + unit would be urged straight from it or straight towards it, according as the charge on it is + or −, it follows that the lines of force are straight lines radiating from the particle as centre.

What are the equipotential surfaces, *i.e.* level surfaces, or surfaces of no work? They must be those over which our + unit always cuts the lines of force at right angles, thus doing no work.

That is, they must be spheres having the particle as centre

and the lines of force as radii. In more complicated systems of electrical charges we may have lines of force curved about in any way whatever. But still the equipotential surfaces, over which our + unit moves without doing any work, will be those that cut at right angles the lines of force.

From what we have said the reader will see that wherever are lines of force, there is rise or fall of potential; and conversely.

A region where are no lines of force must be all at one potential; since there can be no work done in moving our + unit against zero force, and therefore there can be no differences of electrical level. And conversely, where a region is all at one potential, there are no lines of force.

§ 6. **Induction; from a 'Potential' Point of View.**—We will now consider briefly what light the above considerations throw upon the matter of electrostatic induction.

Definition of a conductor.—We must first, however, define what we mean by the often-used term *conductors.* We mean bodies on which electrical charges are perfectly free to move, bodies which can resist no electrical stress due to differences of electrical potential.

By our definitions then it follows that when any conductor is left until the electrical charge has had time to arrange itself, it will be all at one potential; for were two parts at different potentials, the electrification would readjust itself until the two places were at the same potential. This readjustment is so rapid that we can consider it to be practically instantaneous.

The reader may object that one might conceivably have a conductor of which two parts remain at different potentials for the reason that there is no electrical charge to be readjusted. The answer to this is that experiment shows us the universal presence of unlimited quantities of + and − electrifications in all bodies, these being equal in amount if the body be in what we call an uncharged condition. If two parts of the conductor be at different potentials, then there is unlimited + electricity ready to flow from the higher level to the lower, and unlimited − electricity ready to flow from the lower to the higher level; and a flow takes place until all is at one potential.

Hydrostatic analogy to electrostatic inductions.—We may with advantage discuss, in connection with the matter of electrical induction, a hydrostatic analogy. We shall consider the *sea-level*

as analogous to the *electrical level or potential of the earth* ; a *hill-side* as analogous to a region through which there is a *fall of electric potential*, or what we may term an *electric hill* ; and a *trough of water* as (very roughly indeed) analogous to an *electrical conductor.* In a still rougher sense we may sometimes take a *tower* as analogous to a + *electrical charge* ; and a *well* as analogous to a — electrical charge. We may now note the following facts.

(*a*) The water in the trough will always be at one level, or water will have no more tendency to flow in one direction than in the other, in whatever position the trough be placed.

(*b*) If the trough be placed upon a hill-side, the whole will be above the level of the sea ; and water would flow from any part of the trough down to the sea, if connected therewith by a pipe.

(*c*) Further, the water will rise at one end and will sink at the other ; still remaining level. It will be shallower at the end that lies uphill, and deeper at the end that lies downhill.

(*d*) If instead of a trough we have a trench of unlimited depth, and connected this with the sea-level, water will flow out until all is at the sea-level. The water will then be at the sea-level, but will be deep below the surface of the hill.

(*e*) If the hill-side now sank, with the trench still cut in it, to the sea-level, the water in the trench would be now *below* the sea-level ; and water would flow from the sea into the trench until the level was again raised to sea-level.

(*f*) Let there be a trough of water whose surface is at the sea-level. Now let this trough be raised up on to a hill-side and there be put in a sloping position. The water will dispose itself as in (*c*) above. Now let a partition be put in the middle of the trough, parallel to the ends, and let the whole be replaced in its original position. It is obvious that in the one half we shall now have the surface *above*, and in the other half the surface *below*, the sea-level. On removing the partition the original state of things is restored, and the surface is once more all at sea-level.

(*g*) Let us build a tower on the sea bottom so that its top reaches the sea level. If the sea-bottom were to rise up, the top of the tower would now be above sea-level.

(*h*) Let us dig a well on the hill so that its bottom reaches sea-level. If the hill sinks to sea-level, the bottom of the well will be below this.

We will now consider the facts in electrical potential, or electrical level, that are analogous to the above. Both by the wording of the statements and by the correspondence of the lettering (a', b', c', &c.), the reader will be able to see how the two sets of facts answer the one to the other respectively. We must, however, again repeat that, though such analogies are interesting, too great stress must not be laid upon them; above all we must not *argue* from analogy.

Let us suppose that there is a room whose walls &c. are all connected with the earth, and are therefore at the same electrical level, or potential, as the earth; this we take for our arbitrary *zero* of potential. Further, let there be a +ly charged body A suspended in this room.

There will be a fall of potential, or an electrical hill, from this body A, down to the walls, &c. Such was the case, if only we change the sign of the charge, in Chapter IV. § 14.

If any insulated uncharged conductor B be placed between the walls and the body A, this body B will also be at a + potential; it will take work to bring a + unit of electricity from the earth up to B, in consequence of the presence of A. So that A has given rise to an electrical hill running down to the walls; and all insulated uncharged bodies placed, as it were, on this hill, will be at a + potential; though of course at a lower potential than is A, since it will take less work to bring a + unit of electricity up to B than up to A.

The corresponding electrical facts are as follows.

(a') A conductor such as B will always be at one electrical potential over all, wherever it be placed.

(b') When the insulated conductor B is placed between A and the walls, *i.e.* on an electrical hill, the whole is above the electrical potential of the earth; and + electricity would flow from any part of this conductor to the earth, if it be connected therewith by a wire.

(c') Further, we shall find — electricity at the end nearer to A, and + electricity at the end nearer to the walls. This readjustment, or separation of + and — electricities, will be just such as will leave B all at one potential; or will be such that a + unit of electricity will move on B in any direction indifferently, and this in spite of the presence of A (*see* Chapter IV. § 14, Experiment (i.)).

(d') If we connect B to earth by a wire, + electricity will

flow out of B until it is all at zero potential. It will then be charged with — electricity, though all at zero potential. Since B is all at one potential, this flow of electricity to earth will take place from any point on it indifferently (*see* Chapter IV. § 14, Experiment (ii.)).

(*e'*) If A be now discharged, or reduced to the level of the earth's potential, our electrical hill disappears. It is now found that B is at a — potential; and + electricity will flow from the earth into it until it be again at the earth's potential.

(*f'*) Let an uncharged insulated cylinder, capable of division about its centre (*see* Chapter IV. § 14, Experiment (iii.)), be placed between A and the walls. It will now exhibit a + charge at one end, and a — charge at the other. If the two halves be separated and A be removed or discharged, we shall find these halves to be above and below zero potential respectively. On again connecting the two halves the whole resumes its original condition of zero potential.

(*g'*) When a body is in a region of — potential let us give it such a + charge as will raise it to zero potential. If the region rises in potential, the charge given to the body will cause it to be now above zero potential.

(*h'*) So when a body is in a region of + potential let us give it such a charge of — electricity as will reduce it to zero potential. If the region now sink to zero potential, the body with its — charge will be below zero potential.

From what we have said above it is clear that all cases of *induction* can be regarded as *electrical redistributions that take place on conductors when placed in regions of varying (or unequal) potential: these redistributions being such as to maintain the conductor at the same potential all over.*

Note.—Charging the gold-leaf electroscope by induction.—In Chapter IV. § 14 (iv.) we indicated how we could charge the gold-leaf electroscope inductively. It is now time to explain more exactly what occurs.

Let the electroscope be uncharged, and let a +ly excited rod be approached. The electroscope will be in the field between the rod and the earth, and it will be at a + potential. There will, therefore, be a field of force between the leaves and the wall; and + electricity will be ready to flow from any part of the instrument to earth. The leaves will, therefore, move from each other towards the walls, or will 'diverge with + electricity.' If now we touch the instrument with the hand, we reduce it to zero potential, and the leaves collapse; it now has a — charge sufficient to keep it at zero potential

in the presence of the + charged rod. If the hand be removed, and then the rod, the instrument will be at a − potential in virtue of its − charge, there being now no region of + potential to be counteracted ; and the leaves will 'diverge with − electricity.' The approach of a − charged rod will cause its − potential to be still greater, and the leaves will diverge more. But the approach of a + charged rod will again cause the region about the electroscope to rise in potential. Hence, according to the strength of charge of the + ly excited rod and the nearness of its approach, the electroscope will become less −, or zero, or even + in potential ; and its leaves will diverge less, collapse, or 'diverge with +,' in the three cases respectively.

§ 7. **Necessity of distinguishing Sign of Charge and Sign of Potential.**—Where we have an isolated charged body, there the potential of the body is + or − according as the charge is + or − in sign. For we do + or − work respectively in moving our test + unit of electricity from the earth to the body. But if we have more than one charged body we cannot predict the sign of the potential of a body from its charge. Thus if we have a pail charged +, a ball hung inside it may be at a + potential even if it be itself charged − ; for, owing to the presence of the pail, it may require + work to bring our test + unit of electricity from the earth up to the ball.

So again, in Chapter IV. § 14, Experiment (ii.), the conductor C D was at zero potential although it had a + charge on it.

§ 8. **Further on Capacities.**—When a body is charged with a quantity + Q, there is a complementary quantity − Q on the surrounding surfaces.

If these surroundings are so far off that this charge − Q exerts no perceptible force on our test-charge of a '+ unit' as we carry it from the earth up to the body, then the work done on our '+ unit' as we so carry it will be independent of the surroundings; it will depend only on the charge Q of the body, and on the shape and size of the latter. And the potential of the body, measured by this work, will also depend only on its shape, size, and charge.

In such a case the body is said to be *isolated*, and it has a *definite capacity.*

But if the surroundings be not so far removed as to make the above condition hold, then the work done on our '+ unit' will depend in part on the position of the surroundings. In such a case the potential of the body depends not only on its shape and size and on the charge given to it, but also on the position of

ırrounding surfaces. The body is then not isolated ; and we :ust consider the *whole system* of body and surroundings.

Note.—A little consideration will show that if a body previously 'isolated' ≥ brought near (*e.g.*) to the walls of the room, most of the complementary - Q will now be on the part of the walls near to the body. This will have the ffect of lessening the work required to bring our + unit up to the body. In ther words, *the nearer approach to the walls* will *increase the body's capacity.*

CHAPTER VI.

ELEMENTARY DISCUSSION OF CONDENSERS.

(For further, see Chapter X.)

§ 1. **General Ideas. Apparatus used.**—In this chapter we consider the alteration of the capacity of a conductor as the distances and positions of surrounding conductors alter. Or, more accurately, we consider the electrical capacity of the system of conductors taken as a whole.

We shall use the following apparatus.

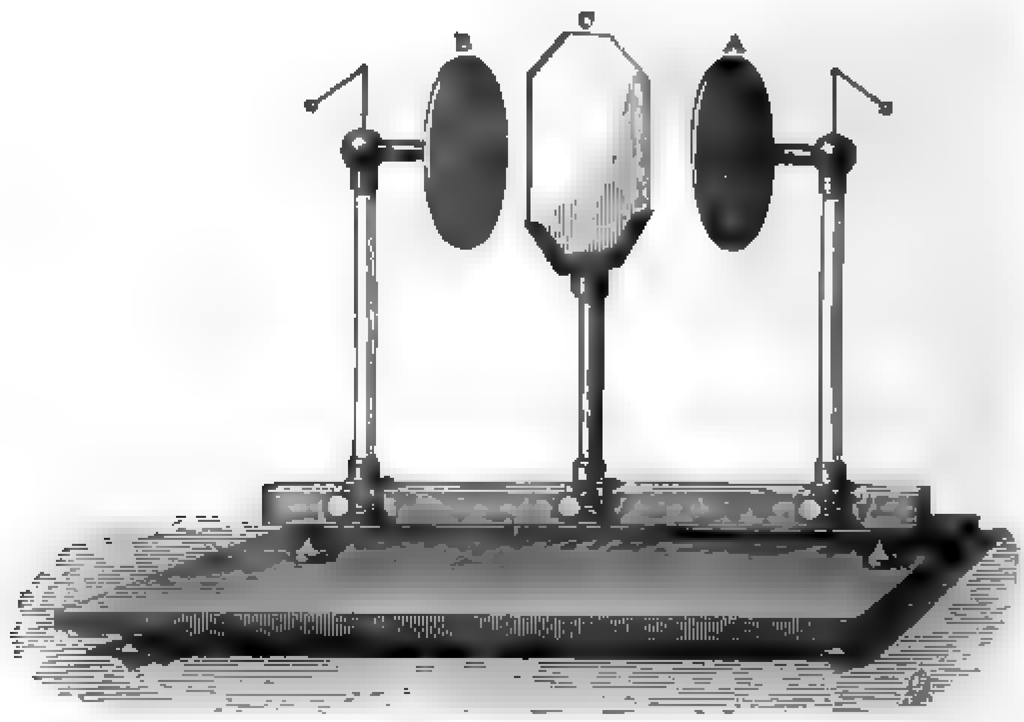


FIG. 1.

(i.) *The sliding condenser plates.* (*Æpinus's condenser.*)—A and B are two discs of metal, having well-rounded edges. They are

insulated by being fixed on to elbow-bent rods of glass. These rods are set in stands that run, guided by rails, along a board three or four feet in length.

(ii.) *A form of Peltier electrometer.*—Insulated inside a glass shade, the method of insulation being indicated in the accompanying figure, is a frame of wire *a b c d.* Balanced on a steel point is a magnetic needle *n s*, provided at the ends with gilt pith-balls.

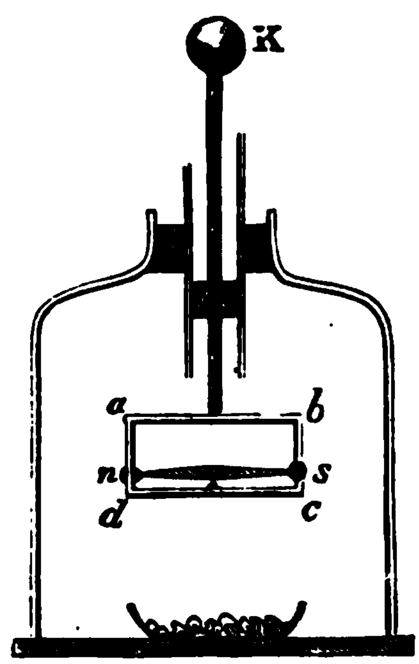


FIG. ii.

The apparatus is so placed that the needle, as directed by the earth's field, rests with the pith-balls just lightly touching the sides of the brass frame.

When the knob K is connected with a conductor at a + or – potential, the frame and needle all become of a + or – potential with respect to the earth ; there is an electric field, and the needle is urged away from the frame towards the walls.

Or, in ordinary two-fluid language, if we connect K with a conductor that we have charged + or –, we shall have the needle urged away from the similarly charged frame towards the walls of the room ; and it finally rests in such a position that the couple due to the electrostatic field balances the couple due to the earth's magnetic field.

(iii.) We shall also use a – charged ebonite rod with which to test the sign of the charge that may be at any time causing the needle of the electrometer to be deflected ; this testing having been explained earlier.

We may add that the electrometer can be made more or less sensitive by weakening or strengthening the magnetic field acting on the needle, by means of a controlling magnet, as explained in Chapter XVII. § 9.

(iv.) *Source of electricity.*—For charging the plates we shall use an electrophorus or frictional machine ; and shall assume that it is so regular in its action that it is of a constant potential. That is, the machine will continue to give a charge to a conductor, cease to do so, or receive a charge back from the conductor, according as this latter is below, up to, or above, this fixed potential of the machine respectively.

The analogy would be a water-supply kept at a certain fixed level.

§ 2. **Experiments with the two Condenser Plates.**—In the following experiments the two plates A and B are connected with the electrometers E_A and E_B respectively, and are insulated, unless the contrary is stated.

Experiment.—(i.) Let B be at some distance, say ·5 *mètre*, from A, and let both plates be initially uncharged. Now charge A with (say) + electricity, until it will receive no further charge from the source. It will be found that

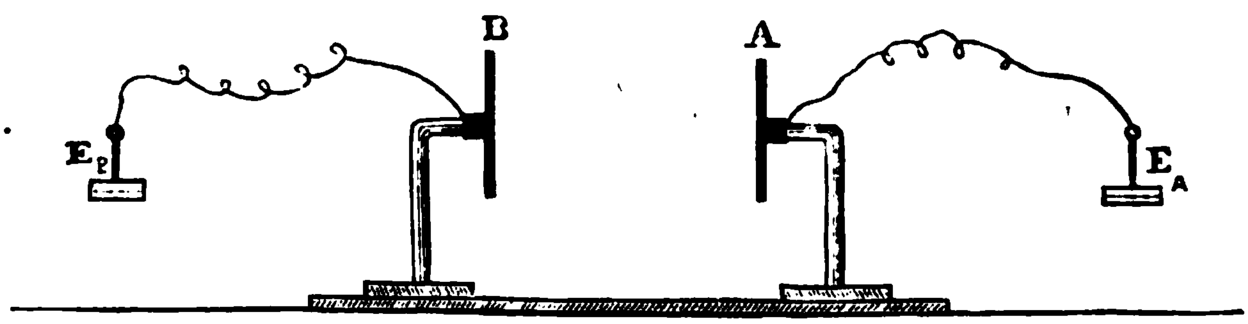


FIG. i.

the needle E_A is deflected with + electricity. At the same time the usual proof-plane test will show that on the face of B turned towards A there is – electricity; while E_B is deflected with + electricity.

In two-fluid language we should say that the + charged body A induces a – charge on the face of B turned towards it, repelling the complementary + to the further side of B and to E_B (*see* Chapter IV. § 14).

In potential language we should say that there is on B and E_B such a redistribution of the total charge (whose algebraic sum is zero) as to leave this system at one potential. Since the whole system is between a + charged body A and the earth, it must be at a + potential, since it would take work to bring our + unit from earth up to B. Hence + electricity tends to run from all parts of the system to the earth; and hence the needle of E_B is urged down an electric field between the brass frame and the walls.

Experiment.—(ii.) Put B to earth, by touching any part of it. It will be seen that E_B returns to *zero* deflexion, while E_A is deflected less than it was before, and further that A will now receive more charge from our source.

In two-fluid language we can say that we have allowed the repelled + to pass away. In consequence of this, A has further inductive action on B; so that there is more – on B than

fore, and more + passes to earth. Corresponding to this larger — charge on B, there is drawn over to the surface of A that faces more of the + charge on A; and, as a consequence, E_A is less deflected, and the whole will receive further charge.

(The reader may here look at the diagram given at end of this section. It represents roughly the effect that the presence of B has on the distribution of the charge on A.)

In potential language we may say that we have put B and E_B to zero potential. There is therefore no field of force between the brass frame of E_B and the walls, and hence the needle has no couple deflecting it.

Now, there was in (i.) on the side of B turned towards A a certain — charge. This by itself would have made B of a — potential; but, owing to the presence of the + charge on A, B was yet at a + potential. In (ii.) we have B at zero potential, since it is to earth. It seems clear, then, that there must now be a larger — charge on B than there was before, since it is as close to the + charge of A as it was before and is yet at zero potential.

For a similar reason to that indicated in Chapter V. § 8, *note*, the presence of this larger — charge near A will lessen the work required to bring a '+ unit' up to A. In other words the potential of A will be in this case (ii.) lower than it was in case (i.).

Experiment.—(iii.) Re-insulate B, and approach it nearer to A. We shall find that E_B again diverges with +, while E_A falls a little in its deflexion.

And further, as we should expect from the falling off of the deflexion of E_A, we can give A more charge from our source.

In two-fluid language we have a further separation of + and — on B, the + being repelled to the regions (such as E_B) remote from A. There is also in A a further concentration of the charge on the face turned toward B, and consequently a weakening of the charge in E_B. We should thus expect to be able to give more charge to the whole, A and E_A.

In potential language we have moved B higher up the 'electrical hill' that exists between A and the earth, or into a region of higher potential than before. If, before, its charge of — electricity was such as to keep it just at zero potential, this charge will be insufficient to keep it at zero now that it is nearer to the + charged A. It will therefore again become of a + potential, or + electricity

will be ready to flow from it to the earth. The electrometer needle is again in an electric field, and is again deflected.

At the same time the potential of A will be lowered for a reason similar to that given in case (ii.).

Experiment.—(iv.) Next put B to earth. We find that E_B is no more deflected, E_A falls slightly.

The discussion of this will be similar to that of Experiment (ii.).

Experiment.—(v.) Again insulate B and remove it further from A. The needle E_B is deflected with − this time, while E_A is deflected more strongly with +.

In two-fluid language the removal of A and B from each other causes some of the + and − electricities, concentrated on the opposed faces of A and B respectively, to pass away from these faces to the other portions of the conductors, and so to E_A and E_B.

In potential language the potential of A will be raised, for a reason similar to that given in cases (ii.) and (iii.). There will be a stronger field between the wire frame of the electroscope and the walls, and the needle will be more deflected.

With respect to B, the − charge that kept it at zero potential when in the nearer presence of A, is too much for this purpose when A is removed to a greater distance; it renders B − in sign of potential; and the − charged needle of the electrometer is in an electric field running down from the walls to the frame, and therefore moves up this field away from the brass frame.

It is important to note that by charging A when B is near, and by then removing B, we can get in A a charge of a much higher electrical potential, or level, than that of the source from which we charged A initially.

Experiment.—(vi.) When A B are very close and B is to earth, the charge that can be given to A [roughly measured by counting the number of sparks given] will be seen to be very large indeed as compared with the charge that can be given to A when isolated.

The proof-plane, if it can be used, will show that this charge is mainly on the face of A turned towards B; while on the opposed face of B is an opposite charge of equal magnitude (*see* § 4).

If B be pushed still nearer, a brilliant blue spark will be seen to span the gap between A and B, and a strident 'crack' will be heard.

If a person holding B touch A, this discharge will pass through him and will give him a shock. Such an experiment must, however, be tried with caution.

The accompanying figures show roughly the cases of A charged when isolated, and of A and B acting as a condenser, respectively. The crowding of the signs + and − is meant to indicate the concentration of charge.

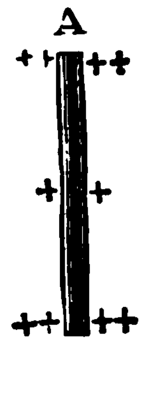


FIG. ii.

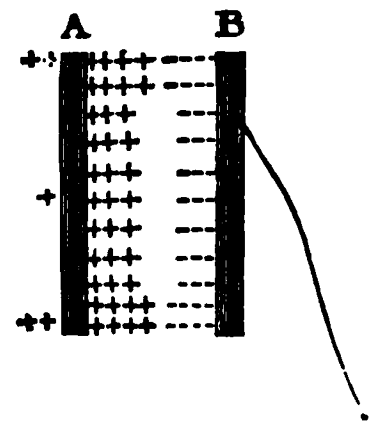


FIG. iii.

§ 3. **Discussion of the Terms 'Bound' and 'Free.'**—We have seen that when a body is charged with a certain quantity, say + Q, of electricity, there is induced on surrounding objects an equal quantity of opposite sign, or − Q (*see* Chapter IV. § 16). The distribution of this quantity + Q on the conductor depends on its form and on the relative positions of surrounding bodies.

It is found to be part of the essential nature of 'electricity' that a + or − charge does not exist alone; it is always the one or other side of an electrical field.

Now the charge on the plate A (*see* figure at end of § 2) induces on the whole an equal and opposite charge on the walls, &c., and the greater part of this will be on that surface of B which is opposed to A; since B is by far the nearest part of the earth that is presented to A.

If we now cut off B from the earth and insulate it, without otherwise disturbing the system, and then put A to earth, what will happen?

To begin with we may say roughly, but as we shall see in § 6 not quite exactly, there will pass away from A that part of the charge that can, by so passing away, get at and neutralise the corresponding part of the induced − charge. If B were still to earth, then all the charge would pass from A, and the two sides of the field (the equal and opposite + and − charges) would close in and leave no field or charge; the + and − would entirely 'neutralise' each other. But B has, we suppose, been insulated again before A is put to earth. Hence, to speak roughly, that part of the charge on A which answers to the charge induced on B, will have no tendency to pass away, since it cannot thereby get at the − charge on B, which is the opposite side of its field. But if we stood on B, insulated from the earth, and then touched A, we should withdraw from A all that part of the charge answering to

the — charge induced on B; while on A would remain that part of its charge which has its corresponding induced charge on the walls and ceilings, &c., from which we are now insulated. When B is insulated, therefore, we can divide the charge on A into two portions; *one* part can be withdrawn by a person in connection with the walls, ceiling, &c., and unconnected with B, but cannot be withdrawn by a person connected with B and insulated from the walls, &c.; the *other* part can be withdrawn by a person connected with B and unconnected with walls, &c., but not by a person insulated from B. The former is *free* with respect to any person connected with the walls, &c., the latter is *bound* with respect to such a person.

This is the origin of the terms *bound* and *free*; the reader will see in how relative a sense they must be understood.

§ 4. **Conditions affecting the Magnitude of the 'Bound Charge.**—When B is very close to A we may neglect the 'free' charge on A as relatively insignificant (*see* § 3). The 'charge' of the condensing system, composed of A and B, will then mean the 'bound charge' residing on that face of A that is turned toward B; the inseparable accompaniment of an equal and opposite charge on B will be understood, but not generally mentioned.

The magnitude of the charge can be experimentally shown to depend on several conditions.

(i.) *On the size of the plates* A *and* B.—If these are equal in area, and if the distance between them be very small as compared with the diameter of either, then the charge will be (approximately) directly proportional to the area of the plates.

(ii.) *On the distance between the plates* A *and* B.—The nearer the plates, the greater the charge. And, with the condition just given, the charge is inversely proportional to the distance between the plates.

(iii.) *On the difference of potential between the plates* A *and* B.—We have not yet explained exactly in what units we measured electrical level or potential; but we have sufficiently indicated the nature of the unit employed. For further, the reader must wait for Chapter X. §§ 9, 19, 25, &c. We may, however, here state that the charge is directly proportional to the difference of the potentials of A and B.

(iv.) *On the nature of the dielectric* (*see* note).—If the charge is

be a certain magnitude Q when the dielectric is air, then *cæteris paribus* it will be $\sigma.Q$ when the dielectric is some other substance. This σ is (as we shall see in Chapter IX.) the *specific inductive capacity* of the particular dielectric. If then Q represent the numerical value of the bound charge, V_1 and V_2 the potentials of the plates A and B, and K be a quantity including the area of the plates, the distance between them, and the specific inductive capacity of the dielectric, and called *the capacity of the condenser*, then we shall have the formula . . . .

$$Q = K.(V_1 - V_2);$$
$$\text{or } Q = K.V_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

when the outer coating is put to earth and so is at zero potential.

Note.—'*Dielectric.*' Any non-conducting medium, be it solid, liquid, or gas, that intervenes between two conductors, is called a *dielectric*. It is across *dielectrics* that electrostatic attractions and repulsions are exerted; and it is in dielectrics that fields of electrostatic force exist. The usual dielectric is *air*; but in certain instruments *glass*, or *ebonite*, is the dielectric.

§ 5. **An Isolated Body considered as the Limiting Case of a Condenser.**—If the plate B be at zero potential, then the formula becomes

$$Q = K.V$$

where V is the potential of A.

Now, as stated in Chapter V. § 8, it can be shown (*see* Chapter X. § 19) that when B and all other surrounding bodies are at a distance from A very great as compared with the dimensions of A, then any further change in their positions and distances will make no difference in the capacity of A; that is, under such conditions the value of the capacity K depends only on the size and shape of A.

We thus arrive at our old formula for isolated bodies, viz.:

$$Q = K.V; \text{ or } V = \frac{Q}{K}.$$

§ 6. **Alternate Discharge.**—Let us have two equal plates A and B very close together, B being put to earth, and A being charged to its fullest extent; and let us call the charge on A, *unity*.

The total induced charge answering to the charge $+1$ on A will be -1; and of this the larger part by far will be on B. Let

this larger part be called m, where m is some large proper fraction such as $\frac{99}{100}$; the other $\frac{1}{100}$ being on the walls, &c. Then as long as A is untouched we can say that a charge $+1$ on A 'binds' a charge $-m$ on B, in the sense that we cannot withdraw it from B as long as A is insulated.

In the same way, since the plates are equal and similarly situated, a charge -1 on B would 'bind' a charge $+m$ on A. Or, more generally, a charge Q on either would bind a charge $-m$Q on the other.

In the case we have supposed, the charge $+1$ on A binds $-m$ on B. This charge $-m$ on B can bind only $+m \times m$, or $+m^2$, on A.

Hence, if we insulate B and touch A, putting it to earth, we withdraw an amount $+(1-m^2)$ from A, and leave on it $+m^2$ bound by the $-m$ on B. The reader may put *e.g.* $\frac{99}{100}$ for m.

Insulating A and touching B, we withdraw from B what A does not bind; that is, since the $+m^2$ left on A will bind $-m \times m^2$, or $-m^3$, on B, we leave $-m^3$ on B and withdraw $m - m^3$ from B, and so on.

Each time that one plate is put to earth, part of the charge on the other plate becomes 'free.'

This process of alternately putting either plate to earth is called the *alternate discharge*.

We give here a table.

Withdrawn from A +	On A +	Binds	On B −	Withdrawn from B −
	1			
$1 - m^2$			m	
	m^2			$m - m^3$
$m^2 - m^4$			m^3	
	m^4			$m^3 - m^5$
$m^4 - m^6$			m^5	
	m^6			$m^5 - m^7$

Note.—As regards the exact meaning of m, the reader will understand the following explanation better when he has read Chapter X.

Let us consider the plate A only. There is on the inside surface next to B a bound charge Q; and on the outside surface, opposite to the relatively distant walls, a relatively insignificant charge Q′. The charge Q raises the inner surface, and the charge Q′ raises the outer surface, to a potential V_A, which is necessarily the same for the two surfaces. If then K is the capacity of the inner surface, and K′ the capacity of the outer surface, we have the relation $V_A = \frac{Q}{K} = \frac{Q'}{K'}$. We may call K the 'bound capacity' of A, or the capacity of the condensing system, its magnitude depending on the shape and size of A and also on the position &c. of B. We may call K′ the 'free capacity' of A—or rather of its outer surface; its magnitude depends on the shape and size of this outer surface, but on nothing else provided that the walls &c. are so remote that the outer surface of A may be considered to be 'isolated' with respect to them.

From the relation given above we easily obtain the result that

$$Q = \frac{K}{K+K'}(Q+Q').$$

Remembering then that Q + Q′ is the total charge on A, and that the − charge on B equals Q in magnitude, we see that $m = \frac{K}{K+K'}$.

Experiments.—(i.) With the plates A and B, each connected with a Peltier electrometer, the alternate discharge can be shown; though the arrangement is not one adapted to give indication for more than a few discharges. As we touch A we receive a spark, and E_B, previously undeflected because at zero potential, will be deflected with − electricity, B and E_B being now at a − potential, while E_A will fall to zero deflection. If we now touch B, E_B falls to zero and E_A is deflected with +, and so on.

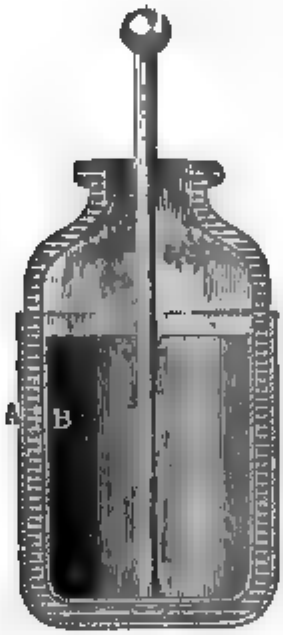
(ii.) A Leyden jar (see next section) is mounted on an insulating stand. While it is being charged the outer coating is put to earth, and then the whole is left insulated.

We can, by *alternately* touching the knob connected with the inside, and the outside coating, draw off sparks of + and − electricity in turns. The signs of these alternate charges can readily be tested as in Chapter IV.

§ 7. **Leyden Jars.**—When it is required merely to store a large charge at the potential of the source, and when there is no need to alter the distance between the plates, the condenser usually takes the form of a jar of glass having inside and outside coatings of tin-foil. A knob gives connection with the inside of the jar; this knob should, if possible, pass up from the inside without any connection with the neck of the jar, in order to insure good insulation.

Since the charge, *cæteris paribus*, is proportional to the differ-

ence of potential of outside and inside, it should be our object to make this difference as large as possible. As a rule the highest available potential is that of our source, and with this we connect the knob; the lowest available is the zero potential of the earth, and so we put the outside coating to earth. A larger charge would be obtained if we could put the outside to a − potential.



When the outer coating is to earth, the charge of the jar is determined by the formula

$$Q = K \cdot V$$

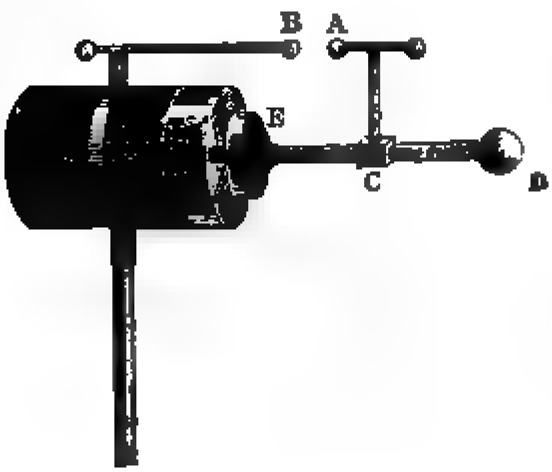
where K is the capacity of the jar, and V is the potential of the inner coating.

Glass is used for several reasons. It is easy to make glass jars; glass has a high specific inductive capacity (*see* Chapter IX.); and the jar can be made thin, or the distance between the plates small and therefore the capacity of the jar large, without a discharge taking place through the glass.

For the calculation of K, as well as for the units in which V is measured, the reader is referred to Chapter X.

§ 8. **The Unit Jar.**—If we can fix the capacity of a jar and the difference of potential between its two coatings, then we have fixed the charge in that jar.

A unit jar is a small Leyden jar of some convenient shape; usually so made as to admit of easy cleaning and drying, and to insulate well.



Connected with the outside is a knob B, and connected with the inside is a knob A, whose distance from B can be regulated by sliding the piece A C along the graduated rod D E.

It is proved experimentally that the distance between A and B across which the jar will just discharge itself is directly proportional to the difference of potential between A and B.

(It depends also on the curvature of the knobs, and on the medium between A and B; but we suppose these conditions to remain constant.)

Hence, if we keep the distance between A and B fixed, the difference of potential between A and B at the moment when the jar discharges itself in a spark across the gap A B will also be fixed. Let us now put the outer coating of the jar to earth, put A at a fixed distance (that we will call 1) from B, and charge the jar by placing the knob D near the prime conductor of a machine; and let us suppose that it takes a difference of potential measured by v_1 for a spark to pass from A to B.

Then on charging the jar we shall shortly perceive a brilliant discharge to pass from A to B; this discharge leaving the jar uncharged. At the moment of this discharge there was on the inner coating a charge $+ Q_1$ determined by the relation

$$Q_1 = K (V_A - V_B)$$

or, by hypothesis, $$Q_1 = K v_1.$$

There was, on the inner surface of the outer coating, a charge $- Q_1$; and there *had passed to earth* a quantity $+ Q_1$.

Hence, every time that the jar discharges itself, the $+ Q_1$ and the $- Q_1$, that are opposed to each other, neutralise each other; while there *has passed* to earth a quantity $+ Q_1$.

If we put A and B at a distance apart twice as great as before, or at a distance 2, then we have $Q_2 = K . v_2 = K . 2 v_1 = 2 Q_1$.

Let us now see how the unit jar is used.

We connect the outside coating of our small unit jar with the inside of a large Leyden jar whose outer coating is to earth.

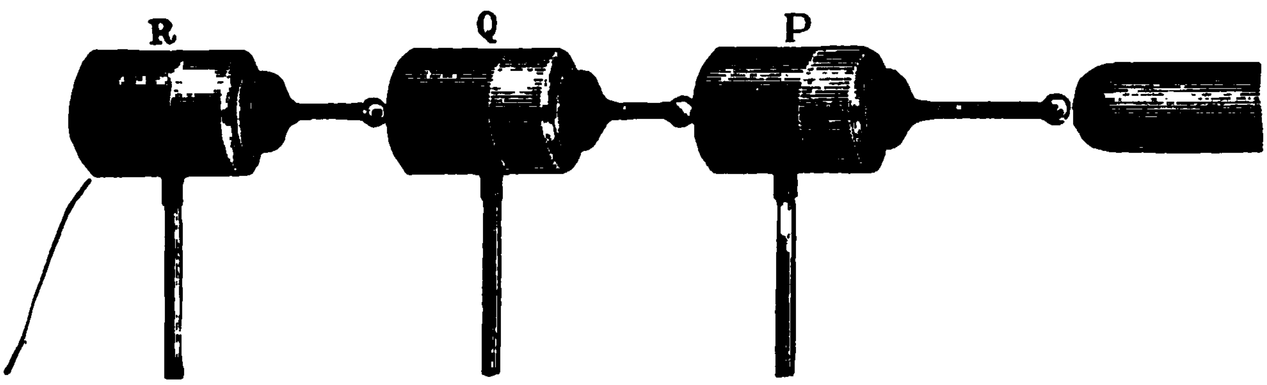
If we now work the machine, then each time that the unit jar discharges itself we know that a certain quantity $+ Q_1$ of electricity *has passed away* from the outside of the unit jar into the large jar. And so, by counting the sparks of discharge of the unit jar, we are able to give the large jar a charge represented by Q_1, $2 Q_1$, $3 Q_1$, &c.

Comments and notes.—(i.) We ought to have the unit jar so small as compared with the jar to be charged, and the knobs A and B at such a distance apart, that a considerable number of discharges of the unit jar may take place by the time that we have given the required charge to the large jar. For we cannot stop charging the moment that a spark passes, and therefore there will always be a *fraction of* $+ Q_1$ *in excess* given to the large jar.

(ii.) We have said that, at each discharge of the unit jar, a quantity + Q has passed into the large jar. This is not accurately true. The outside of the unit jar is a small conductor at a considerable distance from surrounding bodies, and the inside of a large jar is a relatively large conductor having the outer coating of the jar very near it and opposed to it. These two form one conductor at one potential, since they are connected; but the capacity of the first is negligibly small as compared with that of the second. The charge + Q distributes itself between these two portions of the conductor in the ratio of their capacities. Hence, we may say that *very nearly* the whole charge, that would have passed to earth if the outside of the unit jar had been to earth, will have passed into the large jar.

(iii.) It requires a certain difference of potential v for a spark to pass between A and B. If then A is connected with the prime conductor of the machine, and B with the inside of a large jar, we cannot charge this latter up to the full potential of the prime conductor, but to a potential less than this by the amount v.

§ 9. **Cascade arrangement of Leyden Jars.**—P, Q, and R are three Leyden ars mounted on insulated stands, arranged 'in



cascade' as indicated in the figure. These jars must be equal in capacity. To insure this we may take one jar of many that are presumably equal, and then by comparison choose two or more others equal to it. If we give a certain charge to our jar chosen, and measure the potential of the inside (when the outside is to earth), this potential ought to fall to *one-half* when the knob is connected with that of an uncharged equal jar. This method is employed in Chapter IX. § 4.

But for rough experimental purposes it will be sufficient to choose the jars as follows. Let an electrical machine be worked for some time so that it has got into a constant condition. Then charge the jars in question with a *unit jar* interposed, as shown in the last section. If the jars refuse further charge after the same number of discharges of the unit jar have taken place, then their capacity will be approximately equal.

Having arranged our three (or more) jars as shown, let us work the machine.

When no more charge will pass, then the inside of the first jar will be at a potential V_1; which will be that of the prime conductor, or something less, according as the knob is or is not in contact with the prime conductor. The outside of the first jar and the inside of the second jar (in contact with the former) will be at some lower potential V_2. The outside of the second jar and the inside of the third will be at a still lower potential V_3; and the outside of the third will be at zero potential, which we may call V_0 or zero-V, as we please.

Now, as shown in the last section, if there be in the first jar a 'bound' charge $+Q$, then there has passed into the second jar also a quantity $+Q$, and from the outside of this into the third jar also a quantity $+Q$, if we neglect the trivial 'free' charges on the outside of the first jar and on the knob of the second jar, and so on.

That is, in the cascade arrangement we have of necessity equal charges in the jar. But by Chapter VI. § 4 (explained further in Chapter X.) we have

$$\begin{cases} \text{for first jar } Q = K.(V_1 - V_2) \\ \text{for second jar } Q = K.(V_2 - V_3) \\ \text{for third jar } Q = K.(V_3 - V_0). \end{cases}$$

So that, since Q and K are the same for each jar, we see that

$$V_1 - V_2 = V_2 - V_3 = V_3 - V_0 = \tfrac{1}{3}(V_1 - V_0).$$

Hence the total fall of potential that is possible, viz. the fall measured by $V_1 - V_0$ (or by V_1, since we take V_0 as our zero), has been broken up into three equal falls.

(This is somewhat like the breaking up of a large waterfall into three smaller ones.)

Now let us consider the charge Q' that is given to *one* of these jars when its outside coating is to earth and its inside coating is at the potential V_1 of the prime conductor; *i.e.* its charge, when it has been charged in the usual way. By our formula we shall have

$$Q' = K.(V_1 - V_0).$$

But by what has preceded we see that this equals $3Q$. Hence $Q' = 3Q$.

We see then that the sum of all the charges given to the jars, when these are arranged in cascade, equals the charge given to a single jar when this is treated in the usual manner.

The above reasoning can readily be extended to any number *n* jars. For further on the 'cascade arrangement' the reader is referred to Chapter X. § 30.

§ 10. **Nature of the Leyden Jar Charge.**—It is now time to investigate, as far as we are able, what is meant by the expression 'an electric charge.' All that we have hitherto observed indicates that neither '+ charges,' nor '− charges,' can exist by themselves; but that there must always co-exist two equal charges of opposite sign. In fact, it would seem that in all electrostatic phenomena there must be a + charge and an equal − charge, separated by some insulating medium called a *dielectric*. It is probable that in the case of two conductors thus separated by a dielectric it is the surfaces of the dielectric in immediate contact with the conductors that are in the condition that we have called '+ charged' and '− charged' respectively; though it is usual, and perhaps more convenient, to speak of the conductors themselves as so charged.

Various experiments tend to show that the dielectric is, under these conditions, subjected to a *stress*; and to this it yields to a greater or less degree, becoming deformed or *strained*.

The conductors appear to play somewhat the following part. They mark out the portion of the dielectric that can be converted into an electric field and can be put under a stress; and they allow this stress to be imposed or taken away with rapidity. If the stress be continued for a sufficient time, it is found that all solid dielectrics become strained (or distorted) to an appreciable distance from the conducting surfaces; and that then the strain cannot at once be removed, or the 'condensor' cannot be at once discharged. Some of the stress remains and must be allowed to be relieved gradually. This gives rise to the 'residual charges' discussed below.

Experiments.—(i.) *Leyden jar with moveable coatings.*—A is a Leyden jar so constructed as to be separable into three portions; B the glass jar, D the inside coating, and C the outside coating.

The jar is put together, charged as usual, and then placed on an insulating stand. The inside is then removed, a small charge coming away with it, and

a small charge being 'set free' on the outside coating. Then the glass is lifted out of its outer coating and set down on the table, and finally the outer coating may be handled and removed. In all this we notice but very slight discharges on touching D and C respectively.

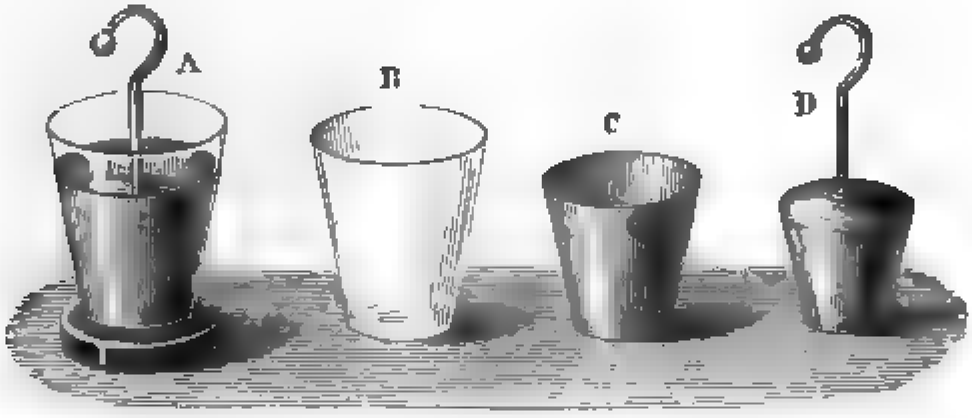


FIG. i.

On carefully putting the whole together again (on the insulating stand), a discharging rod (fig. ii.) will show us that the whole charge of the condenser has been (approximately) unaffected by the above process. Hence, since the coatings B and C were separately put to earth by handling, the charge of the condenser must have resided in the glass. We could not, however, discharge the glass by itself; it was necessary to have the metal coatings as distributors.

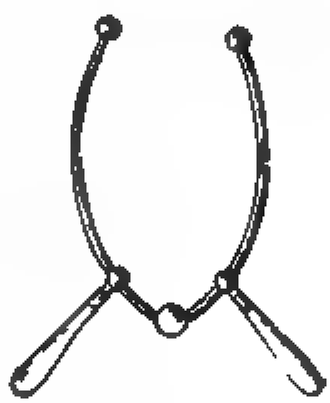


FIG. ii.

(ii.) *Residual charge.*—The penetration of the charge, from the surfaces inwards, is shown by the following.

If a Leyden jar be discharged and then left for a time, a second small discharge can be obtained, and so on.

That the interior of the glass is actually strained or distorted is shown by the fact that its optical properties, when it is under the electrical stress, undergo an alteration similar to or identical with that produced by mechanical stress.

When the electrical stress is excessive, the material may give way and a hole may be made; through this a discharge takes place, since the air which then separates the two charges is far weaker to withstand the stress than is glass.

§ 11. **Various Effects of the Discharge.**—The Leyden jar gives us a means of observing the effects of electrical discharge. There is nothing essentially different between the discharge between the prime conductor and the floor and walls, where was collected the induced charge of opposite sign, and the discharge between the inner and outer coatings of the Leyden jar. Only the latter

gives us a means of storing up quantities, ready for discharge, enormously greater than we should get did we discharge the prime conductor only.

I. *Mechanical effects.*

Piercing card, &c.—In several of these experiments we make use of a discharging table. Its construction is evident from the figure. One rod, A, is connected with the outside C of the jar, or battery of jars, and the other, B, can be connected with the inside D of the jar

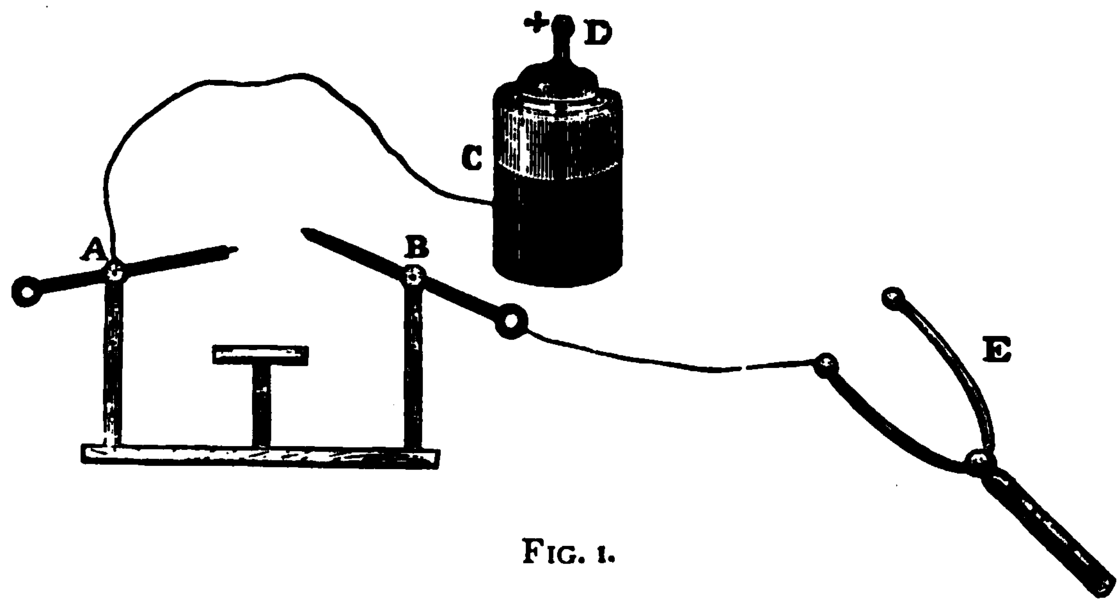


FIG. 1.

or battery by means of an insulated discharging rod E. The battery is charged, a piece of card placed between the points of the rods A and B, and then connection made.

If the pierced card be examined, it will be seen that the hole has been apparently formed by an explosion from inside, due to the sudden expansion of air, or to sudden vaporisation of water in the inside of the card. Whatever the cause, it may help the reader to remember that 'the passage of a spark' is an expression hardly authorised by our knowledge; the discharge may take place from + to −, or from − to +, or may be a series of alternate discharges, or may be of some dual nature not represented by any of the above suppositions.

II. *Magnetic effects.*

Experiment.—(i.) In fig. ii., A B is a wire along which the discharge can be effected. If we, for convenience, agree to say that by the 'direction of a discharge' we mean that direction in which the + charge would move in order to meet the − charge, then in fig. ii. the discharge is from B to A. This expression, 'direction of discharge,' is not here assumed to have a real physical meaning, but is employed in order to save confusion.

n s is a piece of steel placed at right angles to the wire B A, and underneath it. After discharge it will be found that the steel is magnetised in the way shown. That is, *if we swim with the + electricity and face the steel bar, the induced north pole will be found at our left hand.*

Experiments of this sort, and similar ones made with steel filings on glass, show us that round a wire bearing a discharge are lines of magnetic force: they are circles round the wire as axis, and their + and – direction is clearly given by the italicised rule above. (Compare also Chapter XVII. §§ 1 and 2.)

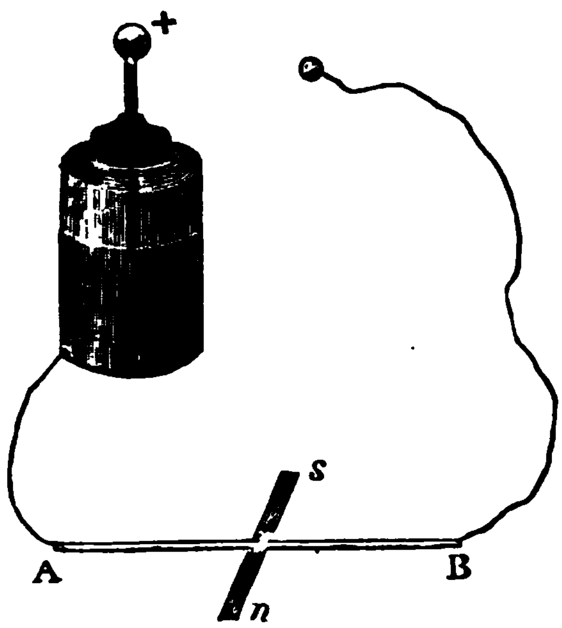


FIG. ii.

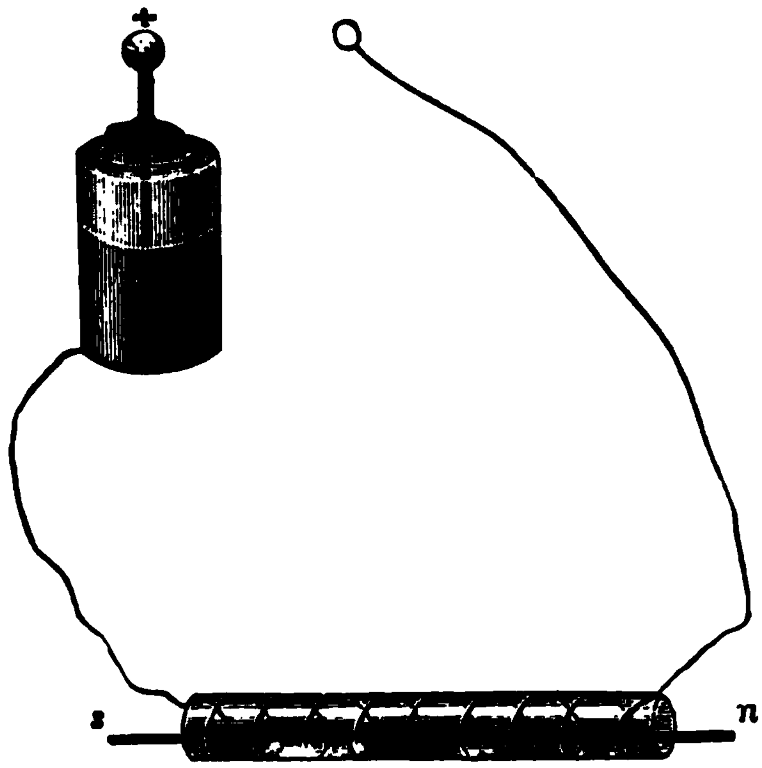


FIG. iii.

The reader can verify this rule when he has determined the sign of the charge on the prime conductor of his machine.

(ii.) We may (*see* fig. iii.) intensify the effect by coiling a wire round a glass tube in which lies the steel needle. The above rule will be found true, and it will be seen to be equivalent to saying that *if we face one end of the spiral coiled round the tube, the needle will have been made n or s at that end, according as the discharge so viewed is against or with movement of the hands of a watch.* A reference to Chapter XVII. § 2 will make the rule clear.

This experiment is one of the many that link together the electrostatical and electrodynamical divisions of our subject.

III. *Heating effects.*

Experiment.—(i.) The brilliant light of the discharge, and the fact that discharges over the dust of various metals scattered over a glass surface give the well-known spectrum of each metal respectively, indicate that in the discharge there is an elevation of temperature sufficient to volatilise metals and other bodies, and to raise the vapours to brilliant incandescence.

By experiments easily contrived we may reduce a platinum wire to fused drops; or may volatilise gold-leaf between two well dried, and therefore insulating, cards.

Experiment.—(ii.) *Ignition of gunpowder.*—If we place gunpowder on the small table of the discharger, between the points of the discharging rods, we find that the discharge merely scatters it.

But if we interpose between A and C a substance, such as wet string, which

conducts worse than does a metal, we find that the discharge will ignite the powder. We conclude that this retards the discharge and gives sufficient heating effects without such violent mechanical disturbance.

IV. *Chemical effects.*

The subject of chemical decompositions may be left until we come to it under the head of 'Phenomena of electric currents,' in Chapters XI. and XII.

It will suffice here to say that if we keep to the convention given above, as to which we shall call the direction of discharge, then the chemical phenomena given later on in Chapter XI. could be repeated with similar though less striking results if we substituted a series of discharges for the continuous currents employed in that chapter.

§ 12. **Induction Effects of the Discharge.**

Note.—This section may be omitted until the student has read Chapter XXI.

We will first give a general account of the phenomena that are the subject of this section, and will then describe how these phenomena may be observed.

If a discharge be effected along a wire B A, it is found that at the moment of discharge there passes along a wire C D, placed parallel to B A and near to it, an induced discharge.

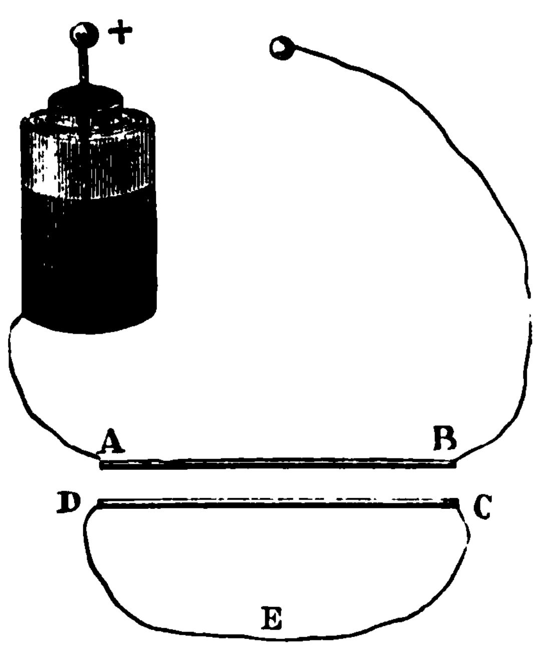


FIG. 1.

Further examination, in which we are aided by results of experiments with the more manageable currents given by voltaic batteries, shows us that the phenomenon is not a simple one.

It is found that in C D are induced *two* rushes of electricity; one answering to a discharge of + electricity from D to C, one in the contrary direction.

It would seem that these two induced rushes of electricity are equal in total amount of electricity; but that the rush from C to D, or that whose direction is the same as that of the inducing discharge from B to A, is the more violent in character.

If we interpose a sufficient gap in the circuit C D E, we find that the *direct* induced discharge (or that passing from C to D) will leap over the gap in the form of a spark; while the *inverse* induced discharge (or that passing from D to C) will not pass, but will be suppressed.

Comparing with the results obtained in Chapter XXI., where we employ electrical currents over whose increase and decrease we have complete control, it would seem that as the discharge from B to A *begins*, there is induced a rush in the *contrary* direction from D to C; while as the discharge from B to A fades away, there is induced a rush from C to D in the *same* direction.

It would seem also that the *direct* induced rush from C to D is the more violent of the two, for the reason that the cessation of the inducing discharge in B A is more abrupt than its commencement. The inducing current is called the *primary*; the induced is called the *secondary* current.

Experiments.—In the figure, X represents a screen of card or of glass, on the further side of which is fixed a 'primary' spiral of wire through which a Leyden jar discharge can be passed. On the nearer side of the screen is fixed

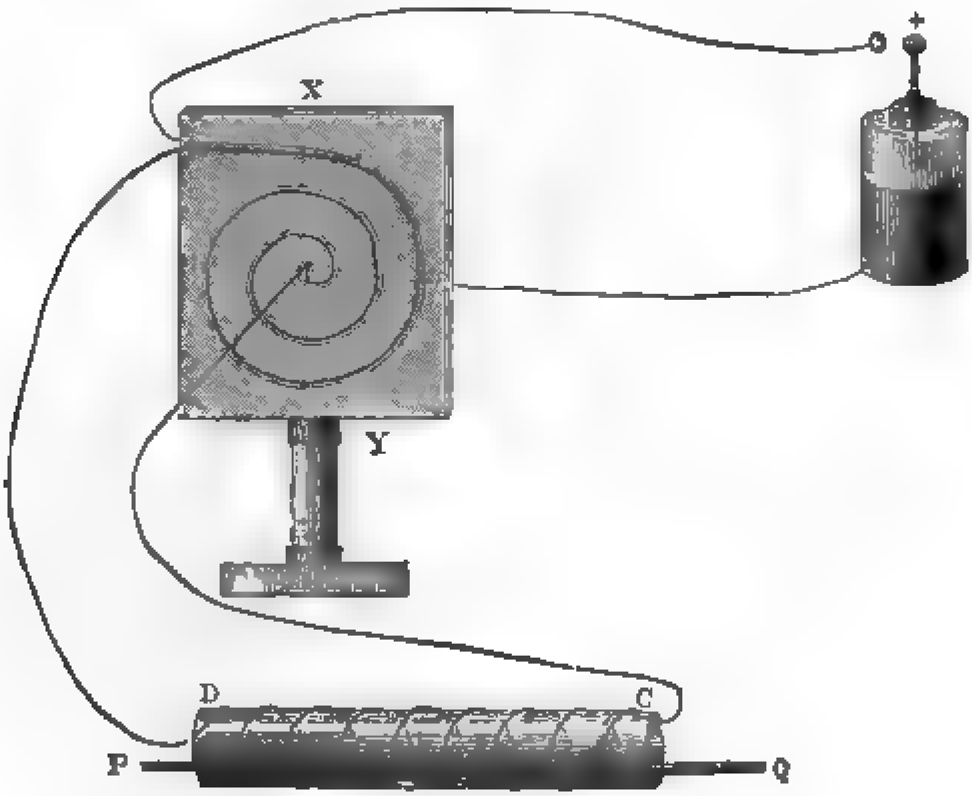


Fig. ii.

another, 'secondary,' spiral of wire, parallel to the other but separated from it by the glass or card. The ends of this last wire are connected with the ends of a wire coiled round a tube, C D.

Then, by placing steel needles inside the tube C D, we can [by the results obtained in the last section] examine the direction of the current induced in the secondary spiral. Thus, if the end P of the needle were found to be a north pole, it would mean that the current passed in the general direction from

P to Q; since then, if one swam with the current and looked at the needle the north pole would be to the left (*see* the rule given in last section).

We shall find that when there is no break in the secondary circuit there is a weak magnetising action making the end P the north pole. This implies a *total* magnetising action as of a current in the opposite direction to that in the primary circuit. Without further experiment this would indicate merely a weak inverse current induced in C D. Other experiments show us that as regards quantity of current in C D there is really a zero total, though the inverse current is the more effective as regards magnetising powers. But if we make a small break in the secondary circuit, such as will just allow a bright spark to pass, we find a relatively powerful magnetising effect, making Q a north pole. This indicates that now there is in C D a powerfully induced direct current.

Other experiments, which we do not describe here, are needed to bear out fully our statements as given above.

§ 13. **Wheatstone's Spark-board.**—This was a piece of apparatus designed to investigate whether the spark of discharge had any appreciable duration, whether the discharge passed from one coating to the other or from both simultaneously, and with what velocity it traversed such conductors as copper bell-wire The apparatus is sketched in the accompanying diagram, in whic the reader is supposed to be viewing it from above.

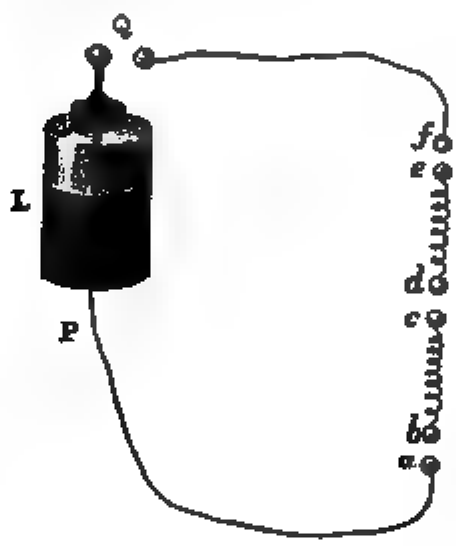


FIG i

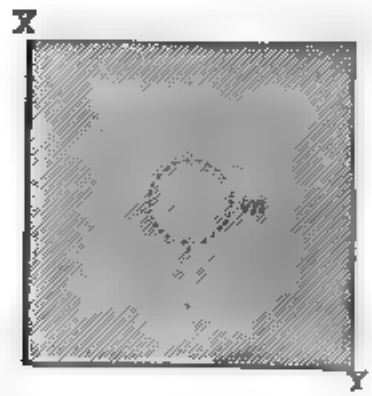


FIG. ii.

In fig. i., L is a Leyden jar. The discharge takes pla through the circuit P *a b c d e f* Q; so that sparks appear at tl three breaks *a b*, *c d*, and *e f*.

Between *b* and *c*, *d* and *e*, are coils of wire of the same knov thickness and length, say 1,000 *mètres* of common copper be

wire. Between *a* and P, *f* and Q, there should be wire of the same thickness and length respectively. In fig. ii., *m* is a small plane mirror revolving on a horizontal axis that lies parallel to the line of the three sparks *a b*, *c d*, and *e f*.

Above it is a horizontal screen of ground glass, marked out in degrees in a manner indicated later on.

The mirror can be made to revolve rapidly, its rate of revolution being indicated by clockwork or by the musical note produced by its striking a fine metal wire.

If the discharges take place very rapidly, then a person looking down from above will see some of the sparks reflected in the mirror as it passes through the position proper for reflection of the sparks to the eye ; others of the sparks will not be seen, owing to the mirror being unsuitably situated at the moment of their occurrence.

The rotation of the mirror will of course distort, by 'drawing out' in the direction of the motion any spark that lasts an appreciable time ; *i.e.* that lasts while the mirror has moved appreciably.

Now it is found that, for moderate rates of revolution of the mirror, we see on the screen X Y merely three points of light,

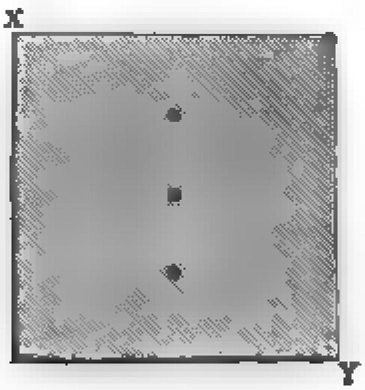


FIG. iii.

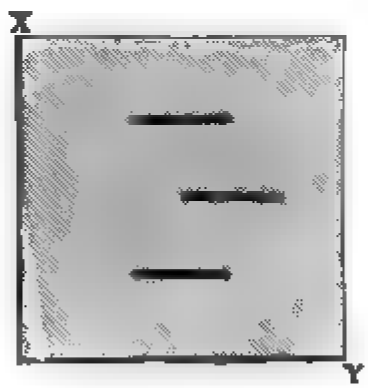


FIG. iv.

lying parallel to the sparks of which they are the images, as represented in fig. iii.

But when the mirror revolves with sufficient rapidity, we see these points 'drawn out' into three lines of equal length, covering a number of degrees on the glass screen that is definite for each definite rate of revolution of the mirror, as indicated in fig. iv.

Further, we see that the bright lines answering to the sparks

a b and *e f* have their ends lying in a direction parallel to the line of sparks, while the bright line answering to the spark *c d* is displaced as shown.

This shows (i.) that the duration of the spark is such that the mirror turns through an appreciable angle while it is reflecting; and (ii.) that, whereas the sparks *a b* and *e f* occur simultaneously, the spark *c d* occurs later than either.

The fact that the spark has *duration* was perhaps obvious beforehand; we cannot conceive of 'no duration.' But this apparatus gives us a measure of the duration of a spark under any particular conditions of distance between *a* and *b*, and nature of the intervening gas.

But the fact that the lines answering to the sparks *a b* and *e f* are still similarly situate shows that these occur simultaneously, and that the discharge is of the dual nature indicated before, and that when we speak of its 'passing from one coating of the jar to the other' we are but using a convenient convention.

§ 14. **The Condensing Electroscope.**—Before concluding this chapter on *condensers*, we must describe the 'condensing electroscope,' a piece of apparatus in which the principle of experiment (v.) of § 2 in this chapter is made use of. If the reader will turn to the figure to Chapter XI. § 3, he will there see the apparatus in question represented.

On the plate N of a gold-leaf electroscope is placed another plate M, separated from the first by a thin insulating film of shellac.

When M is put to earth, the plate N considered by itself will have a capacity very large as compared with its capacity when 'isolated.' Hence, if it and the gold leaves are raised to a very low potential when M is to earth, the charge that it has received will suffice to raise it to a potential many times greater when M is removed. Thus the leaves, which did not stir when at some small difference v from zero, may diverge widely when this has been multiplied to (say) 500 v.

We can, in fact, multiply any potential v by the fraction,

$$\frac{\text{capacity of N, when M is on it, and is to earth}}{\text{capacity of N when isolated}}.$$

§ 15. **Various Forms of Electrical Discharge.**—We have many times alluded to the passage of a 'spark' or 'disruptive electrical

discharge.' It may not be out of place to enumerate a few of the forms in which this discharge may take place.

I. *The ordinary spark.*—When we approach the finger, or any other conductor, to the prime conductor of a machine, or to any other conductor that is of a high potential but has not a very great capacity, the discharge usually takes place in the form of a jagged spark of no very great luminosity. If the opposed surfaces be of very gentle curvature, as *e.g.* two spheres of large radius, the spark will be straighter and more brilliant. In both cases there will be heard a sharp crack or other report.

II. *The Leyden jar discharge.*—In the Leyden jar we have stored up a quantity of electricity that is very great as compared with that stored on any ordinary isolated conductor. In Leyden jar discharges the spark is usually very brilliant and dense in appearance, and much straighter than in case (i.) above. Moreover, the sound is very strident.

III. *The brush discharge.*—If any projecting part of the prime conductor of a machine be of very sharp curvature, or still more if it be pointed, the discharge from this part will be in the form of a beautiful faintly luminous brush, and this discharge will be nearly or quite silent.

IV. *Discharge 'in vacuo.'*—When the discharge takes place through a tube or other vessel in which the air or other gas is very rare, the whole tube is filled with a faint luminosity, and there is no sound.

CHAPTER VII.

INDUCTION MACHINES.

§ 1. **Some further Propositions in the Theory of Potential.**—Still pursuing our system of introducing more and more of the theory of potential as we require it, we shall here discuss certain points that we must make clear before we can rightly understand the theory of *induction machines.*

By *induction machines* we mean those pieces of apparatus by means of which we can obtain continuous supplies of + and − electricity without any friction or chemical action; the essentials being (1) an initial supply of electricity that can act inductively on conductors placed near to it, and (2) mechanical work to effect such movements of the parts of the apparatus as shall give us continuous action. The common electrophorus is the simplest form of such a machine; and the reader can see how it differs in principle from a frictional machine, though in both cases mechanical work is the source of energy.

(i.) *There is no field of force inside a simple closed conductor.*—If we have a simple closed conductor, that is, a closed vessel containing no insulated charged bodies, it can be shown experimentally that there is no field of force inside it, or that it, and all the space inside it, are at one potential.

(ii.) *Concerning partially closed vessels.*—If we experiment with a partially closed vessel, such as a tin pail whose height is greater than the diameter of the mouth, and if it be placed so that those external objects which are opposite to the opening are relatively remote, then we shall find that there is no appreciable field of force in the inside of the vessel.

(iii.) *Insulated uncharged conductors inside a vessel.*—If into the inside of a charged vessel there be introduced an uncharged conductor, there is nothing to alter the fact of the potential being the

same throughout. The insulated conductor is in a region of no field of force, and hence there is no inductive action between it and the charged vessel. The conductor is at the same potential as this vessel and the space inside it.

(iv.) *An insulated uncharged conductor partially inside a vessel.*—Let A be an insulated vessel charged +, and let B be an insulated uncharged conductor partly inside and partly outside the vessel.

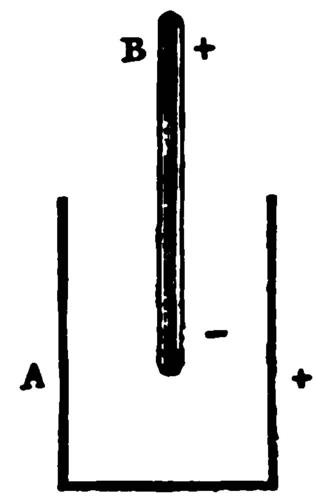


FIG. i.

The conductor B must be at one potential; and yet the one end is more remote from the vessel than is the other end.

As in Chapter IV. § 14 (i.), and in Chapter VI. § 2 (i.), there will be a redistribution of electricity on B, there being a − charge on the end inside A, and the complementary + charge on the end outside A; the whole of B being at a potential below that of A but above that of the earth. There will be also inside A a portion of its + charge equal to the − charge induced on the lower end of B.

All this can be shown experimentally. It is what we should expect from Chapter VI. § 2 (i.).

(v.) *An earth-connected body inside a vessel.*—If the body B be put to earth, it will be at zero potential. As in Chapter VI. § 2 (ii.), we shall find that it now has a − charge greater in amount than the − charge in the last case.

(vi.) *Two vessels at different potentials; a conductor inside each of them.*—If we have two vessels A and B at potentials + V and − V, there being an insulated conductor (initially uncharged) inside each of them, and if these conductors be joined by a conducting wire, it is not difficult to predict what will take place.

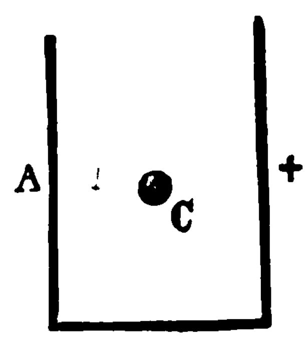


FIG. ii.

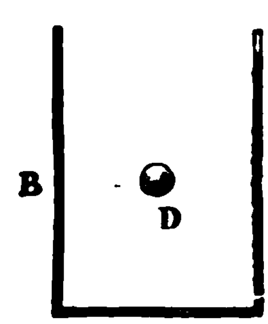


FIG. iii.

The conductor C will, before joining, be at + V, and D will be at − V. Hence, when they are brought to the same potential by joining, there will be a discharge between C and D; C will become charged −ly, and D will become charged +ly.

Further, if we now disconnect C and D and lift them still insulated from the vessels, and drop C into B and D into A, they

will give up the above-named charges, and the difference of potential between A and B will be still further increased.

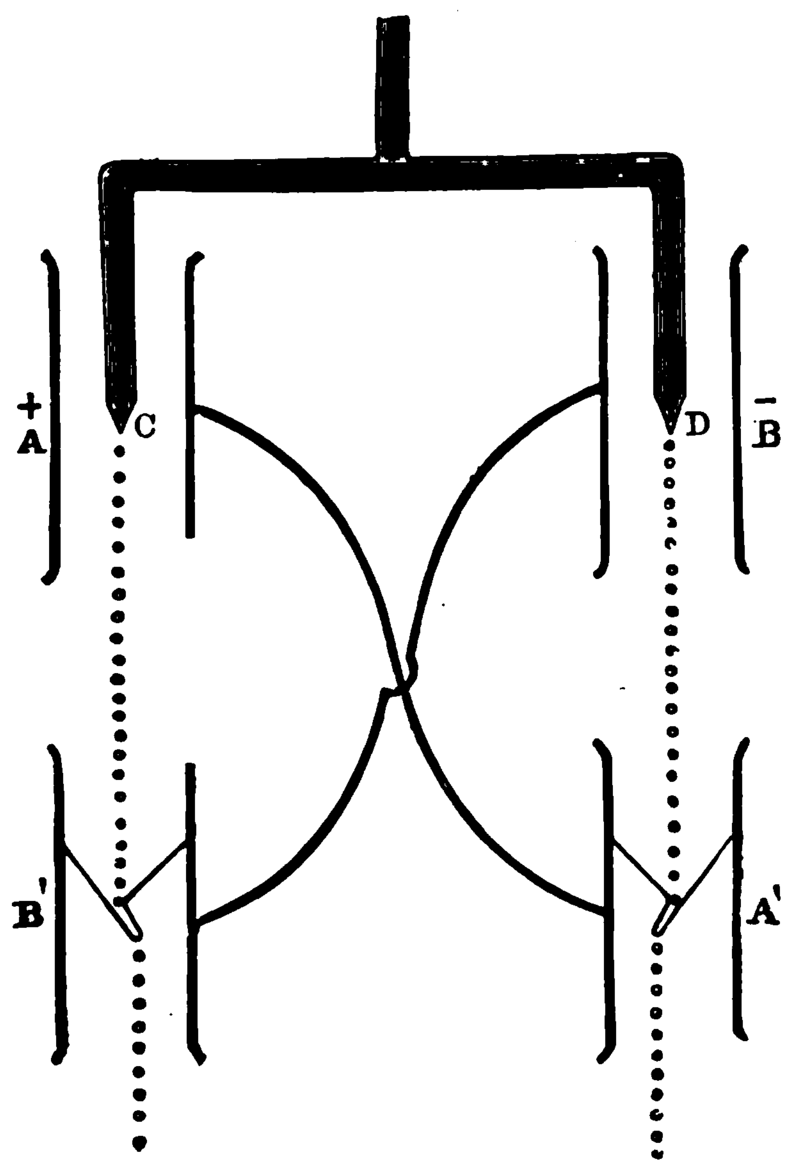
§ 2. **Application to Induction Machines.**—The above shows us that we have a means . . . .

(i.) Of getting an unlimited number of electrical discharges (viz. between C and D), ...

(ii.) Of indefinitely increasing the difference of potential between two (hollow) conductors, ...

at the expense of mechanical work. For we have merely to repeat indefinitely the cycle of operations indicated above; and we shall thus get a series of discharges between C and D, each more vigorous than the last, and shall be increasing the difference of potential between A and B.

It remains to discuss some convenient arrangements designed to repeat continuously and rapidly such a cycle of operations.



Note.—In what follows we shall use the symbol ΔV to denote *difference of potential.*

§ 3. **Sir W. Thomson's Water-dropping Accumulator.**—In the arrangement to be described in this section there are two points to be noticed.

First: we make no use of the discharges spoken of above; we only aim at increasing the ΔV between two (hollow) conductors that are initially at a small ΔV.

Secondly: instead of two vessels we have four, connected in pairs so as really to form two.

The accompanying figure is a sectional sketch of the apparatus in question. The two pairs of vessels, A A′ and B B′ respectively, are at a certain ΔV as indicated by the + and − signs.

In A and B are the ends C and D of water-pipes leading from a supply that is at the zero V of the earth. So that C and D will be charged inductively with − and + electricities respectively; the orifices being so small that the water breaks away in *detached drops*, so that these drops are continually carrying away − and + charges respectively. They are caught by funnels *in the interior* of B′ and A′ respectively; and so they give up their charges to these vessels, before dropping away to waste. The reader will see that it is essential for the water to fall *in drops*.

Hence B′ and B are being continually charged with −, and A′ and A with + electricity; and therefore the Δ V between A and B will continually increase.

It is important to notice that this increase of Δ V, both in this apparatus and in the Holtz, Voss, and other machines, proceeds on *compound interest* principles. For as the Δ V between A and B increases, so will the respective inductive actions of A and B on C and D increase; and so will the drops carry away charges of increasing magnitude.

The reader should also notice that we cannot avail ourselves of the discharge between C and D spoken of in § 2 (i.); for, since C and D are permanently connected, the rearrangement of electricities between C and D is continuous and not disruptive.

The question will occur, 'Whence do we derive our energy?' The answer is as follows.

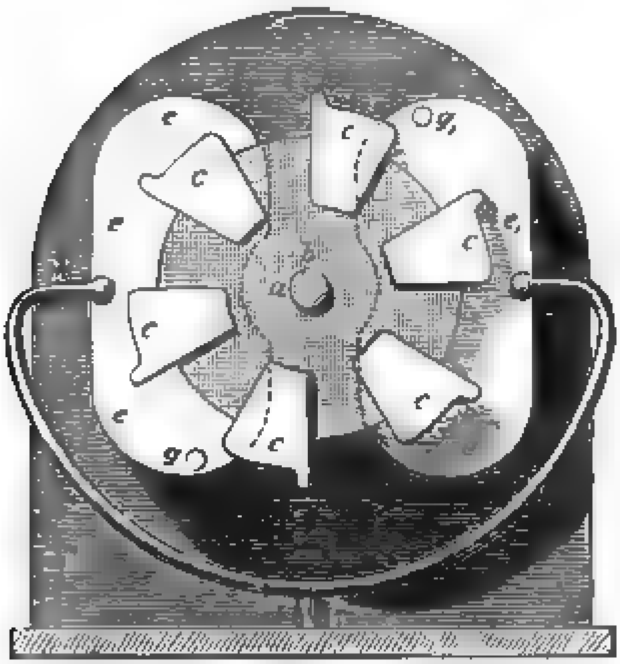
The raised water supply represents a certain quantity of mechanical potential-energy; and, as the water drops to a lower level, we should get heat developed equivalent to the potential-energy that is lost. But, owing to electrical attractions, the water drops are pulled upwards, and so fall more gently than they otherwise would; thus giving out less heat. The electrical potential-energy gained is the equivalent of the heat thus lost. In fact, we may say that the energy used is part of that expended in the original raising of water to the level of our supply (for 'Energy,' see Chapter X.).

§ 4. **Varley's Induction Machine.**—In this piece of apparatus we have *two* hollow vessels, and metal carriers instead of drops of water.

In the figure the insulated hollow vessels $e\,e$, $e_{,}\,e_{,}$, are represented with their front halves removed, so as to expose to view the

carriers, &c. In reality they are flat hollow vessels through which the carriers $c\,c$ pass.

These carriers are mounted on an insulating disc of ebonite. When each carrier has just entered an armature (the hollow pieces $e\,e$ and $e_{,}\,e_{,}$ are called *armatures*), it touches a projection g or $g_{,}$, and so gives up any charge it has to that armature. When halfway through the armature, it becomes momentarily connected with the diametrically opposite carrier by means of a metal piece $h\,h_{,}$. This metal piece is insulated, and its ends h and $h_{,}$ pass through holes in the armatures, without touching them.



Action. — Initially the armatures must be at different potentials; we may suppose one to be at $+ v$ and the other at $- v$.

When a pair of carriers are connected by the conductor $h\,h_{,}$ they will be reduced to the same potential, and therefore will become charged $-$ly and $+$ly respectively.

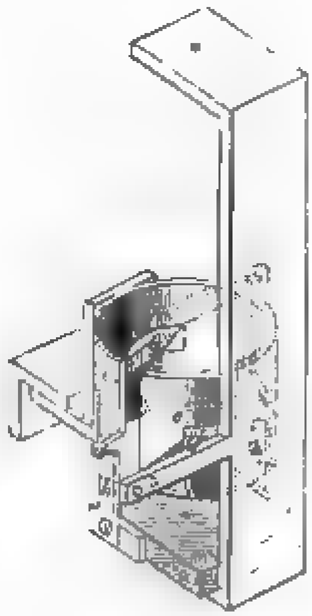
As the disc turns, these carriers are again insulated; and so they carry away the charges acquired, and deliver them up to the other armatures respectively by contact with the knobs $g_{,}$ and g.

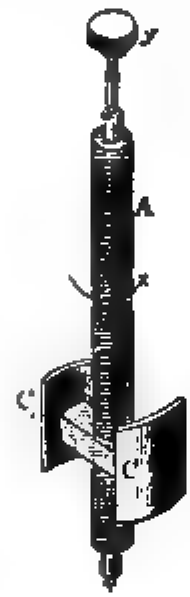
Thus the Δ V of the two armatures will continually increase. When the Δ V is large enough we may introduce a break into the conductor $h\,h_{,}$; we shall then see the electrical readjustment between each pair of opposite carriers take place disruptively, a spark leaping over the break made.

§ 5. **Sir W. Thomson's Replenisher.**—This is a small piece of apparatus used to keep up the potential of the Leyden jar with which the needle of the quadrant electrometer is connected. We here desire a slow action, and do not aim at powerful action.

The figure shows that the armatures $e\,e_{,}$ are hardly to be called 'hollow vessels'; they are merely metal plates bent into a

cylindrical form. They do not therefore act so powerfully on the carriers $C C_{,}$, since these are never enclosed by them ; nor do the carriers give up *all* their charge to the armature when in contact with it. The action, therefore, is less vigorous, but sufficient for the purpose. After § 4 it will be sufficient to point out one or two details.





(i.) One armature is permanently put to earth.

(ii.) The conductor that connects the two carriers when they are inside the two armatures is insulated. It is continuous ; a break being only for the purpose of obtaining phenomena of discharge, which are not wanted here.

The reader should, by reasoning out the action, discover for himself why $h\,h_{,}$ may be uninsulated when e and $e_{,}$ are insulated ; while it must be insulated if either e or $e_{,}$ be put to earth.

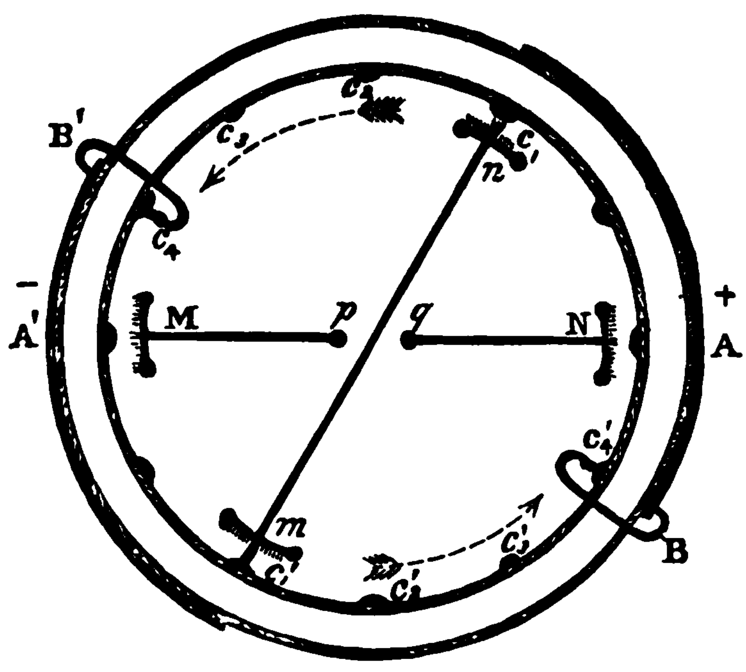
§ 6. **The Voss Machine.**—With regard to the more complex induction machines, the Voss, the Holtz, and others, it may be said that no mere verbal description is enough to enable a student to understand the action ; any such description should be read by him with the machine actually before him.

Further, the theory of these machines is by no means in a satisfactory condition. This is due mainly to two causes. In the first place, we no longer have hollow armatures into which the carriers pass, and concerning which the theory is simple ; the armatures are now plane pieces of tin-foil or of paper, and the carriers merely come opposite to these. And, in the second place, there is little doubt but that the extremely rapid movements in these machines give rise to actions concerning which our information is imperfect ; actions depending upon the relation between the velocity of these movements on the one hand, and the rate at which electrical charges spread on the badly-conducting surface of the varnished glass on the other. These difficulties will be

better understood when the following description has been read and the machine has been seen.

In the Voss, as (*see* § 7, fig. i.) in the Holtz, there are two glass plates mounted on the same axis and very close to one another; one of these being fixed, the other being capable of very rapid rotation upon the axis. On the former are fixed the sheets of tin-foil or of paper that serve for armatures, and the varnished surface of the latter (supplemented in the case of the Voss by metallic carriers fixed on to the glass) takes the place of the carriers whose functions were explained in §§ 3 and 4.

Since it is not possible to represent in any one simple diagram the action of the real machine with its double plate, we here give a *representative* diagram by means of which the actual machine may be understood. It is supposed that, instead of two plates, we have two co-axial glass cylinders, the outer one being fixed, and the inner one revolving as shown by the arrows. The diagram shows such an arrangement in section, the two cylinders being represented by two thin circles of glass. Hence, the reader must imagine that all the lines (whether long as A, or short as c_1), with the exception of the metal pieces M N, *m n*, B, and B′, are but 'end-on' views of strips that in reality run down the whole length of the cylinders. A and A′, the armatures, are sheets of tin-foil; while c_1, c_2, &c., are metallic strips fixed on to the glass. M N and *m n* are stout brass rods; and B and B′ may be made of thick brass wire.



For the present we will neglect the piece M N; and will show how, when the inner cylinder is rotated in the direction indicated, any initial ΔV between the armatures A and A′ will be soon increased up to the limit set by the conditions of insulation in the machine.

Suppose that initially the armatures A and A′ have some

[illegible]ght ΔV; they will act inductively upon the carriers that come [o]pposite to them. Now, at one point in their course, as the [in]ner cylinder revolves, each pair of carriers as c_1 c'_1 are connected [wi]th one another through the metallic piece $m\ n$, whose extremities [ar]e provided with fine wire brushes. In consequence of this, c_1 [w]ill pass on charged $-$, and c'_1 will pass on charged $+$.

When the two carriers come opposite to the other armatures, [A]$'$ and A respectively, they make contact with these through the [me]tal pieces B$'$ and B; these pieces reach round the ends of the [c]ylinders, and have wire brushes at their extremities. The question [n]ow occurs, will these carriers charge the armatures to greater $+$ and $-$ potentials as did the carriers in §§ 3 and 4? The difficulty here arises from the fact that the carriers do not now pass into the interiors of the armatures, and so do not give up all their charge.

We may perhaps reason somewhat as follows. The carrier c_1, while still uncharged, is brought very near to the armature A; and we may assume that it is thereby raised to at any rate more than half the potential of A. That is, it is raised to some potential m V, where m is greater than $\frac{1}{2}$. It is then brought to zero-V by connection with the opposite carrier. It must therefore have received a charge that would, in the absence of A, have lowered it to $-m$V. Next it comes into the same position with respect to A$'$; and would therefore, if uncharged, have been lowered to a potential $-m$ V. Being charged, however, it will acquire on the whole a potential of $-m$V $-m$V, or $-2m$V. But we have supposed m to be greater than $\frac{1}{2}$. Hence $-2m$V is numerically greater than $-$ V; and therefore $+$ electricity will flow from armature A$'$ to carrier c_1, or $-$ electricity in the other direction; and hence, finally, the potential of A$'$ becomes lower than before, or its $-$ V is numerically increased.

A like action will meanwhile have taken place with respect to c'_1 and A.

So far then we have shown how the armatures are raised to, and maintained at, a great ΔV.

Now consider the brass pieces M and N, which are represented as furnished with combs and as having an air-break $p\,q$ between them. (It must here be noted that the 'combs' with which M N and $m\ n$ are provided should in reality run parallel to the axis of the cylinder. They are drawn as in the diagram so that they may

be seen ; whereas strictly they should be drawn 'end-on.') Bo
the metal carriers, and the varnished surface of the glass whi
serves as a continuous series of insulated carriers, will act indu
tively upon the brass combs ; + electricity from M, and — electr
city from N, will pass convectively from the points of the combs o
to the carriers and glass surfaces, tending to reduce the regions th
lie opposite to the two combs to a smaller Δ V ; while the 're-
pelled' — and + charges will give a stream of sparks across the
air-space *p q*.

If there be no break at *p q*, and if the combs have *very* sharp points, the opposite pairs of carriers and of glass surface will be reduced to almost the same potential ; and the piece *m n* will be idle, M N taking its place. If there be a break at *p q* it requires a certain Δ V to give a discharge across this break ; and the opposite pairs of carriers cannot certainly be reduced to anything less than this Δ V ; so that *m n* will now act as above described. The limit to the striking distance *p q* is given by the Δ V of the armatures; the Δ V between M and N being always less than this. When the distance *p q* is too great, M and N are idle.

The action, or idleness, of the pieces *m n* and M N can readily be detected by working the machine in the dark ; the presence or absence of glows and brush-discharges giving the required information.

The combs with which the piece *m n* is provided thus play with respect to the glass surface the same part as the brushes play with respect to the metal carriers.

§ 7. **The Holtz Machine.**—The Holtz machine was invented before the Voss ; but, as its theory is less simple, we have chosen to discuss it later.

The main differences between this machine and those that we have discussed above are that here the armatures are charged *inductively*, and not directly, by the carriers ; and that we have as 'pairs of carriers' no metal pieces, but simply the diametrically opposed portions of the varnished glass surface.

Fig. i. gives a general view of the simplest form of Holtz ; and this figure will serve, *mutatis mutandis*, to give an idea of the Voss. The larger and hindermost plate A A is fixed. In it are cut two windows F and F′ ; and along the edges of these windows are pasted the armatures *p* and *p′*, consisting of two sheets of tin-

l or of varnished paper. From the armatures project two inted tongues of paper, n and n'; these touch lightly the back rface of the smaller revolving plate B B (which is the nearer to in the figure), the points of contact being opposite to the indows F F′ cut in the fixed plate. The front plate B B revolves as indicated by the arrows.

On the nearer side of the front plate B B, and opposite to the rmatures $p\,p'$, are the combs O O′ of the conductors C C′. These

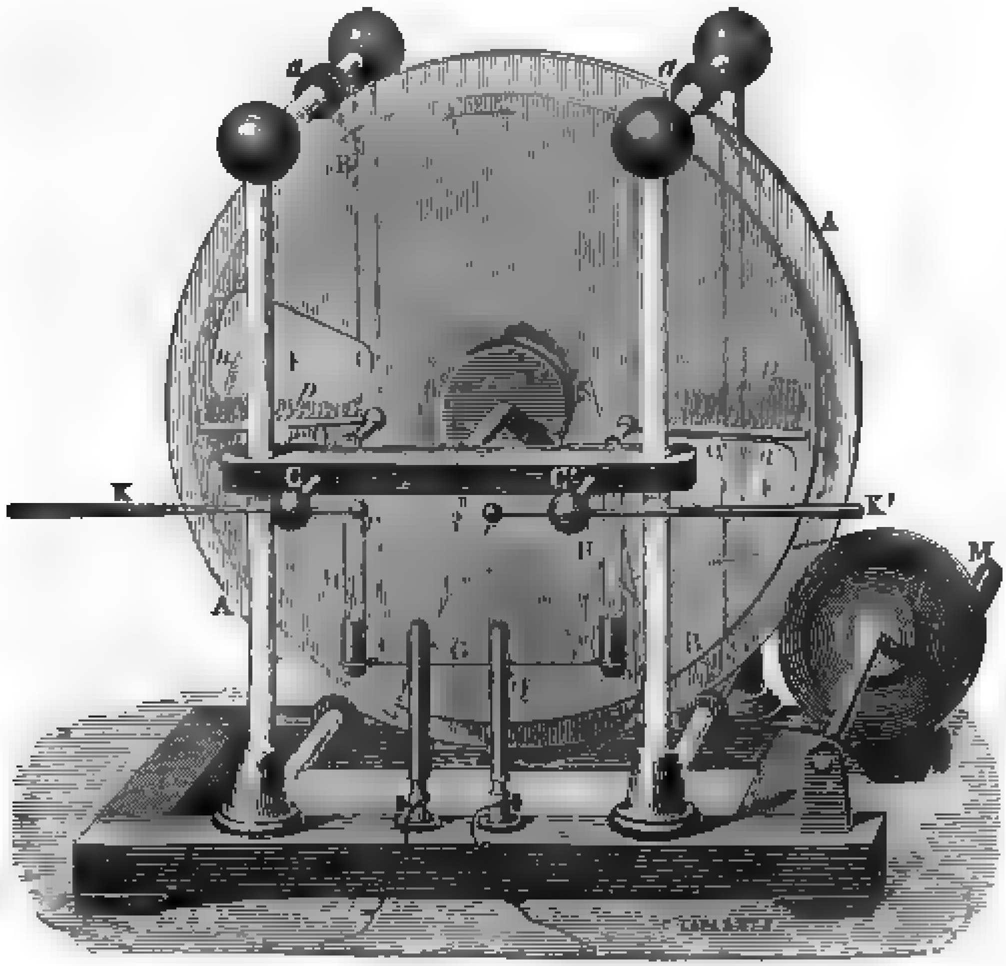


Fig. 1.

conductors are put into contact by means of the pieces K and K′, until the action resulting from the revolution of B B has increased the small initial Δ V of the armatures to a sufficient magnitude. When this has been effected, an air-space may be introduced at $r\,r'$, when a discharge will take place across.

As in all induction machines, this discharge will be nearly continuous, and will consist of a luminous brush accompanied by small sparks; unless, by connection with the condensers H H′

or by some other means, the capacity of the prime conductors be greatly increased. When this is done, the character of the discharge alters; it now takes place at intervals in the form of a dense and brilliant spark.

In fig. ii. we give a representative diagram. The lettering of this, similar to that of the figure to § 6, differs from that of fig. i. above; but the reader will find no difficulty in recognising the various parts. Here we have the *sections* of two co-axial cylinders, of long strips of tinfoil which form the armatures, and of long open strips which form the windows cut in the outer fixed cylinder. As in the case of the Voss, the combs should run parallel to the axis of the cylinder, *i.e.* to the edges of the windows and armatures; they should have been viewed end-on instead of as here drawn.

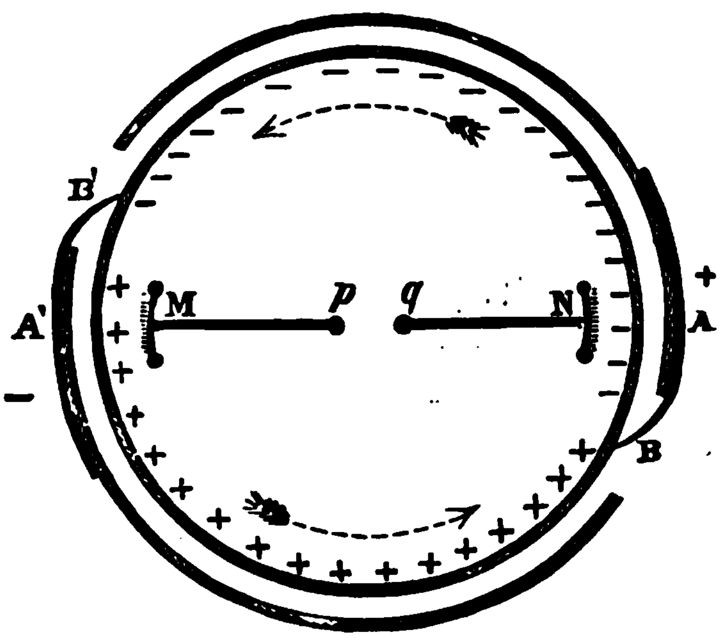


Fig. ii.

As in the other machines discussed above, there is given initially to A A′ some slight ΔV. The conductors are then put into contact, no air-space being left at $p\ q$; and the inner cylinder is turned as indicated by the arrows. The opposite pairs of what we may style 'glass-surface carriers' are thus charged inductively, as usual.

Let us consider the − charged glass surface that comes opposite to B′. In virtue of its − charge, and of the presence of the − charged armature A′, this portion of glass surface will acquire a negative potential greater in magnitude than that of A′ and B′. There will therefore be a field of force running from the armature A′ towards the 'carrier,' since this is at a negative potential of greater magnitude. Hence we shall have + electricity passing off the tongue B′ on to the back surface of the inner cylinder; while the armature A′ is consequently left at a negative potential numerically greater than before.

When our 'carrier' glass-surface comes opposite to the comb M, it will be charged + as usual; and the + charge that was

-bound' at the back of the cylinder will become 'free.' When the same portion comes in turn opposite to B, this tongue will be charged to a higher + potential, in part directly by this free + charge on the back of the inner cylinder, and in part inductively by the + charge on the inner surface of the cylinder, in the manner described above.

When the ΔV between A and A′ is great enough, an air-space may be introduced at pq; and sparks will now strike across.

But it is to be noticed that if this air-space be great, requiring a large ΔV to drive a discharge across it, then the charges on the glass surfaces will fail to be neutralised and reversed as they pass the combs; and therefore, as can be seen from the theory of action given above, the machine will become less efficient, or will even cease to act, just when we make the greatest demands upon it.

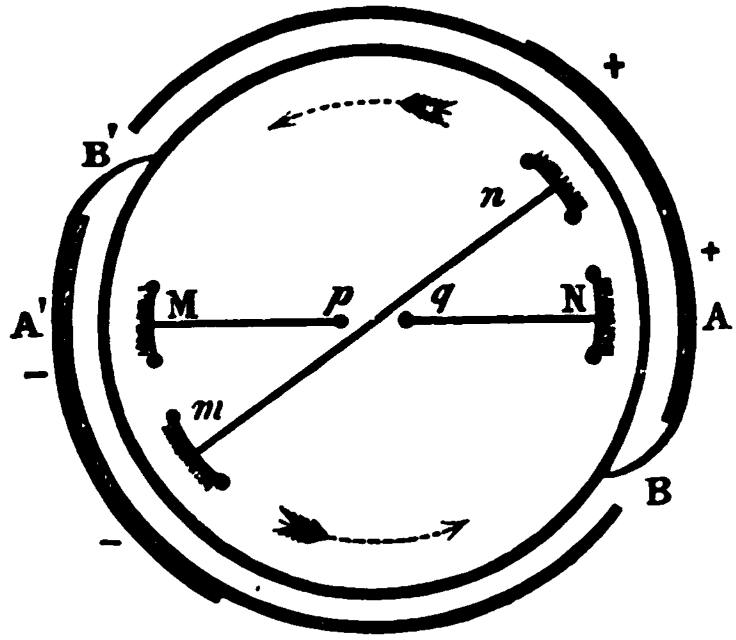


FIG. iii.

To obviate this defect, the armatures may be extended, and a continuous metal piece $m\,n$, furnished with combs at the ends, may be introduced, as shown in fig. iii. This form of Holtz is more like the Voss. The piece $m\,n$ with its combs will now insure the reversal of charge upon the glass surfaces; and, if the gap pq be great, but not too great for the insulation of the machine, the ΔV of the armatures A A′ will increase until a spark can strike across pq.

Notes on the Voss and Holtz.—(i.) It may be remarked that in both the above machines it is only a small portion of the revolving surface that is employed to raise the armatures to, and maintain them at, the high ΔV required. In the Voss this part is played by the metallic carriers; these in the actual machine are small metallic discs which bulge out at their centres sufficiently to make contact with the brushes at m and n. In the Holtz the same function is performed by that ring of glass surface which lies in the immediate neighbourhood of the circle traced out on the revolving plate by the sharp-pointed tongues.

(ii.) We have not in §§ 6 and 7 pointed out the *compound interest* of the Voss and Holtz, this having been already discussed in § 3.

(iii.) The reader is again warned that possibly the action of these ma especially as regards the feeding of the armatures, depends in some present unknown upon the velocity of rotation. Hence the above disc which ignores this velocity as affecting the nature of the action, may be tive.

CHAPTER VIII.

ATMOSPHERIC ELECTRICITY.

§ 1. **Franklin's, and Other Early Experiments.**—In a 'course' such as the present, it is impossible to give a history of the successive experiments and discoveries by which little by little our present knowledge has been attained.

But we cannot pass over without a brief notice the classical 'kite' experiment of Franklin.

Having successfully imitated in miniature the phenomena of thunder and lightning with his powerful Leyden jar batteries, Franklin wished to establish the common nature of the large and the small phenomena by obtaining from the thunder-cloud an ordinary electric spark. He had noticed the action of pointed conductors, and considered that the discharging of an insulated conductor by the presenting to it an earth-connected pointed conductor was due to the point 'drawing' the charge from the body (*see* Chapter IV. § 15). So he conceived the idea of flying a kite, provided with metallic points, near to or in a thunder-cloud.

Such a kite was flown. The end of the string was tied to a key, and the key was fastened by an insulating silk cord to a tree. Apparently Franklin had not considered sufficiently the part the string had to play; for it was when dry a bad conductor, and no results were at first obtained. When, however, the string had been wetted and so rendered more conducting by an accidental fall of rain, abundant sparks could be drawn from the key.

It is clear that if the string had been put to earth, the thunder-cloud would have been slowly discharged by a current of oppositely charged air proceeding towards it from the points of the kite.

§ 2. **Lightning Conductors.**—When a cloud charged with + or − electricity comes over any portion of the earth, there is induced opposite to it a − or + charge respectively, of a magnitude

equal to that of the cloud's charge. The stress thus produced will, if of sufficient intensity, be relieved by a disruptive discharge; a 'flash of lightning' will pass, and the equal and opposite charges will disappear.

But if on any portion of the earth there be erected one or more metallic points *well connected with earth* there will be a slow discharge of the convection-current nature, with the effect in general that the ΔV between the cloud and the earth does not rise high enough for a disruptive discharge to take place.

During this quiet discharge there will in the dark be observed (as in the smaller experiments performed with electrical machines) a glow or a brush of pale blue light on the points. This phenomenon, when it occurs on the extremities of a ship's masts, has usually been called 'St. Elmo's fire.'

If the conductor be not well connected with earth, its presence may be a cause of danger, as determining the locality of the now necessary disruptive discharge. The same may be said of a conductor whose point is not sufficiently sharp.

For a good and effective conductor the following points should be attended to.

(i.) The point should be sharp and of some metal not liable to be corroded.

(ii.) The lower end should be put well to earth, *e.g.* be led to a stream or pond, or into a pit filled with metallic refuse or coke, or into a marshy piece of ground; it being remembered that dry soil does not conduct well.

(iii.) All detached metallic masses should be connected with the conductor. Otherwise we may get disruptive discharges where there are gaps in the connection.

(iv.) The conductivity of the conductor should be sufficiently great to obviate any chance of fusion.

(v.) The height should bear a certain relation to the area to be protected.

The reader will observe that a 'lightning conductor' aims in reality at *preventing* discharges of a sudden nature, rather than at receiving these discharges.

Some trees, such as pines, act as natural dischargers, the numerous points of the leaves acting as so many imperfect lightning conductors.

Wire network used as a protector.—The inside of a hollow conductor is (as will be further explained in Chapter X.) free from any action due to charged bodies situated outside. Hence, it has been proposed to protect buildings by covering them with an earth-connected system of wires, and so making them approximately hollow conductors. If, however, a disruptive discharge did take place, such wires (which would naturally not be very thick) might be fused, and further, since in a discharge the conditions are no longer statical, such a discharge might, even with the wires intact, pass partly through the body of the building.

§ 3. **Return Shocks.**—Any abrupt change of electrical condition makes itself felt physiologically as a shock. Hence the sudden readjustment consequent on a lightning discharge may produce fatal results in the vicinity of the actual discharge.

Experiments.—(i.) Place near the prime conductor of a machine another conductor imperfectly connected with earth. As the prime conductor is gradually charged we shall perceive small sparks passing over the gaps left, the other conductor being acted upon inductively. If we now discharge the prime conductor suddenly, we shall perceive a much stronger spark pass over each gap as the second conductor suddenly loses its induced charge.

(ii.) If a person hold his face near a charged conductor he will experience a slight shock when this latter is discharged. Or if a person stand on the insulating stool and hold the prime conductor he will receive a shock when this is discharged, although the spark is not 'taken from' him directly.

It is for the above reason that rule (iii.) of § 2 should be observed.

§ 4. **The High Potential of Thunder-clouds.**—The differences of potential as directly caused by natural actions are very small. They are mainly due to the friction of the winds over the earth, to evaporation, and to the sun's action in producing differences of temperature. The first two causes are the more important.

Experiments.—(i.) Direct a strong current of air by means of a pair of bellows on to the plate of a delicate gold-leaf electroscope. The leaves will diverge.

(ii.) Heat a platinum capsule and place it on the plate of the electroscope. Now drop into it some aqueous solution of copper sulphate. A divergence of the leaves indicates that the vessel is charged −. It is necessary, as a rule, to use a condensing electroscope, the platinum vessel being on the upper plate, and the lower plate being temporarily put to earth during the evaporation. Pure water gives no results.

It must, however, be stated that many consider the electrical separation to be due to friction between the heated vapour and the metallic vessel.

The question naturally arises, 'How can we account for differences of potential so great as to give us flashes of lightning that are sometimes perhaps several miles in length?'

Note.—The writer is inclined to think that the difference of potential required to produce flashes of lightning (whose length was roughly calculated from the duration of the peal of thunder or by some other means) has been over-estimated. The calculation of the difference of potential required is based on experiments connecting the difference of potential between two brass spheres with the length of spark that will pass between them. In the latter case the spark is a truly disruptive discharge, there being nothing to help it on its way (so to speak) except a little dust in the air. But, in the former case, who can be sure of the nature of those discharges whose great length has been roughly calculated? They may be discharges only partly disruptive, and partly passing from particle to particle of water.

We here give the theory that explains how a high potential may be attained.

The reader will understand it better when he has read Chapter X.

When visible cloud is first formed from true vapour, the water is in very minute spheres. As these coalesce the cloud gets denser and denser, and it is possible that in a heavy thunder-cloud each sphere may contain thousands or millions of the very small spheres first formed.

Now each original sphere we may suppose to have acquired a charge from the air and from the vapour from which it was formed, this charge being due to one of the causes named.

If the charge of each small sphere was Q, and its radius was r, then its potential (as we shall see in Chapter X.) was measured by

$$\frac{Q}{r} \qquad (1)$$

Now, let n such drops coalesce into one larger drop. To find the radius R of this drop we have merely to state that its mass equals the sum of the n small masses.

Hence, if d be the density of water, we have

$$\frac{4\pi d R^3}{3} = n \times \frac{4\pi d r^3}{3}$$

$$\text{or } R = r.\sqrt[3]{n}$$

Hence the new potential will be (from Chapter X.)

$$\frac{n\,Q}{R} = \frac{n\,Q}{r.\sqrt[3]{n}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Comparing (1) and (2) we see that

$$\frac{\text{New potential}}{\text{Old potential}} = \frac{\dfrac{n\,Q}{r.\sqrt[3]{n}}}{\dfrac{Q}{r}} = \frac{n}{n^{\frac{1}{3}}} = n^{\frac{2}{3}}.$$

Thus, if n were 1,000,000, we have

New potential = 10,000 × (old potential).

Besides this we shall in general have the charge tending to pass to the outside of a cloud as a whole, and this would still further decrease the capacity and increase the potential.

When the flash passes, the drops will, as a rule, more readily coalesce, there being no longer the electrostatic 'repulsion' between them. This may be one cause of the sudden fall of rain observed to follow a powerful flash.

§ 5. **Potential at a Point in the Atmosphere.**—Observations on the phenomena of thunder-storms are occasional; and are, as a rule, of necessity very inexact.

There are, however, daily observations of a far more exact nature that are made, viz. observations of potential that exist between the earth and points in the atmosphere at different heights above the earth.

(*a*) Before discussing this matter we will, even at the risk of repeating somewhat, examine what is meant by *the potential at a point in the atmosphere.*

We have, in Chapter V. §§ 2 and 3, given some explanation of what is meant by 'potential' and how it is measured. The reader must wait until he has read Chapter X. to understand fully what is given in a general way in Chapter V.; he can then turn back to the present chapter and read it again. But the passages referred to are sufficient to show that we measure the potential at a point in the atmosphere by seeing how much work it takes to raise + unit electricity from our arbitrary zero of potential, the earth, up to the point in question.

(*b*) A small conductor, elevated above the earth, is at the

potential of the region where it is situated when it has no charge on it; since it would require the same work to bring the + unit up to the body from the earth as it would to bring the + unit up to the same place before the conductor was put there.

(*c*) If an *earth-connected conductor* be situated at a region A in the atmosphere that is at a different potential from the earth, a charge of opposite sign to this potential will be induced on the conductor, as explained in earlier chapters (*see* Chapter VI. § 2 (ii.), &c.). If this charge can escape, as it will off a point, or still better off a flame, this convectional escape can only cease when the region is reduced to the zero potential of the earth. The charge induced on the conductor will indicate by its magnitude the potential of the region at A.

(*d*) If a conductor be situated at A, and be *connected with an insulated conductor* situated elsewhere, *e.g.*, nearer the surface of the earth where there is a different potential, induction will take place somewhat as in the last case, a charge being induced of opposite sign to the potential at A.

If this charge can escape, as by means of a flame, this convectional escape will take place until the *whole* conductor is at the potential of the region at A; for there cannot be equilibrium until this is the case. But it must be noticed that, if the conductor be large enough, this escape *may* sensibly alter the original potential of the region A.

Notes.—The above reasoning is of a general nature. It can however be shown, by reasoning somewhat beyond this Course, that—

(i.) When there is a charge on the surface of a conductor, this means that the conductor is at a different potential from the region surrounding it; and conversely.

(ii.) That any such charge is urged normally outward from the conductor.

(iii.) That when there is no such charge, this means that the conductor is at the same potential as the region surrounding it; and conversely.

§ 6. Methods of Measuring the Potential at a Point in the Atmosphere.

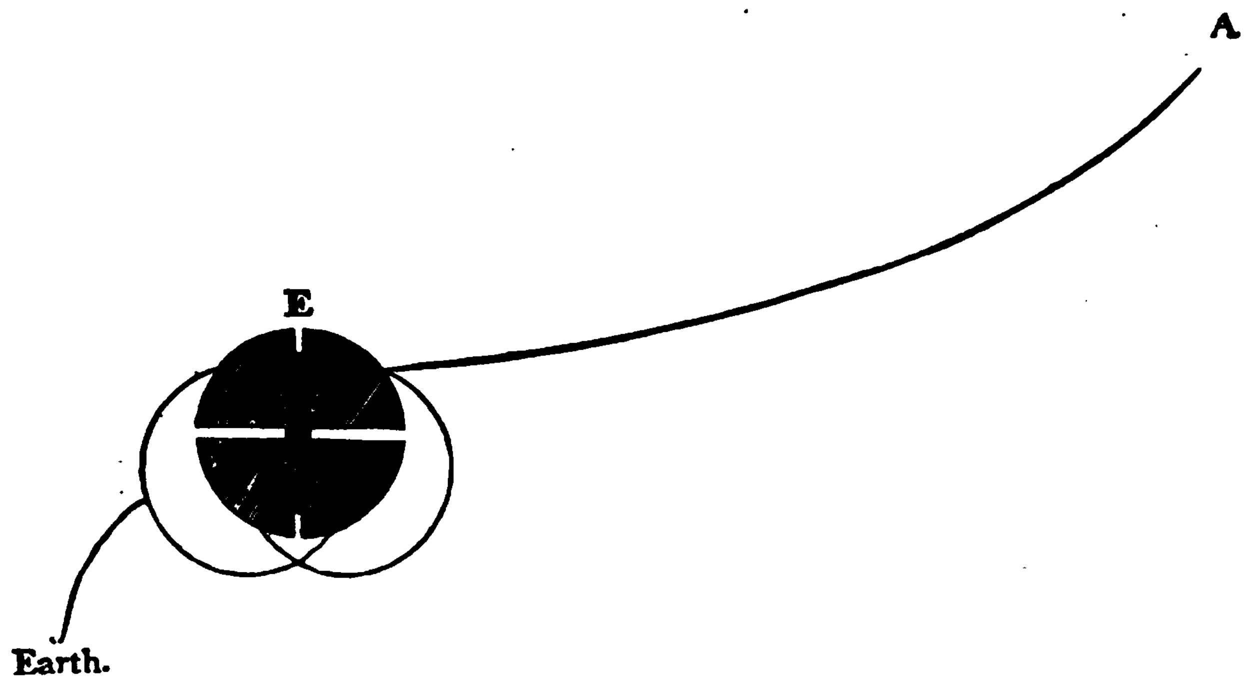
(i.) One method, formerly used, was based upon (*c*) of the last section.

A ball was elevated into the region in question; was temporarily connected with earth; was again insulated; was lowered;

and finally the charge induced (of opposite sign to the potential in question) was examined.

(ii.) But the best method is as follows.

E represents a quadrant electrometer (*see* Chapter X. § 33, for full description and complete figure). One pair of quadrants is put to earth, and from the other pair is raised a wire ending in a flame or slow-match at the point A, whose potential is required. Thus we have an insulated conductor consisting of the slow-match, wire, and pair of quadrants. As explained in (*d*) of the last section, there will be a convectional discharge by means of the slow-



match until the end of this latter is uncharged, and is, therefore, at the potential of the region A (*see* last section (*b*)). When this is the case, then the wire and the pair of quadrants with which it is connected will all be at the potential of the region A. The needle of the electrometer will then indicate by its deflexion the difference of this potential from the zero potential of the other pair of quadrants.

(iii.) Or we may employ the dropping of water to carry off the induced charge, and so to reduce the conductor to the potential of the region where the water breaks into drops.

If with the pair of quadrants there be connected an insulated tank, and if a pipe from this reach to A, and if there the water fall away in drops, we shall attain the desired end just as well as we did by the use of a flame or slow-match.

§ 7. **Results of Observations.**—Some results arrived at by such observations are here given. In ordinary circumstances the potential of the atmosphere under a cloudless sky is + ; and it

increases as we move higher from the earth. There are variations during the day, and these are fairly regular for all cloudless days.

Clouds may be charged to a + or – potential; and hence, when clouds pass, the points in the atmosphere which lie between them and the earth will also have a + or – potential respectively.

Whatever the potential of the atmosphere, there will be found on the exposed surface of the earth a *charge* of opposite sign, the *potential* of the earth being of course zero.

§ 8. **Sheet-Lightning and other Phenomena.**—With respect to the *Aurora Borealis*, references to papers on this subject will be found in the Preface, p. viii.

Sheet-lightning may, in some cases, be a true brush or glow discharge. In other cases this name is wrongly given to the visible reflection of a spark discharge that itself occurs out of sight.

Forked-lightning.—The lightning-spark is nearly always zigzag in its course. It is supposed that a greater resistance is created in the direct line of a spark, and thus the discharge is diverted along the line of least resistance to one side or the other. In very rare cases the discharge is straight; so that we conclude the crookedness of the usual path to be due in some way to the air through which it passes.

Globe-lightning.—There is one form of discharge of which the explanation is at present very imperfect. It is now sufficiently well established that sometimes a *globe* form of discharge is seen. This appears like a fiery globe moving slowly, or even at times resting stationary. Its movements are sometimes very erratic. It appears at times to follow conductors; it is said to break through non-conducting structures such as walls; and all accounts agree that it ultimately disappears with a powerful detonation. Planté has produced on a small scale similar appearances by aid of a voltaic battery, which gave him large quantities of electricity and a high difference of potential; but it is not certain that there is a true connection between the two phenomena.

Some believe this 'globe' form to be a kind of Leyden jar highly charged; but this explanation does not appear to make matters clearer.

No doubt in some cases the phenomenon was really a heated aerolite; but in other cases its nature seems undoubtedly to have been that of an electric discharge.

CHAPTER IX.

SPECIFIC INDUCTIVE CAPACITIES.

§ 1. **Definition.**—We stated in Chapter VI. § 4 (iv.) that the charge of a condenser depended, *cæteris paribus*, on the nature of the substance that was between the two plates; or, in usual language, on the nature of the *dielectric*.

We propose in this chapter to discuss at some length the relative efficiencies of different substances as dielectrics.

If we have two condensers of *exactly similar construction*, one having air as the dielectric and the other having some other substance A as its dielectric, we shall find that for any given difference of potential between the plates the two condensers have unequal charges. Further, we shall find that the ratio between the two charges is constant, whatever the form of the condensers and difference of potential between the plates, provided that all conditions are the same for the two condensers.

Thus we have for the substance A a constant number representing its efficiency as a dielectric as compared with air. We call this number the *specific inductive capacity* of the substance A, and usually denote it by the symbol σ.

Or we define as follows.

$$\text{The specific inductive capacity of a substance A} = \frac{\text{Capacity of the condenser when A is the dielectric}}{\text{Capacity of the condenser when air is the dielectric}}.$$

We here give a table of the specific inductive capacities of a few substances.

These results must, however, be regarded as approximate only.

Air	1·00	Paraffin	1·98
Glass	1·90	India-rubber	2·22
Sulphur	2·58	Gutta-percha	2·46
Shellac	2·74		

The expressions *inductive power*, and *dielectric constant*, are used as synonymous with specific inductive capacity.

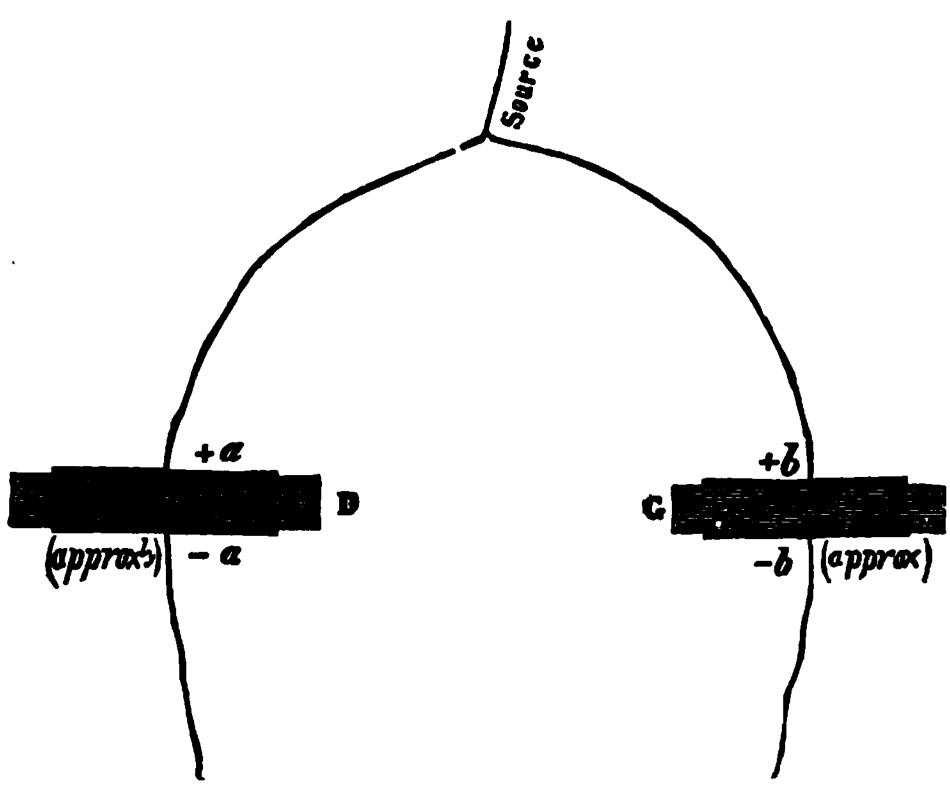
§ 2. **Variation with Time.**—It was soon found that for solid dielectrics the time for which the experiment lasted had a marked influence on the value obtained for σ.

If, in Chapter VI. § 2 (i.), we interpose a plate of ebonite I between the two condenser plates A and B, we have initially a certain inductive action of the plate A on B ; this being indicated by the divergence of the needle of the electrometer E_B.

But we shall observe that this action gradually gets less, as the plate I gradually yields to the electric strain. There is a slow separation of electricities on I similar to the practically instantaneous separation that occurs in a conducting plate. In fact, with time the ebonite plate I acts more or less as a conducting plate ; and then, like a conducting plate, it *screens* B from the action of A to a greater or less degree (*see* Chapter X. § 17).

Or again we can, by waiting, charge a Leyden jar from a source of constant potential with a larger charge than it would receive in the first instance ; the *specific inductive capacity* of the glass appearing to increase with time. As explained in Chapter VI. § 10, there is a penetration of the charge, and we find residual charges.

The true *specific inductive capacity* would be that obtained before any such action on the dielectric had taken place.



Earth. FIG. i. Earth.

§ 3. **Cavendish's Method.**—Cavendish determined, with a degree of accuracy that is surprising when we consider the apparatus that was at his disposal, the value of σ for various dielectrics.

His general method was as follows. He prepared a series of condensers constructed with glass and tin-foil. These he employed as arbitrary standards, with which he compared condensers in which

various other dielectrics were used. In this way he obtained the inductive capacity of these materials as compared with that of the standard glass. Then he determined the ratio borne by the inductive capacity of this glass to that of air, and so obtained finally the *specific* inductive capacity (*see* definition) of the dielectrics employed.

In fig. i. D represents in section the condenser in which the dielectric was the substance whose specific inductive capacity was desired. G represents one or more standard condensers. The upper plates of both were charged from the same source, and therefore to the same potential; and the under plates were put to earth or were at *zero* potential. If the charges upon the upper plates were $+a$ and $+b$ respectively, then those upon the lower plates would be (very nearly) $-a$ and $-b$ respectively; the charges on the upper plates exceeding those on the lower by negligible 'free' charges.

When the condensers were charged, matters were arranged as in fig. ii. It is not difficult to see that if a and b were equal, then the charges $+a$ and $-b$ would (very nearly) neutralise one another. The wire connecting these plates communicated with a delicate electroscope; and thus it could be seen whether there was neutralisation or no. Standard condensers were grouped together until such neutralisation was observed. When this was the case the capacity of G was equal to that of D. For, by our formula,

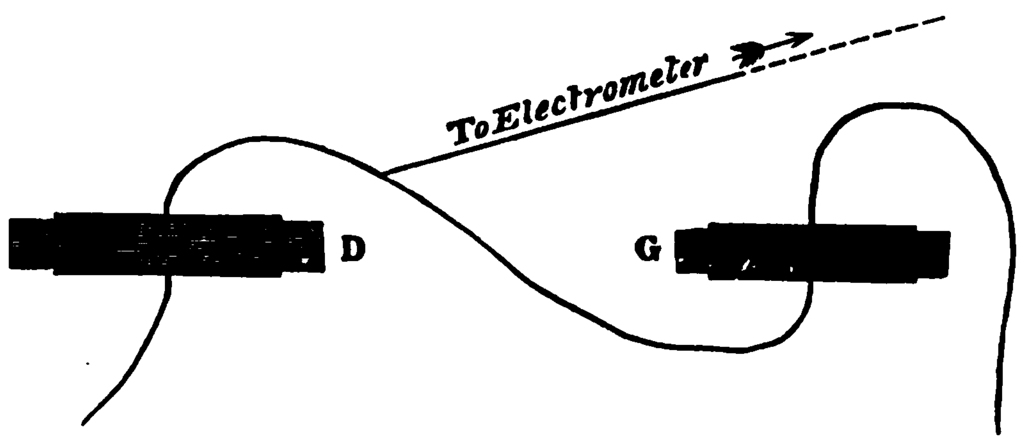


FIG. ii.

$$\begin{cases} a = K' \,(V - V_O) \\ b = K \,\ (V - V_O) \end{cases}$$

where K′ and K were the capacities of the two condensers D and G respectively. And since $(V - V_O)$ was the same for both, it followed that a could only equal b if $K' = K$. Then, from a knowledge of the dimensions of the two condensers, Cavendish determined the ratio of the two inductive capacities, or the ratio that would have existed between the charges a and b had the condensers been similar in all respects save in the nature of the dielectric

In somewhat the same manner he compared the capacities of these standards with that of a large sphere 'isolated' (*see* Chapter V. § 8 &c.) in the middle of the room. From this comparison he was able to calculate what would have been the capacities of the standards had

air been their dielectric, and thus he finally found the ratio that the charge *a* would have borne to that of an exactly similar condenser having air as the dielectric.

§ 4. **Faraday's Method.**—(*a*) The figures represent, complete and in section, the apparatus used by Faraday; this being a condenser of a convenient and symmetrical form. The outer coating consisted of a hollow brass sphere P Q, that was always put to earth. The inner coating was a brass globe C. This could be charged by means of the knob B connected with it. A represents a layer of shellac serving to insulate the wire connecting B and C.

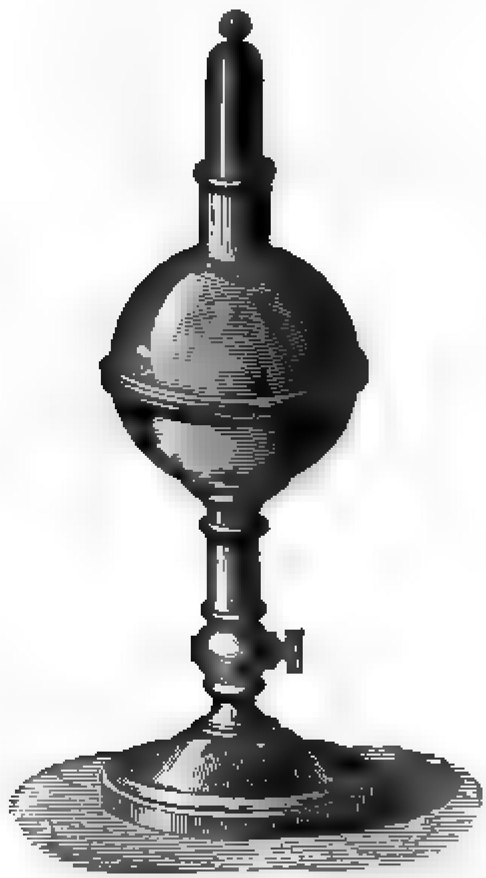


Fig. i.

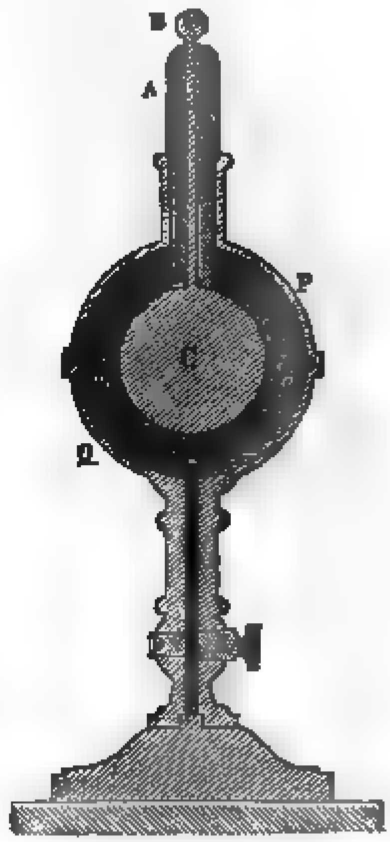


Fig. ii.

Further, the space between P Q and C was air-tight; b means of a tap any gas could be introduced, and P Q could b pulled apart so as to admit of the introduction of various dielec trics into the space between C and P Q.

(*b*) The potentials of B at different times could be *compared* (nc *measured* in absolute units) by touching B with an insulated ba of a certain size, communicating this charge to a torsion balanc (*see* Chapter IV. § 12 (i.)), and observing the torsions needed in th different cases to keep the needle deflected at constant angle.

Note.—As this method of comparing potentials is obsolete, we merely *state* that they can be so compared; we do not go further into the matter.

(*c*) For his experiments it was necessary to have two such condensers that were of exactly equal capacities when filled with air.

To test whether this were the case, he charged the one and measured the potential of its knob B. He then connected this knob B with the knob B′ of the uncharged condenser; thus causing the charge to be divided between the two condensers. He then measured the potential of B again. If the jars were of equal capacity the new potential should be half the old. In fact, in mathematical symbols we have—

$$v = \frac{Q}{K} \;.\;.\;. \text{ in the first case,}$$

$$\text{and} \quad v' = \frac{Q}{2K} = \tfrac{1}{2} v \;.\;.\;. \text{ in the second case,}$$

if the capacities are equal (*see* Chapter VI. § 4).

(*d*) Having provided himself with two similar condensers, he filled one with the substance whose *specific inductive capacity* was required. (For convenience he only *half* filled it, and calculated accordingly.) He then charged the other, or air-condenser, and observed the potential v of its inside coating. The knob B was then connected with the knob of the condenser which had been filled with the dielectric in question and had been left uncharged. The new potential v' of the compound condenser was then measured. As before stated, the outside coatings were to earth. Let K be the capacity of the air condenser, and σ K that of the similar condenser filled with the dielectric in question, where σ is the required specific inductive capacity.

Then we have—

$$v = \frac{Q}{K}\,;\; v' = \frac{Q}{K(1+\sigma)}.$$

$$\text{Whence} \quad \frac{K(1+\sigma)}{K} = \frac{v}{v'}\,; \text{ and } \sigma = \frac{v - v'}{v'}.$$

§ 5. **Modern Methods.**—For the reason given in § 2, methods have been devised to obviate the errors due to penetration of charge. These methods employ rapidly alternating + and − charges, this reversal of charge occurring even as often as 12,000 times per second in some of Gordon's experiments. The condensers

thus never remain charged for more than a fraction of a second; and the charge has, so to speak, no time to penetrate into the body of the dielectric.

Let us suppose that, when this reversal of charge takes place with a certain rapidity, the value of σ is measured; and let us further suppose that, on increasing the rapidity of reversal, no change is made in this value of σ obtained. When this is the case it may fairly be said that errors due to penetration of charge have been obviated.

Gordon's experiments.—Mr. Gordon has made a series of elaborate experiments on the specific inductive capacities of various solid dielectrics, in which the principle of rapid reversal of charge was employed.

As it is not possible to give in a brief space a satisfactory account of the apparatus and methods employed, the reader is referred to Gordon's 'Physical Treatise on Electricity and Magnetism,' where he will find a full description given. It may, however, be mentioned that there are two considerations which may modify considerably the value of the results obtained; the first objection is of undoubted weight, the second is possibly without foundation.

(i.) In calculations concerning the circular brass plates which formed the plates of the condensers, no allowance was made for the variations in distribution which occur within a considerable distance of their edges. These condenser-plates should have been provided with *guard-rings* (*see* Chapter X. § 32).

(ii.) Further, it may reasonably be asked whether, with these rapid reversals of charge, the conditions are truly statical. Whether, in fact, we are obtaining what we desired, viz. the true electrostatical specific inductive capacity of the dielectric. It is *assumed* that these extraordinarily rapid swingings to and fro of electrical charges leave the theory of the condenser unaltered. If this be really the case, it must be because the rapidity with which statical electric equilibrium is attained immeasurably transcends the rapidity of the reversals.

CHAPTER X.

ELECTROSTATIC POTENTIAL.

§ 1. **Introductory.**—In the present chapter we purpose dealing more fully with the conceptions of *electrostatic potential*, *fields* and *lines of electric force*, and *work of electric charge* and *discharge.*

We recommend the student to read again carefully the following portions of the earlier chapters of this course.

(i.) Chapter II. § 8, &c. Here he should find no difficulty in making for himself all such necessary changes in the wording as the substitution of 'a + unit of electricity' for 'a + magnetic pole of unit strength,' &c.

(ii.) The whole of Chapter V.

(iii.) In Chapter VI., all that relates to the formulæ connecting K, V, and Q.

(iv.) Chapter VII. § 1 ; and Chapter VIII. §§ 5 and 6.

He will then be in a better position to understand on what points he needs more exact knowledge and fuller discussion.

§ 2. **Definition and Measurement of Work.**—In addition to the mechanical principles and terms briefly mentioned in Chapter II., we must now assume a knowledge of the principle of *work.*

The student should clearly understand

(i.) What 'work' means; and its essential difference from 'force.'

(ii.) What 'positive' and 'negative' work mean respectively.

(iii.) In what sense, and under what general conditions, we can say that 'when a body is displaced in a field of force from a position A to a position B the work done is independent of the route followed between these two positions.'

In the C.G.S. system the *unit of work* is called the '*erg*,' and is the work done when a force of one *dyne* is overcome through a distance of one *centimètre* along the lines of force.

§ 3. **Dimensions of Work.**—We see that the unit of work [W], or the *erg*, involves the fundamental units in the following way.

$$[W] = [F] \times [L] = [M] \times [A] \times [L] = [M] \times \frac{[V]}{[T]} \times [L];$$

$$\text{of finally} \quad . \quad . \quad . \quad [W] = \frac{[M] \times [L]^2}{[T]^2}.$$

§ 4. **Energy and Conservation of Energy.**—(*a*) If we have a supply of heat we can by a properly constructed engine *do work* and use up (or cause to disappear) some of the heat.

(*b*) If a cannon-ball be moving, we can cause it to *do work* such as the raising of weights, until it comes to rest.

(*c*) If we have a body of water at a higher level, we can cause it to *do work* by means of a water-mill until it is all at the lower level.

(*d*) If an elastic be stretched, we can make it *do work* until it has returned to its unstretched condition.

(*e*) If two substances have chemical affinity for one another, we can let them combine and give out *heat*, and can then cause this heat to be used up in *doing work*, this going on until the chemical affinity is 'satisfied.'

In each of the above cases we had a condition of things such that we could *get work done* (measured in *ergs*, *foot-pounds-weight*, or in any other unit involving the essential product of *force overcome* into *distance through which it is overcome*) at the expense of losing this advantageous condition of things.

Now when a body, or a system of bodies, is in such a condition that it can *do work*, then it or they are said to possess energy. In fact, energy may be sufficiently accurately defined as *capacity for doing work*, and it is measured by the work that can be done, or it is measured in *ergs*.

Where the energy is possessed (as in cases (*a*) and (*b*) above) in virtue of mass and velocity combined, it is called **kinetic energy**. Where it is possessed (as in the other cases) in virtue of position, it is called **potential energy**. In case (*e*) we may call it *chemical potential energy*.

One of the greatest generalisations of modern times, the most powerful weapon we possess in discussing the problems of physical science, is the great law of **conservation of energy**. This law,

abundantly proved by direct experiment and corroborated by masses of more indirect evidence, means that *energy is indestructible*. This means that in a self-contained system—(or a system not acting upon nor acted upon by any other system)—the total amount of energy is constant, though it may undergo transformation, as, *e.g.*, from the form of mechanical-energy into the form of heat-energy.

The student should study carefully the principle of 'conservation of energy' as given in books on mechanics, and should make himself familiar with cases of transformations of energy.

§ 5. **Work against a Constant Force.**—Where the field of force acting on a particle is uniform, the calculation of work done is simple enough.

It is merely necessary to know the force in *dynes* and the distance between the initial and final positions of the particle, as measured along the lines of force, in *centimètres*. Then the product gives us the work in *ergs*.

All work done against gravitation on the surface of the earth comes practically under this head; since, for distances small as compared with the radius of the earth, we may consider the field to be constant.

§ 6. **Work where the Force varies as the Distance.**—Another case of work may here be considered, though it is of no direct interest in our present subject.

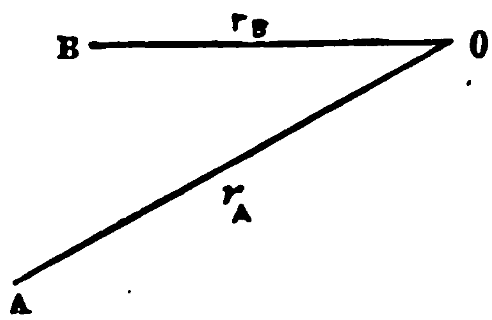
When a piece of india-rubber is stretched from the length it has when under no tension into some new length, we overcome a gradually increasing force. This force is proportional to the extension. If then we start with zero force and end with a force of F *dynes*, the extension being *b centimètres*, it is not difficult to see that

the work done $= (\frac{1}{2} \text{F} \times b)$ *ergs*.

In this case we may call $\frac{1}{2}$ F the *average* force overcome.

§ 7. **Work, where the Force varies as the Inverse Square of the Distance, or as** $\frac{1}{r^2}$.—In our present subject we are concerned mainly with forces that act towards or from *centres* of force, the magnitude of the force, due to any such centre, varying inversely as the square of the distance from that centre. A single magnetic pole, or a single sphere charged with electricity, are simple cases of such centres of force.

We will first discuss this important case without making any reference to the *nature* of the force, whether it be electrical, magnetic, or of any other description; and then we will apply the general results obtained to electrostatics.



Let O represent a centre of force, and let us suppose that there is a particle that is repelled *from* this centre with a force that varies as $\dfrac{1}{(\text{distance})^2}$.

If at 1 *centimètre* the force be measured by F *dynes*, then at distance r_A it will be $\dfrac{F}{(r_A)^2}$, and at distance r_B it will be $\dfrac{F}{(r_B)^2}$, *dynes*; r_A and r_B being measured in *centimètres.*

What work then will be done in moving the particle from A to B? This is evidently no simple matter of arithmetic, for we cannot by any arithmetical means find the *average* force as we did in § 6.

We might work out the problem here without going into any advanced mathematics. But, as the calculation is one which belongs essentially to the *integral calculus*, and is a very easy matter to settle with the aid of this powerful mathematical weapon, we prefer in an elementary Course to give merely the result.

This result is that

$$\begin{array}{c}\text{Work done in moving the}\\ \text{particle from A to B}\end{array} = F.\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \textit{ ergs.}$$

If A be so remote from O that the force at A is quite inappreciable as compared with that at B, or so remote that $\dfrac{1}{(r_A)^2}$ and $\dfrac{1}{(r_A)}$ are practically *zero* as compared with $\dfrac{1}{(r_B)^2}$ and $\dfrac{1}{(r_B)}$, then we may consider A to be at an infinite distance from O. We thus obtain from our formula the result that

$$\begin{array}{c}\text{Work done in bringing the particle}\\ \text{from an infinite distance up to B}\end{array} = \frac{F}{r_B} \textit{ ergs.}$$

The reader is here warned to remember

(i.) That F is the force acting on the particle when at unit distance from O.

(ii.) That r refers to the distance from O up to which the particle is brought, and is *not* the distance through which it is moved.

§ 8. **Potential, and Difference of Potential.**—If the reader will turn back to Chapter V. he will there see explained the general meaning of potential, and the measurement of the same with respect to some chosen zero. The explanation was, however, incomplete, because we had not then considered the question of *Work*.

Referring to the diagram given in § 7, we now understand that there is between A and B a certain *difference of potential*; and that it is measured by the work done on some sort of *unit particle* in bringing this from A to B.

If the field of force be one due to gravitation, and be considered to be practically uniform and acting with a force of 981 *dynes* on 1 gramme mass, or with 1 *dyne* on $\frac{1}{981}$ gramme, then we do *one erg* work in raising our mass of $\frac{1}{981}$ gramme through 1 *centimètre* vertically. Or we could theoretically measure *in centimètres* the difference of *gravitation potential* or *level* between two points by finding the work done in *ergs* in raising a particle of $\frac{1}{981}$ gramme mass from one point to the other.

If the field of force be electrical, then we take as our unit particle a + *unit of electricity*; and we measure the difference of electric-potential between the points A and B by the work that is done on this *test-particle* in moving it from A to B.

If the work is found to be negative, we say that the final position has a lower potential than the initial position.

Note.—The reader must observe that all fields of force can be considered from the potential point of view, and that we may speak of gravitation-potential as well as of magnetic- or electric-potential.

§ 9. **Application of § 7 to the Measurement of Electric-Potential.**—Referring to the diagram given in § 7, let us now consider the case of electrostatic forces.

Let us take as our test-particle (*see* above) a + unit of electricity; and let there be situated at O a quantity of electricity measured by Q units, this giving us an electrical field of force.

By the laws of electrostatics, summed up at the end of § 13, Chapter IV., the force exerted upon our + unit by a quantity of

will at unit distance be measured by Q *dynes*; and, at other distance r from the centre O, by $\frac{Q}{r^2}$ *dynes*.

Let V_A and V_B represent the numerical value of the potentials at A and B respectively; these being measured by the number of *ergs work* done in bringing our + unit from infinity up to the points A and B respectively. Then the formulæ given in § 7 tell us that—

$$\text{The absolute potential } V_A \text{ of the point A} = \frac{Q}{r_A}.$$

$$\text{The absolute potential } V_B \text{ of the point B} = \frac{Q}{r_B}.$$

$$\text{The difference of potential, } V_B - V_A, \text{ between the points A and B} = Q\left(\frac{1}{r_B} - \frac{1}{r_A}\right).$$

An example or two will make the meaning clearer.

(i.) Let there be + 24 units of electricity at the point O; let r_B be 4 *cms.*, and let r_A be 6 *cms.*

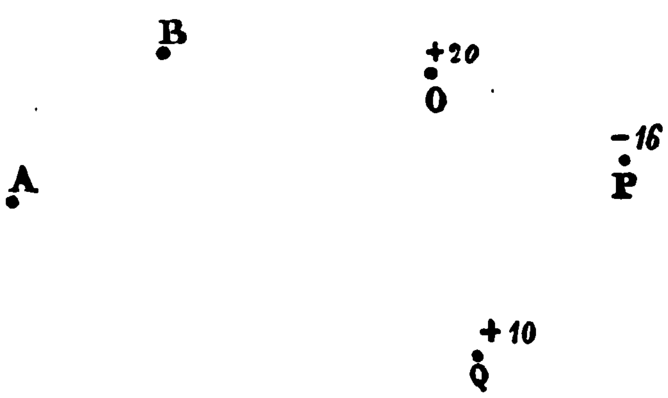
Then at unit distance (or 1 *cm.*) from O our + unit will be repelled with a force of 24 *dynes*; at B with a force of $\frac{24}{4^2} = \frac{3}{2}$ *dynes*; and at A with a force of $\frac{24}{6^2} = \frac{2}{3}$ *dynes*.

The potential of B will be greater than that of A by an amount measured by

$$V_B - V_A = 24\left(\frac{1}{4} - \frac{1}{6}\right) = 2.$$

Or it will take 2 *ergs work* to move our + unit from A up to B.

The absolute potential of B will be measured by $\frac{24}{4} = 6$; and that of A by $\frac{24}{6} = 4$; or it will take 6 *ergs* to move our + unit from infinite distance up to B, and 4 *ergs* to move it from infinity up to A.



(ii.) Let there be several quantities of electricity, as + 20 at O, − 16 at P, + 10 at Q. These will give rise to a somewhat complex field of force. But still the results arrived at in § 7 and in the present section enable

us to find easily the potentials of any points as B and A in the field, and the difference of potentials.

For the total work due to this system of electric charges will be the algebraic sum of the works due to each charge separately.

Thus the absolute potential of B will be measured by $\frac{20}{BO} + \frac{10}{BQ} - \frac{16}{BP}$; and that of A by $\frac{20}{AO} + \frac{10}{AQ} - \frac{16}{AP}$. Hence the difference of potential, $V_B - V_A$, is easily found by subtraction.

(iii.) Where there are a whole series of quantities $q_1, q_2, q_3 \dots$ situated at distances $r_1, r_2, r_3 \dots$ respectively from a point B, then the potential of B as due to these quantities will be $\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots$; which is conveniently represented by $\Sigma \frac{q}{r_B}$. Here, of course, attention must be paid to the *signs* of the quantities q, &c.

$$\text{And} \quad V_B - V_A = \Sigma \frac{q}{r_B} - \Sigma \frac{q}{r_A}.$$

(iv.) Where the quantities are distributed continuously over a conductor, or over any surface, this adding up of $\frac{q_1}{r_1} + \dots$ &c. becomes in general a matter for the application of the *integral calculus.*

Note.—It may occur to the reader that if r be zero, or if we touch the charge Q, then the potential becomes infinite, since $\frac{Q}{O} = \infty$. In answer to this we may remark that it is physically impossible to be at zero distance from any finite mass, or electrical or magnetic quantity. We can be at zero distance only from a geometric point. Hence, we cannot have Q finite and r zero simultaneously.

§ 10. Equipotential Surfaces.

(i.) If we so move our + unit of electricity as not to move up or down the lines of force, then we do no work against the electrical forces. Hence, by definition, all the positions thus arrived at must have *the same potential.*

Let us consider the case of a simple field of force due to a single charge of Q units situated at the point O (*see* § 7, figure). If we move our + unit over any sphere that has O as centre, we

do no work; and hence every point on such a sphere has the same potential.

Thus if we consider the sphere that has O as centre and that passes through the point A, it is clear that all points on this sphere have the same potential as A; a potential that is measured by $\frac{Q}{r_A}$ *ergs*.

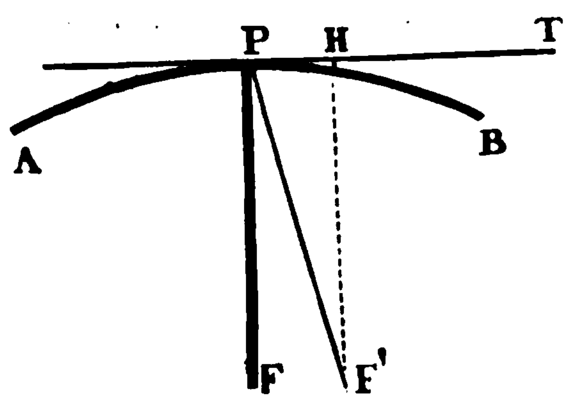
It is necessary to do work to the amount of $Q\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$ *ergs* to move our + unit of electricity from *any* point on the sphere passing through A to *any* point on that passing through B.

(ii.) *Equipotential surfaces in general.*—Where there are many centres of force, there both the lines of force and the equipotential surfaces will be complex, and will not have the simple forms that they had in the last case.

But in all cases certain general results hold good. The equipotential surfaces are always perpendicular to the lines of force (*see* § 11); they may be mapped out into a series of 'marked' surfaces such that we do one *erg* work in moving our + unit between two consecutive surfaces; these marked surfaces will be closer together where the field is stronger, further apart where the field is weaker, and at a constant distance where the field is uniform.

§ 11. **Lines of Force are Perpendicular to Equipotential Surfaces.**—This has been already proved implicitly; or rather it is a consequence of the very definition of an 'equipotential surface.' But it may be well to give a formal demonstration.

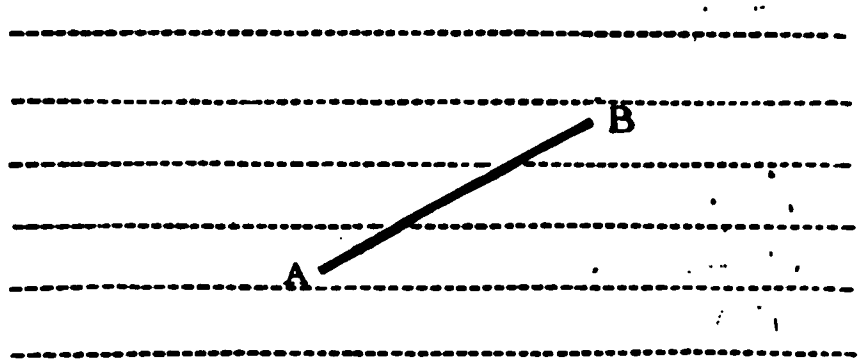
Let P F be a line of force; and let A P B be a section of the equipotential surface that passes through P; and let P T be a section of the tangent-plane to this surface, at the point P.



Then shall P F be perpendicular to the tangent P T. For if possible let it lie as P F′; making with P T an angle less than 90°. We can now resolve the force P F into two components, along the tangent P T and perpendicular to it respectively. For the former component we have P F cos F′ P T; and this is *not* zero, since F′ P T is not 90°. Since then there is a tangential component, we should do work in moving our + unit along

the surface near P. But this is contrary to hypothesis, since A P B is an equipotential surface. Therefore, F P must be perpendicular to A P B.

§ 12. **Field-strength; and Rate of Change of Potential.**—The diagram represents a portion of a field of force; this portion being so small that we may consider it to be uniform, or the lines of force to be parallel (*see* § 13, coroll.). Let A and B be any two points in this field, and let them be separated by a distance of A B *cms.*



If A B be not perpendicular to the lines of force, *i.e.* if A and B be not on the same equipotential surface, there will be a force on a + unit of electricity acting along the line A B. We may represent the force on a + unit that acts from B to A by the symbol F^B_A; this will be measured in *dynes*, and may be + or − as it acts from B to A or from A to B. [See § 1 (i.); and compare Chapter II. § 8.]

By the definition of 'field-strength,' &c., it is clear that F^B_A measures that component of the field-strength that has the direction B A.

Then, by the definition of work, we have

$$\text{The work done in moving our + unit from A to B} = F^B_A \times AB \textit{ ergs}.$$

But, by the definition of potential, this work measures the difference of potential between A and B; or we have $(V_B - V_A) = F^B_A \times AB$.

Whence
$$F^B_A = \frac{V_B - V_A}{AB}.$$

Now since A B is measured in *centimètres*, this last expression represents the change of potential per unit length, or the *rate of change of potential*, along A B.

So we may translate this formula $\left(F^B_A = \frac{V_B - V_A}{AB}\right)$ into words somewhat as follows.

In a field of force, the component in any direction of the force acting upon a + unit, i.e. the component in any direction of the total field-strength, is measured by the space-rate of change of potential in that direction.

When the potential is constant there is zero field-strength, and conversely.

It is very important to remember that a field of force implies varying potential, and conversely.

Examples.—(i.) In the inside of a charged vessel we may have a potential of very great magnitude; but it is constant throughout, and we have zero field of force.

(ii.) Half-way between two equal charges of the same sign we have a certain potential; but there is a point of zero force.

(iii.) Half-way between two equal charges of opposite signs we have zero potential, but not zero field of force.

§ 13. **The Mapping Out of Lines of Force; Simple Case.**—Since we can draw a line of force through every point in the field—such a line being defined as the direction in which a + unit of electricity is urged—it follows that the lines of force are infinite in number.

But a little further consideration will show us that here, as in the case of equipotential surfaces, we can mark a certain number out of this infinite crowd of lines, in such a way as to represent the various strengths of different parts of the field.

Let us as before consider the simple case of a single charge of Q units collected at the point O (*see* fig. § 7); and let us consider first a spherical surface, described about O as centre, with radius 1 *centimètre*. At any point on this sphere the force exerted on a + unit will be Q *dynes*; or the strength of the field will be measured by Q.

The reader must remember that in our present case the lines of force, which are infinite in number, radiate from O; and that they cut at right angles the surfaces of all spheres which have O as centre.

Now let us mark out (we may suppose that we paint them *red*) such a number of lines of force that exactly Q of them pierce each 1 *square centimètre* of this spherical surface. Then the number of lines piercing unit area placed perpendicularly to the lines of

ce does, over this spherical surface, represent numerically the ength of the field.

At 2 *centimètres* distant from O these marked lines will have read out over a sphere of four times the area; and therefore ere will be $\frac{1}{4}$ Q lines piercing each 1 *square centimètre* of the here whose radius is 2 *centimètres*. And, in general, at any stance r from O the lines will have so thinned out that the imber piercing each 1 *square centimètre* of the sphere, whose dius is r, will be $\frac{Q}{r^2}$.

But $\frac{Q}{4}, \frac{Q}{9}, \ldots \frac{Q}{r^2}$, also represent the strengths of the eld at distances 2 *centimètres*, 3 *centimètres*, r *centimètres*, from O respectively. Therefore these marked lines will represent umerically the strength of the field at *any* point, by the number of them that at that point cut 1 *square centimètre* held perpendicular o them.

The reader should carefully study this method of marking out he lines. He will see that it consists in marking out such a number that they do over *one* equipotential surface (viz. over the sphere of unit radius) represent, by the number piercing each 1 *square centimètre*, the field-strength at all points over this surface. He will see also that the result found to hold is that these lines so chosen will indicate the field-strength at *any* distance from O, in just the same manner.

Of course where the force on + unit is less than 1 *dyne*, or the field-strength is less than unity, there will be less than 1 line piercing each 1 *square centimètre*. We must then take several *square centimètres* and divide the number of lines piercing them by the number of *square centimètres* taken.

Corollary. Uniform field.—Hence it follows that in a uniform field the marked lines of force are parallel and equidistant from one another.

§ 14. **General Case.**—We cannot in an elementary Course *prove* that the same result holds in the general case; but at least the above discussion will prepare the reader for the following statement. It may be well to point out that 'equipotential surface,' and 'surface lying perpendicularly to the lines of force' are synonymous expressions.

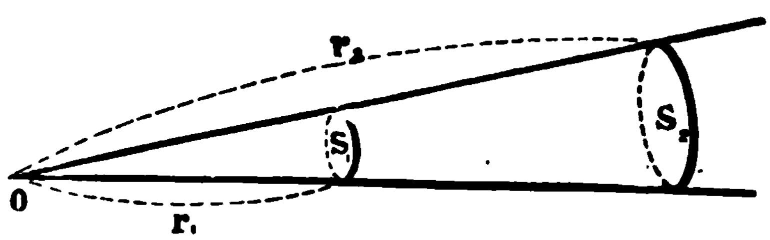
'*If we mark out such a number of the lines of force that on any one equipotential surface they measure, by the number piercing each* 1 *square centimètre, the field-strengths all over that equipotential surface, then will these same marked lines in any part of the field measure the field-strength by the number piercing* 1 *square centimètre held perpendicularly to the lines of force at the place in question.*'

It is to be noted that when we use the expression 'number of lines of force,' we refer to these *marked* lines.

§ 15. **Total Number of marked Lines of Force.**—In the simple case given in § 13, since we mark Q lines of force for each 1 *square centimètre* of the sphere of 1 *centimètre* radius, and since such a sphere has an area of 4π *square centimètres*, it follows that we have a number $4\pi Q$ of such marked lines.

And if there be any system of quantities of electricity + or —, and if the whole system of quantities be surrounded by an envelope, then the total number of + lines piercing this envelope will be $4\pi\Sigma Q$; where ΣQ is the algebraic sum of the quantities.

§ 16. **Tubes of Force. '$F\sigma$ is Constant.'**—We may, out of the infinite crowd of lines of force, construct hollow tubes bounded by the lines. Such tubes are called *tubes of force.*



Now lines of force cannot cross; for, if they did, it would mean that at the point where they crossed there were *two* resultant forces, which would be absurd. Hence, if such a tube once include any number n of the marked lines, it will always include these same lines, neither more nor fewer.

Let us consider the simple case of § 13, and let us suppose the area of the perpendicular cross section of such a tube to be σ_1 *square centimètres.* Then if the tube enclose n lines of force, there are n lines of force piercing σ_1 *square centimètres*; or the average force on our + unit will be measured by $\frac{n}{\sigma_1}$, according to the results of § 13. If we name this average force F_1, then the product of (*average force over cross section*) into (*the cross section*) will be

$$F_1 \times \sigma_1 = \frac{n}{\sigma_1} \times \sigma_1 = n.$$

If we take another section σ_2 of the same tube we have for erage force the value $\frac{n}{\sigma_2}$; and the product $F_2 \times \sigma_2$ still equals n. ence we have for the same tube of force the result that $\sigma_1 = F_2\sigma_2 = \ldots = n =$ constant; which is more shortly ted as—

$$F\sigma = \text{constant.}$$

If we take an oblique section σ', and take the force F' resolved rpendicular to this oblique section, then $F' = F \cos\theta$, and $\sigma' = \sigma \sec\theta$, ere θ is the angle that the oblique section makes with the perpencular section. Hence the product $F'\sigma' = F\sigma =$ constant.

We may state this property of tubes of force as follows.

If σ be the area of cross section of a tube of force, and F *be the verage force on + unit over this area σ resolved perpendicularly to , then the product* $F\sigma$ *is constant; it equals n, where n is the number f marked lines of force enclosed by the tube in question.*

We have proved the above only for the simple case of § 13; but e here state that it is true for any field, however complex.

A tube of force in which one marked line is enclosed, or a ube where $F\sigma = 1$, is called a *unit tube of force.*

§ 17. **Statement of some further Theorems on Lines of Force.**

(1) It can be shown that a tube of force terminates against qual and opposite quantities of electricity, $+q$ and $-q$ respecively. This statement includes Faraday's law of electrostatic nduction referred to and discussed in Chapter IV. § 16.

(2) Lines of force and tubes of force do so terminate when hey meet a conducting surface. Any lines of force on the other ide of the conducting surface are either lines that have curved ound part the edge of the conductor, or are independent lines lue to other charged systems.

(3) *Hollow conductor acting as a screen.*—The diagram repreents in section a hollow conductor X Y separating two systems of lectrical charges, *a b c* external, and *p q r* internal.

Now theory and experiment show that no lines of force due to *b c* exist inside X Y (*see* Chapter IV. § 13 (ii.)). Hence, if X Y s to earth the system *p q r* is totally unaffected by *a b c*; and if

X Y be not to earth the only influence of *a b c* will be to raise lower the potential of X Y and of its interior as a whole; they not in either case alter the nature of the field inside.

Again, both theory and experiment show that the external eff of the presence of the system *p q r* inside X Y is to cause th to be on the external surfa of X Y a charge of magnitu equal to the algebraic sum $p + q + r$; but that this e ternal charge is distribute simply according to the shap of X Y and to the distributio of external bodies, not accord ing to the position of *p*, *q*, an *r* (*see* Chapter IV. § 16 (*c*)).

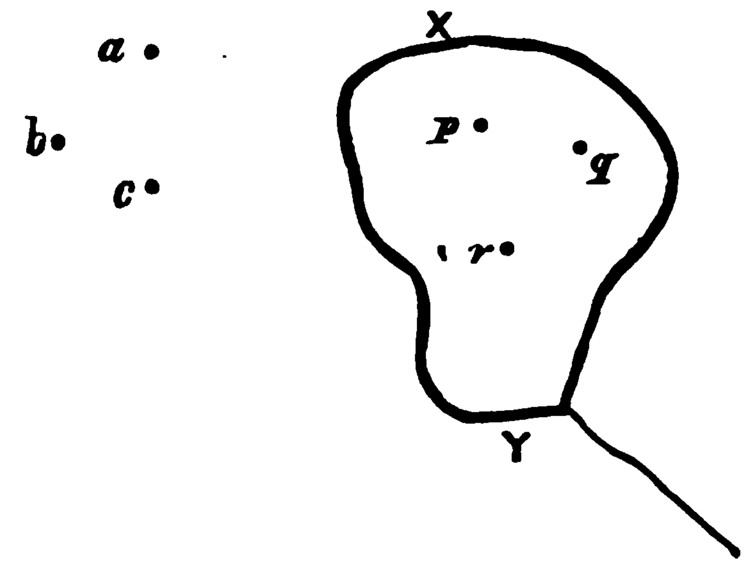


FIG. i.

Hence X Y acts as a *screen* between the two systems.

So we may use large conducting plates put to earth as fairly complete screens. A wire-gauze case will completely screen an electroscope (*see* note).

(4) *Potential at an external point due to uniformly charged concentric spheres.*—It can be shown by the integral calculus that if we have a series of concentric spheres of centre C, having charges Q_1, Q_2, Q_3 respectively, then the potential at an external point P due to these charges will be the same as if they were all collected at the common centre C. Or

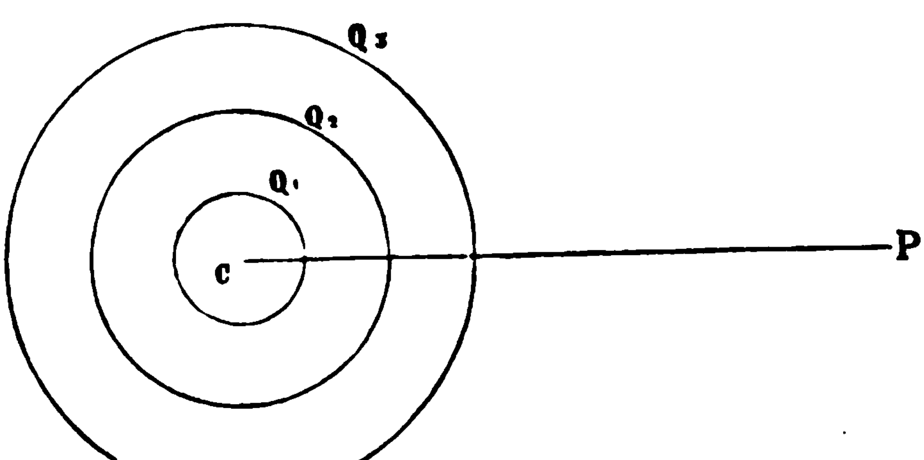


FIG. ii.

$$V_P = \frac{Q_1 + Q_2 + Q_3 + \ldots}{PC}.$$

Note on insulated spherical screens.—We will consider here two simple cases of uninsulated and uncharged screens, showing what effect their presence has upon the potential of an external point. The reader can easily draw for himself the simple diagrams that represent the two cases considered.

..) Let there be a charged sphere, or a charged particle, A; and let there a external point P at a distance R from A. Then if the charge on A be iits, the potential V_P of P is measured by $\frac{Q}{R}$. If now an insulated and un-ged spherical screen B be placed round A *concentrically* with it, between it the point P, we know both by theory and experiment that the field at P is only to the 'free' charge that has been 'repelled to the surface of B; the rge + Q on A, and the induced charge − Q on the inner surface of B, ether give a zero field at any point P outside. But, by (4) above, the ential at P will be the same as if the charge Q on the sphere B were col-ted at its centre. That is, we have still $V_P = \frac{Q}{R}$. Hence, when A and B : *concentric*, the presence of the insulated spherical screen B makes no ference in the potential of external points.

(ii.) Next, let B be no longer concentric with A; but let the distance from P the centre of A be R, while to the centre of B the distance from P is r. Then thout the screen we have $V_P = \frac{Q}{R}$ as before. But, with the screen, we have ›w $V_P = \frac{Q}{r}$; since it is only the free charge on the screen that gives a field the external point P.

(iii.) If any charge Q' be given to B, we have merely to add it to the duced charge Q that has been inductively given to its surface; and the otential at P becomes

$$V_P = \frac{Q+Q'}{R}, \text{ or } = \frac{Q+Q'}{r}$$

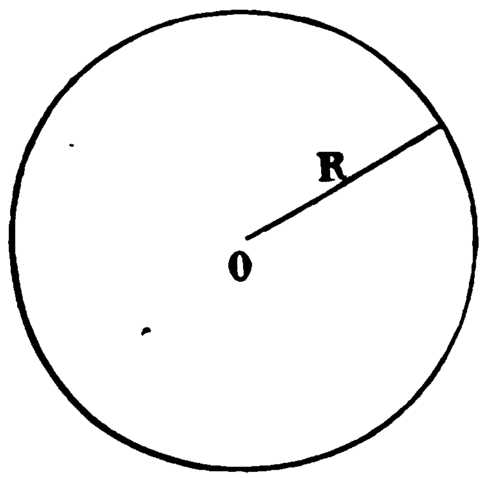
ι the two cases respectively.

§ 18. **The Potential of an 'Isolated' Body.**—Potential is a roperty of a point in space, as explained earlier. But any equi-otential surface has one potential, the same for all points on it. Iow any continuous homogeneous conductor (as a brass sphere r cylinder, a tin vessel, &c.) forms an equipotential surface; for o differences of potential could exist on a surface where electrical eadjustment takes place instantly. Hence we can speak of 'the otential of a conductor.'

The potential may be a hard matter to calculate, owing to the ontinuous nature of the distribution of electricity over the body, hich makes the expression $\Sigma\frac{q}{r}$ of § 9 (iii.) in general a matter or the *integral calculus*. But we can easily deal with the case of sphere.

§ 19. **Potential of an Isolated Sphere.**—The figure represents n section a sphere of centre O and radius R. Let it have on

it a charge Q, which we will consider to be divided into a series of small charges q_1, q_2, q_3, &c., distributed over the surface of the sphere. What is the potential of the sphere? Or, what will be the work required to bring our + unit from infinity up to the sphere?



Now the sphere has the same potential all over it, and inside it. Hence we may find the potential of the most convenient point, viz. of the centre O.

But by § 13 this is $$V_0 = \frac{q_1}{R} + \frac{q_2}{R} + \ldots$$

or $$V_0 = \frac{Q}{R}.$$

Hence, for the sphere we have $V = \frac{Q}{R}$.

Now let us consider what we mean by 'an *isolated* sphere.' There is, on the walls, &c., at a distance (let us suppose) of *r centimètres* from the centre O, a charge equal to $-$ Q. Hence the *true* potential of O and of the sphere will be

$$V = \frac{Q}{R} - \frac{Q}{r}.$$

But if r is so large as compared with R that the latter term $\frac{Q}{r}$ may be neglected, then we have $V = \frac{Q}{R}$.

This shows clearly the meaning of *isolated*; and also shows how, for the same charge, the potential will be smaller as r becomes smaller, or as the walls close in (*see* Chapter V. § 8).

§ 20. **Capacity of an Isolated Sphere.**—By the definition of capacity given in Chapter V. § 4, we have for an isolated sphere

$$V = \frac{Q}{K}.$$

But we have just found that $V = \frac{Q}{R}$. Hence it follows that $K = R$. Or we arrive at the result that

The capacity of an isolated sphere is measured numerically by its radius expressed in centimetres.

Note.—The reasoning just employed is of very general use. Suppose that we find by calculation that

$$\begin{cases} V = l\,.\,Q & \text{for an isolated body;} \\ \text{or } V = h\,.\,Q & \text{for a condenser;} \end{cases}$$

where l and h involve only one or more of the following quantities, viz. dimensions of the plates, distance between them, and nature of the dielectric. We may then argue that, by the definition of capacity given in Chapter V. § 4, and Chapter VI. § 4, it follows that $K = l$, or $K = h$, in the two cases respectively.

§ 21. **Distribution; from the Potential Point of View.**—In hydrostatics it is frequently very useful to consider the distribution of a liquid to be such as will cause every part of the free surface to be at the same level. The more immediate cause of the distribution of a liquid is the action of gravitation-forces; and a level free surface rather follows from this. But still the former view is often convenient.

So, in electrostatics, the cause of distribution is the direct force acting on each elementary portion of the charge. But here also it is often convenient to regard the distribution on a conductor as such as will cause each part of it to be at the same electrostatic potential. This last *must* be the resulting condition, since otherwise we could not have equilibrium.

We will take one case as an example.

'What must be the general distribution on an elongated cylinder that it may be all at one V;' or, we might say, 'that it may take the same work to bring up our + unit to any part of it?'

If the distribution were uniform, it would certainly require more work to bring up our + unit to the central portion, into the presence of the whole charge, than to bring it up to the end, out of the way of most of the charge. But, if the charge be more crowded at the ends, we see how it may now require the same work in the latter case as in the former. Thus, the well-known distribution on an elongated cylinder is such as to cause the whole to be at one potential, though it more directly results from the equilibrium of the tangential components of the electrostatic forces.

§ 22. **Two Spheres of Different Radii.**—Let us consider two spheres of different radii, R_1 and R_2.

(1) If they have the same charge Q, then

$$V_1 = \frac{Q}{R_1};\ V_2 = \frac{Q}{R_2};$$

$$\text{or } \frac{V_1}{V_2} = \frac{R_2}{R_1}.$$

(2) If they have the same potential V, then

$$V = \frac{Q_1}{R_1}; \ V = \frac{Q_2}{R_2};$$

$$\text{or} \quad \frac{Q_1}{Q_2} = \frac{R_1}{R_2}.$$

These results should be translated into words, and noted carefully by the student.

§ 23. **Potential, and Density, distinguished.**—By density of charge, we mean *quantity of charge per unit area*. We use the symbol ρ to signify the numerical value of *density* of charge. We have, by definition,

$$\rho = \frac{Q}{S}$$

where Q is the quantity on the area S; all of course in C.G.S. units.

Now we have seen that, on any conductor but a sphere, the *density* is not constant all over, while the *potential* is. At a point the *density* may be very great, though the *potential* be small.

We will illustrate the difference and the connection between density and potential by the case of spheres.

(i.) Consider two spheres of *equal radius* R. Then, for each, $V = \frac{Q}{R}$; $\rho = \frac{Q}{4\pi R^2}$. Hence, at the same potentials the densities are equal, and both V and ρ are proportional to Q.

(ii.) But now consider two spheres of *different radii* R_1 and R_2, and let them have the same potential V. Then for the two respectively we have $Q_1 = VR_1$; $Q_2 = VR_2$;

$$\text{and} \begin{cases} \rho_1 = \dfrac{Q_1}{4\pi R_1^2} = \dfrac{VR_1}{4\pi R_1^2} = \dfrac{V}{4\pi R_1}; \\[2ex] \rho_2 = \dfrac{Q_2}{4\pi R_2^2} = \dfrac{VR_2}{4\pi R_2^2} = \dfrac{V}{4\pi R_2}. \end{cases}$$

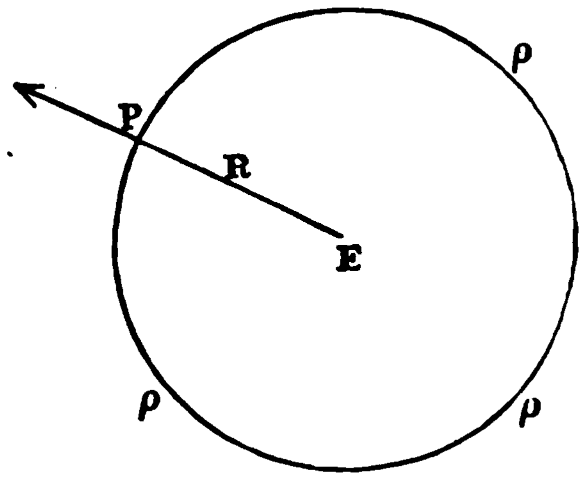
Whence ρ_1 and ρ_2 are not the same, and $\frac{\rho_1}{\rho_2} = \frac{R_2}{R_1}$.

§ 24. **Force on a + Unit, acting Perpendicularly to a Conducting Surface.**

It is for many purposes very important to know how great is the force on + unit at a surface, urging it normally to the surface. The resultant force must evidently be normal, since any tangential force would imply that the conducting surface was not at one potential.

We will take the case of a sphere. Let the diagram represent a

sphere of radius R, and total charge Q; and let P be a point on the sphere. It is required to find the force at P acting on + unit.



Now by the result of calculation, *stated* in § 17 (4), we learned that the action of the charged sphere on a point external to it is the same as if the whole charge Q were collected at the centre E. This is true quite up to the surface; or is true when we take our point P to be just *on* the external surface. Hence on a + unit at P there is acting a normal force equal to $\frac{Q}{R^2}$ *dynes*.

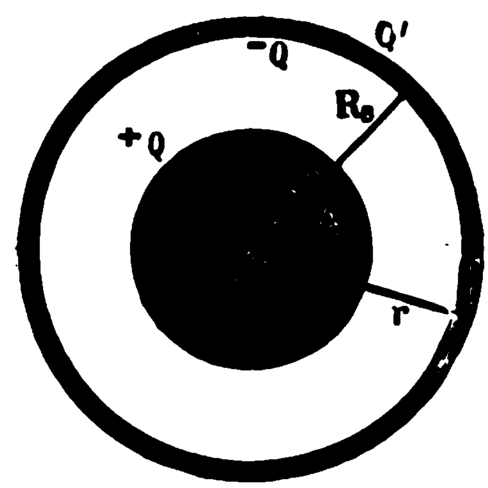
But, if ρ be the uniform density of charge over the sphere, then $\rho = \frac{Q}{4\pi R^2}$, or $Q = 4\pi R^2 \rho$. Hence, for the force F acting on + unit at the surface, we get $F = 4\pi\rho$ in *dynes*.

Now if the radius of the sphere increase without limit, so that the surface at P becomes ultimately *plane*, no change will occur in the expression just found, provided that ρ remains constant.

Hence the formula is true for a *plane* surface on which the density is ρ. It is indeed true for *any* surface, ρ being the density at the point considered. This general result is, in more advanced Courses, proved independently, without reference to a sphere.

§ 25. **Important Case of a Spherical Condenser.**—We will now show how to find the capacity of a spherical condenser.

As in the case of an isolated sphere we found *two* expressions for the *potential* of the sphere, and by equating them found the capacity of the sphere; so here, in the case of two spherical surfaces forming a condenser, we shall find *two* expressions for the *difference of potentials* of the two surfaces, and then by equating them shall find an expression for the capacity of the condenser. The figure represents the *ideal* spherical condenser, in which there are two concentric spherical conductors, the inner being totally enclosed by the outer. In the *practical* condenser we must have an opening through the outer conductor, and an insulated wire and knob connected with the inner; as in Faraday's condenser, Chapter IX. § 4.



Let C be the centre. Let the radius of the inner conductor be R_A *cms.*, and the charge on it be $+Q$; then there will be on the inside surface of the outer conductor a charge of $-Q$, by Chapter IV. § 16. Let the inner radius of this outer conductor be R_B *cms.*, and its outer radius be r *cms.*; and let there be on the outer surface of B a charge Q'.

As we wish to find an expression for the difference of potential $(V_A - V_B)$ between the two surfaces, we will find V_A and V_B separately first. The reader must note that Q is the ***bound*** charge of Chapter VI.

Now V_A is the potential of any point on, or in, A, since A is a conductor containing no insulated charged bodies; and, therefore, V_A is the potential at C.

$$\text{Hence} \quad V_A = \frac{Q}{R_A} + \frac{-Q}{R_B} + \frac{Q'}{r}$$

by the results of § 19.

And V_B will be the same as the potential of a point just on the outer surface of B. But, by § 17 (4), this will be the same as that which would be due to all the charges collected at the common centre C.

That is,

$$V_B = \frac{Q - Q + Q'}{r} = \frac{Q'}{r}.$$

Hence we get

$$(V_A - V_B) = Q\left(\frac{1}{R_A} - \frac{1}{R_B}\right) \quad . \quad . \quad . \quad . \quad . \quad (\alpha)$$

as *one* expression for the difference of potential between A and B.

But, by the definition of capacity, we have also that

$$(V_A - V_B) = \frac{Q}{K}$$

since Q is the 'bound' charge Chapter VI. § 4.

Hence, equating, we have

$$K = \frac{R_A R_B}{R_B - R_A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

The quantity Q', and the outer radius r, have both disappeared from the expression. And this is what we should expect, since Q' has opposite to it on the walls, &c., a charge equal to $-Q'$, and these two charges with the intervening air as a dielectric form *another*

(relatively insignificant) condensing system with which we are not concerned in our present discussion.

The above calculation is based upon the supposition that the dielectric *is air*. For our quantity Q is measured in terms of a unit which repels another unit at 1 *cm.* distance with 1 *dyne* force *in air*; and our expressions for measurement of potential depend on this condition.

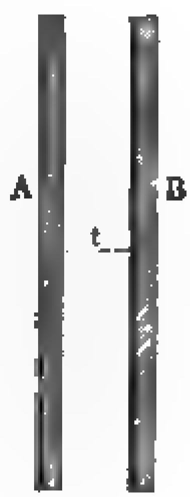
Now, if we have any other dielectric of *specific inductive capacity* σ, then by the definition of σ given in Chapter IX. we must write σ . K instead of K; and, if K′ is the new capacity, we have

$$\left.\begin{aligned} K' &= \sigma . K = \sigma . \frac{R_A \times R_B}{R_B - R_A}; \\ \text{and } (V_A - V_B) &= \frac{Q}{K'} = \frac{Q}{\sigma}\left(\frac{1}{R_A} - \frac{1}{R_B}\right). \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\gamma)$$

The reader will notice that this is the expression that we should arrive at if we assumed, all through our calculations of potential as measured by work done, that in the new medium the quantity Q acted on our + unit with a force of $\frac{1}{\sigma} . \frac{Q}{r^2}$ *dynes* at a distance r, instead of the $\frac{Q}{r^2}$ *dynes* with which it acts on the + unit when air is the medium. Hence we conclude that, in a medium of *specific inductive capacity* σ, the force between two quantities Q and Q′ (measured in the absolute unit of Chapter V. § 1), separated by a distance r, will be $\frac{1}{\sigma} . \frac{Q . Q'}{r^2}$ *dynes.*

We may write the above relation in the form

$$Q = \sigma . \frac{R_A R_B}{R_B - R_A} . (V_A - V_B) \quad . \quad . \quad . \quad . \quad (\delta)$$



§ 26. **The Plate Condenser.**—In the diagram we give a simple section of a plate condenser A B. Let V_A and V_B represent the potentials of A and B respectively; let Q be the 'bound' charges on the two plates respectively; let t be the perpendicular distance between them, measured of course in *centimètres*; let S be the area of the inner surface of either A or B; let ρ be the density of charge on these inside surfaces; let σ be the *specific inductive capacity* of the dielectric.

Now we shall assume that t is so small as compared with the size of the plates that the field of force between A and B is

uniform ; *i.e.* that it consists of lines of force that are parallel and equidistant, which implies that ρ is uniform.

Then by § 12 we have for the field strength F, or force acting on a + unit, between A and B,

$$F = \frac{V_A - V_B}{t}.$$

But by § 24 we have that at the surface of the plate

$$F = 4\pi\rho.$$

Now it is clear that $\rho = \frac{Q}{S}$; since ρ is assumed to be uniform.

Hence we have $V_A - V_B = 4\pi\rho t$

or $$(V_A - V_B) = Q.\frac{4\pi t}{S} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\alpha)$$

But by definition we have $(V_A - V_B) = \frac{Q}{K}$.

Whence

$$K = \frac{S}{4\pi t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

In the above we have assumed *air* to be the dielectric. But if we have another dielectric of *specific inductive capacity* = σ, then the formulæ become

$$\left.\begin{aligned} V_A - V_B &= Q.\frac{4\pi t}{\sigma S} \\ K &= \frac{\sigma S}{4\pi t} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (\gamma)$$

§ 27. **Formulæ for Capacities, &c.**—For convenience we here put together the chief formulæ that have occurred up to the present point in this chapter, adding also one or two more.

The following remarks will apply to some of them, if not to all.

(*a*) All the measurements are in the C.G.S. system of units.

(*b*) Electric quantity is measured by reference to force exerted when *air* is the dielectric. When there is another dielectric of *specific inductive capacity* σ, the force exerted is $\frac{1}{\sigma}$th that exerted through air (*see* § 25, end). The capacity of a condenser with this dielectric will by definition be σ times that of a similar air condenser.

(*c*) An ordinary condenser is really a *double* system, as pointed out immediately after equation (β) in § 25. The 'isolated' body is the limiting case of such a condenser, when this has become a *single* system. The formulæ for isolated bodies can be derived from these for condensers, though it is often more simple to find them independently.

I. *Formulæ connecting V, Q, and K.*

(i.) For 'isolated' bodies, $Q. = K.V.$
(ii.) For condensers, $Q = K.(V_1 - V_2)$.

In the former case Q is the sole charge; in the latter Q is the 'bound' charge.

II. *Capacities of some 'isolated' bodies.*

(i.) For a sphere of radius R, *in air*, $K = R$.

(ii.) For a disc of radius r and of negligible thickness, *in air*,

$$K = \frac{2r}{\pi}.$$

(iii.) For a cylinder of length l and of relatively small radius r, *in air*, $K = \dfrac{l}{2\log_e \dfrac{l}{r}}$.

A *wire* comes under the head of (iii.). The two last formulæ have not been proved in the foregoing.

III. *Capacities of some condensers.*

(i.) For a spherical condenser where the radius of the inside sphere is R_1, and that of the inner surface of the outer sphere is R_2,

$$K = \frac{\sigma . R_1 R_2}{R_2 - R_1}.$$

(ii.) For two co-axial cylinders of common length l, the radius of the inner being r_1, and that of the inner surface of the outer cylinder being r_2,

$$K = \frac{\sigma . l}{2\log_e \dfrac{r_2}{r_1}}.$$

(iii.) For two discs, that surface of either on which is the bound charge being S, and the distance between them being t,

$$K = \frac{\sigma . S}{4\pi t}.$$

This formula applies to any case of parallel surfaces where the dimensions are very large as compared with t. In fact, the formula (i.) above can be transformed into this when R_1 and R_2 are very great as compared with $(R_2 - R_1)$.

§ 28. **Energy of Charging and Discharging.**—When electricity falls from a higher to a lower level, work can be done; we lose a certain quantity of electrical potential energy, and we must have an equivalent in heat or mechanical work, &c. If conversely we raise electricity from a lower level to a higher, we expend this work and gain the above electrical potential-energy which is an equivalent (*see* § 4).

There is no more important matter in our present science of electricity than this question of the application of the law of 'Conservation of energy' to electrical charge and discharge. We will consider the matter under three heads.

Case I. The case where an electrical quantity Q passes between two conductors of fixed potentials V_1 *and* V_2.—Now by definition and measurement of potential, we do on each + unit an amount of + or − work that is measured by $(V_1 - V_2)$ *ergs*. Hence, on the quantity of Q units we do + or − work measured by $Q . (V_1 - V_2)$ *ergs*.

If one potential be V and the other be zero, the work is $Q . V$ *ergs*.

Hence, in this case the energy E of charge or discharge is measured by

$$\left.\begin{array}{ll} & E = Q . (V_1 - V_2) \textit{ ergs} \\ \text{or} & E = Q . V \quad \textit{ergs} \end{array}\right\} \quad \ldots\ldots \quad (a)$$

Case II. The case, more usual in electrostatics, where the potential alters during charge or discharge.—We will consider the simplest case of charging a conductor, that is originally at zero potential, with a quantity Q, up to a potential V.

Here the potential of the conductor rises in arithmetical progression with the charge given, since

$$\text{potential} = \frac{\text{charge}}{\text{capacity}},$$

and capacity is constant. The result, therefore, is that the same work is done as if the *whole* charge were raised to *half* the final potential V. Hence the work of charging, or energy expended,

will be

$$\left.\begin{aligned} E &= \tfrac{1}{2} Q V \\ \text{or } E &= \tfrac{1}{2} K V^2 \\ \text{or } E &= \tfrac{1}{2} \frac{Q^2}{K} \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (\beta)$$

since $V = \frac{Q}{K}$.

This same expression gives the energy of discharge.

Analogies.—(*a*) So, if we reckon the number of *foot-pounds-weight* done in building with P lbs. of bricks a tower of H feet high, it being assumed that the tower is cylindrical or rises in arithmetical progression with the amount of bricks used, we get as a result the same work as if *all* the bricks were raised to *half* the vertical height H ; *i.e.* we get

$$\text{work} = \tfrac{1}{2} P \,.\, H \,.\, \textit{foot-pounds-weight.}$$

(*b*) A similar expression is obtained for work done by water flowing out of a cylindrical reservoir of an average height of $\frac{1}{2}$ H above the lower level.

Case III. Case of discharge of a condenser.—In the ordinary condenser we have two plates of potentials V_1 and V_2 respectively.

In this case the discharge will cause the equal and opposite charges $+Q$ and $-Q$ to disappear, leaving the whole condenser at some potential v due to the 'free' charge. Taking the two charges $+Q$ and $-Q$ separately, we have for the former

$$\text{energy} = \tfrac{1}{2} Q (V_1 - v),$$

and for the latter

$$\text{energy} = \tfrac{1}{2} (-Q) (V_2 - v).$$

Adding the two we get

$$E = \tfrac{1}{2} Q \,.\, (V_1 - V_2) \quad \ldots \ldots \ldots \quad (\gamma)$$

which is the same as result (β), if the one coating of the condenser be at zero potential.

The reader should notice that in the case of the Leyden jar charged to potential V of inner coating, the outer coating being to earth, we have for the discharge of the jar the expression $\frac{1}{2}$ Q V.

§ 29. **Examples in Energy of Discharge.**

(i.) 'A conductor is charged with 8 units to potential 7. What is the energy of discharge?'

$$\text{Here } E = \tfrac{1}{2} Q V = \tfrac{1}{2} \,.\, 7 \,.\, 8 = 28 \textit{ ergs.}$$

'And what was its capacity?'

$$\text{Here } K = \frac{Q}{V} = \frac{8}{7}.$$

(ii.) 'A conductor of capacity 10 is charged with 4 units. What is the energy of discharge?'

$$\text{Here } E = \tfrac{1}{2} Q V = \tfrac{1}{2}\frac{Q^2}{K} = \frac{16}{2 \times 10} = \frac{4}{5} \textit{ ergs}.$$

(iii.) 'A sphere of radius 4 cm. is charged with 12 units. Find the energy of discharge.'

$$\text{Here } K = 4 \ (\textit{see } \S\ 19)$$

$$\text{and } E = \tfrac{1}{2}\frac{Q^2}{K} = \frac{144}{8} = 18 \textit{ ergs}.$$

Two harder examples are worked out in § 35.

§ 30. **Energy of Discharge in the Cascade Arrangement of Leyden Jars.**—We must now ask the reader to turn back to Chapter VI. § 9, and to notice the arrangement there described and the notation employed.

We found that if there were n equal jars in cascade, the difference of potential between the inside and outside coatings of each jar was $\frac{1}{n}$th that of a single jar charged from the same source and having its outside coating to earth, the charge of each jar being also $\frac{1}{n}$th that of the single jar fully charged.

It follows that, in the case of the single jar thus fully charged, we have for the energy of discharge E the value

$$E = \tfrac{1}{2} Q V \textit{ ergs} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{i.})$$

while, for all the n jars of the cascade, we have for the total energy of discharge E′ the value

$$E' = n \times \tfrac{1}{2} . \frac{Q}{n} . \frac{V}{n} = \frac{1}{n} . \tfrac{1}{2} Q V = \frac{1}{n} . E \quad . \quad . \quad . \quad (\text{ii.})$$

This result may be better understood if we give an analogy.

Analogy.—If instead of one tower 100 feet in height we build 10 towers each having $\frac{1}{10}$th of the height, and $\frac{1}{10}$th of the number of bricks of the large one, a little thought will show us that the total work done in the second case will be $\frac{1}{10}$th of that done in the first case; for in the second case the same total weight of bricks will have been raised through $\frac{1}{10}$th of the former average height.

§ 31. **Electroscopes and Electrometers.**—In general these instruments give their indications by movements. These movements are due to the action of a field of force on electrical quantity, the extent of the movement being such that equilibrium is finally arrived at between electrical forces on the one hand and gravitation-, magnetic-, or torsional-forces on the other hand.

We can have movements when there is only one charged body, *e.g.* when this body is situated nearer to one wall of a room than to the other. Here there is a field of force between the body and the walls, and this field is strongest in one direction. Now, besides the effect of a field on a charged body situated in it, we know that the two sides of a field tend to close in; or we may say 'the lines of force tend to shorten.' Hence the charged body will move so as to close up the strongest portion of the field due to it.

(*a*) *In a gold leaf electroscope* each leaf is screened from one wall by the other leaf. Hence, there is no wavering between two walls about equally distant, but each leaf moves off towards the wall from which it is not screened.

This view, or one like it, is more in accordance with present ideas than is the view of 'repulsion between the two leaves.'

(*b*) *Bohnenberger's electroscope.*—Here a single charged leaf moves along the field of force that lies between two brass knobs; these brass knobs being maintained at different potentials by being connected with the two poles respectively of a dry pile.

In both cases here given we *indicate* differences of potential by the extent of the movement; since in case (*a*) the field is stronger according as the leaves are at a larger or smaller difference of potential from the zero potential of the walls, and in case (*b*) the movement of the single leaf in the fixed field of force depends on the potential up to which this single leaf is charged.

§ 32. **Electrometers. The 'Attracted-Disc' Form.**—In electrometers we aim at measuring *differences of potential*, or *absolute potentials* with respect to the earth as zero.

Consider the plate condenser of § 26, and make the same assumptions as to uniformity of field and of ρ as were there made.

It can be shown that the total stress F between the plates is given by the formula

$$F = \frac{S(V_1 - V_2)^2}{8\pi t^2}.$$

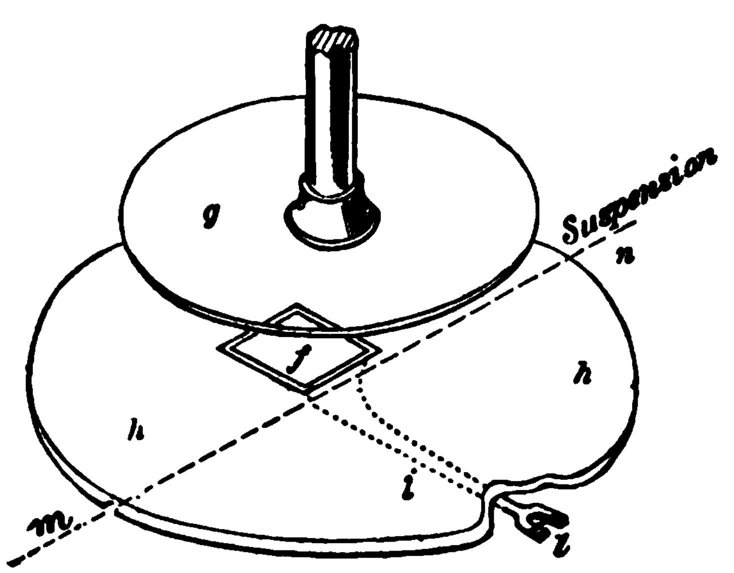
This expression therefore gives in *dynes* the total force with which one plate is urged towards the other ; and, if we measure F by counterbalancing 'weights,' each *gramme weight* being about 981 *dynes force*, we can express $(V_1 - V_2)$ in absolute units.

But in the above apparatus there is error ; for the assumptions made do not hold near the edges of the plates, and this error is not one to be readily allowed for.

Hence, in the actual *attracted-disc electrometer*, Sir W. Thomson adopted a contrivance called a *guard-ring*. The moveable disc was cut out in the centre of a larger disc.

Then, since the fixed and moveable portions were connected and at one potential, and since there was no perceptible break between them, it followed that with respect to the moveable portion of the one disc, and the central portion of the other fixed disc that was opposite and parallel to the former, the assumptions as to uniformity did hold. In fact, only the central portions of the discs were used.

In the figure we have the arrangement indicated. The two plates are *h h* and *g*; and the formula applies to the moveable portion *f* of the one plate and the portion opposite to this of the other fixed plate. The moveable portion *f* is at the end of a light lever whose fulcrum, or axis of suspension, is a wire *m n*. At the free end of this lever is a hair *l*; this is viewed by a lens, and serves to indicate when the plate *f* is flush with the guard-plate *h h*.



The *force* required to keep *f* in its place gives us F. Whence, by knowing the dimensions, &c., of the instrument, we measure $(V_1 - V_2)$.

§ 33. **Sir William Thomson's Quadrant Electrometer.**—The principle of the quadrant electrometer can be illustrated by the Bohnenberger's electroscope.

If the gold-leaf were kept at a constant high potential, while the difference of potential between the two knobs was the variable quantity, then the amount of deflexion of the gold-leaf would indicate the difference of potential between the two knobs.

The next figure gives a sketch, taken looking down directly from above, of the essential parts of the quadrant electrometer.

Let us take a cylindrical box of thin brass about one inch high and about five inches in diameter. Let us cut this into four sectors as indicated in the figure, and let us connect opposite sectors respectively by wire, and support all four sectors on insulating glass legs.

We thus have four hollow brass sectors; in the interior of each of which, as an approximately 'closed vessel,' there is, excepting near the edges, a constant potential. The opposite pairs of these are connected.

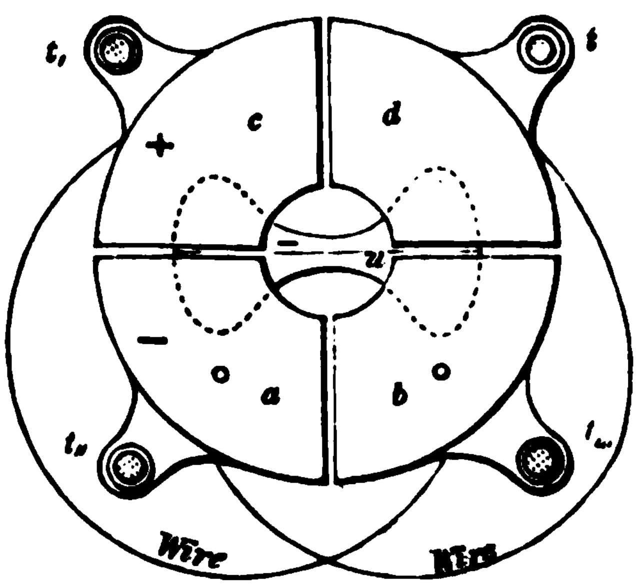


FIG. 1.

By u is represented a light aluminium needle suspended by two parallel silk fibres—this being called *bi-filar suspension.*

Hence there will be called into play, when the needle is deflected from its position of rest, a couple tending to restore it. The value of this couple for any given angle of deflexion can be calculated. This needle swings horizontally in the interior of the sectors; and matters must be so arranged that it may come to rest exactly along the line of one of the slits. The diagram represents the needle thus at rest, unacted upon by any electrical forces.

This needle is maintained at some constant potential V, by being connected with the inside coating of a charged Leyden jar.

If the one pair of sectors c and b be at a potential V_1, and the other pair a and d at some lower potential V_2, there will be a field of force running across the gap or slit from c to a; and from symmetry, an equal field will run across the gap from b to d.

Moreover, this field will be confined to a region only somewh
wider than these gaps; the region in the interior of each secti
will be practically of a uniform potential, or will not be a field
force.

From the symmetry of the whole, the needle will be act
upon by a pure couple due to the electrical field. And, from i
peculiar shape, that amount of the needle which is in the field
force (*i.e.* the portion lying under the slit) will remain constan
since only very small deflexions are employed. We thus contriv
that there shall act on the needle, for any given values of V_1, V_2
and V, a constant electrostatic couple whatever be its deflexion;
provided that this deflexion do not exceed a certain maximum
depending on the construction of the instrument.

This constant couple will deflect the needle until the restoring
couple, due to the twisting of the two parallel silk fibres, is sufficiently great to give equilibrium. If the potential V of the needle be known, and if the 'constants' of the instrument be known, then the difference of potential ($V_1 - V_2$) can be calculated from the observed deflexion.

Fig. 11.

In the next figure we have a sketch of the 'Elliott-pattern' quadrant electrometer; a comparatively simple form of instrument. One of the quadrants is represented as removed, so that the needle may be seen. This needle is maintained at a high potential in somewhat the following manner. From it there hangs a platinum wire, dipping into a glass vessel that contains strong sulphuric acid. Outside this vessel is a coating of tin-foil, so that it is in fact a Leyden jar. Its capacity being very great as compared with the capacity of an isolated body, it serves to maintain the potential of the needle approximately constant for a considerable time.

From the needle rises a light stem, bearing a small mirror.

The usual 'lamp and scale' arrangement gives us, in the reflected spot of light, a very sensitive means of noticing and measuring the deflexions of the needle.

Complete form of the quadrant electrometer.—In the complete form of Sir W. Thomson's quadrant electrometer there are many details of construction that we shall not discuss here. But we must mention two of the most important of these.

(i.) *The gauge.*—Connected with the inside of the Leyden jar (and so with the needle) is an *attracted-disc electrometer.* If the weights or spring be so arranged that the hair (*see* § 32) is in the proper line of sight when the jar, needle, and disc *f* therewith connected, are at some fixed potential V, then any alteration in this potential will be at once detected by the movement of the hair. This is one detail.

(ii.) *The replenisher.*—The other detail is a *replenisher* (*see* Chapter VII. § 5), in which one armature is to earth while the other is connected with the Leyden jar.

We can thus remedy any error in V by turning the replenisher the one way or the other, so as to raise or lower the potential of the jar.

Formula for quadrant electrometer.—It can be shown that the following formula holds for the absolute quadrant electrometer.

$$L = h\left(V - \frac{V_1 + V_2}{2}\right)(V_1 - V_2) \quad . \quad . \quad . \quad . \quad . \quad (\alpha)$$

where L is the electrostatic couple acting on the needle, balanced by and equal to the mechanical restoring couple; V the potential of the needle; V_1 and V_2 the potentials of the two pairs of quadrants respectively, and h a constant depending on the construction of the instrument.

If the needle be charged to a relatively very high potential, then we may neglect $\frac{V_1 + V_2}{2}$ in comparison with V; and we have

$$L = h \, . \, V \, . \, (V_1 - V_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

This relation may be written as follows.

$$V_1 - V_2 = \frac{h \, . \, V}{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\gamma)$$

We see then that we could measure the difference of potential $(V_1 - V_2)$ by observation of the deflexion; provided that we knew

the values of h, V, and L. In practice, however, these quantities are not calculated; but the instrument is graduated by means of a 'standard voltaic cell,' or by comparison with an absolute electrometer of the more simple 'attracted-disc' form.

In instruments devoid of the *gauge* and the replenisher, as well as in the above form, we may write . . . .

$$V_1 - V_2 = b \cdot \tan\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$

so long as the potential V of the needle remains constant. And, for small deflexions, we have . . . .

$$V_1 - V_2 = b.\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad [\textit{nearly}].$$

Here θ is the deflexion; and the constant b may be determined on each occasion of use by means of a 'standard voltaic cell.'

§ 34. Uses of the Quadrant Electrometer.

(i.) *General use.*—The quadrant electrometer is mainly an instrument for the measurement, or comparison, of potentials and potential-differences. We have indicated in the above how it is possible to use simpler forms also, as well as the 'standard' patterns of the instrument, for such purposes of measurement.

It may be added that usually one pair of quadrants is put to earth, so as to be at *zero* potential; the potential of the other pair of quadrants being then measured with respect to the earth as zero.

(ii.) *Use as an electroscope.*—The instrument can also be used for all purposes in which we desire merely a delicate electro*scope*, and not an electro*meter*. For such purposes any delicate form will serve, even though it be so faulty in construction that the law of its action (*see* § 33, end) is unknown. An important example of such a use will be given in Chapter XIV. § 10.

(iii.) *Use in investigating the distribution on a conductor* (*see* Chapter IV. § 16).—In the investigation of distribution upon conductors, of which we said something in Chapter IV. § 16, &c., it would at first sight appear impossible to use the electrometer. For, it would be argued, where we are dealing with a conductor whose potential is constant in every part, how can we employ an instrument that indicates only potential-differences?

A few words of explanation will enable the student to perceive how the instrument may be used in such investigation.

When we lay the proof-plane against any part of the conductor whose potential is V, it will also be at the potential V; but the charge that it acquires will depend upon the density ρ that exists at the part touched. The fact is that the capacity of the proof-plane changes as we change its position on the conductor; and so, though its charge is proportional to ρ, its potential is always V.

But when we remove the proof-plane and place it in an *isolated* position, its capacity assumes some constant value; and its potential will therefore be proportional to its charge, and so will be proportional to ρ.

When the proof-plane is connected with the one pair of quadrants of the electrometer, the combined conductors will have a potential that is directly proportional to the charge, and inversely proportional to their combined capacity. Since the latter remains constant, it follows that the quadrants will assume a potential that is directly proportional to ρ.

§ 35. **Examples in Energy of Discharge, &c.**—(*a*) In all cases where it is required to find the energy of discharge between two electrical systems A and B, the following method will be found to be a good one. First, find the energies of discharge of A to earth, and of B to earth, separately; and add these quantities Secondly, find what will be the energy of discharge to earth of the combined system A and B after these have been connected. It will then follow, by conservation of energy, that the difference between these two results will give the required energy of discharge between the two systems when they are connected with one another. Let us take an example.

'There are 3 Leyden jars A, B, and C, equal in capacity, having their outer coatings connected with earth. A is first charged. Its knob is then connected with the knob of B. It is then disconnected from B, and connected with C. Finally, the knob of B is connected with the knobs of A and C. Find the energies of the several discharges.'

(i.) The initial discharge of A to earth would give energy to the amount of $\frac{1}{2}$ Q V; and the final discharge of A and B together to earth would give energy measured by $\frac{1}{2}\,Q\,.\,\frac{V}{2}$, or $\frac{1}{4}$ Q V. Since energy cannot have been lost, it follows that the difference between these two quantities must measure the energy w_1 of discharge between A and B when these two were connected. Hence

$$w_1 = \frac{1}{2} QV - \frac{1}{4} QV = \frac{1}{4} QV.$$

(ii.) The discharge of A to earth would now give $\frac{1}{2} \cdot \frac{Q}{2} \cdot \frac{V}{2}$, or $\frac{1}{8} QV$. And the subsequent discharge of A and C together to earth would give $\frac{1}{2} \cdot \frac{Q}{2} \cdot \frac{V}{4}$, or $\frac{1}{16} QV$. Hence, energy w_2 of discharge between A and C is given by the difference, or

$$w_2 = \frac{1}{8} QV - \frac{1}{16} QV = \frac{1}{16} QV.$$

(iii.) The discharge of B to earth, and of A and C to earth, would give together $\left(\frac{1}{2} \cdot \frac{Q}{2} \cdot \frac{V}{2} + \frac{1}{2} \cdot \frac{Q}{2} \cdot \frac{V}{4}\right) = \frac{3}{16} QV$. And the discharge of all three together to earth would give $\frac{1}{2} \cdot Q \cdot \frac{V}{3}$, or $\frac{1}{6} QV$. Hence the energy w_3 of discharge when B is connected with A and C is given by the difference, or

$$w_3 = \frac{3}{16} QV - \frac{1}{6} QV = \frac{1}{48} QV.$$

It is to be noticed that all the discharges added together give the energy $\frac{1}{2} QV$, which was that of the original discharge of A to earth. If this result had not followed there must have been some error in the above work; since 'Conservation of energy' demands that no energy be lost.

(β) 'A sphere A of 9 *cms.* diameter is connected by a *thin* wire (*i.e.* one of negligible capacity) with another sphere B of the same diameter. Round this latter, and concentric with it so as to form a spherical condenser, is a larger sphere of 10 *cms.* internal diameter, connected with earth. A charge of 33 units is given to A. Find (i.) in what proportion this charge is distributed between the two systems; and (ii.) the energy of discharge of the two systems separately.'

Let V_A be the potential of A, and therefore also of B.
„ C_A be the capacity of A, and C_B that of B.
„ Q_A be the charge on A, and Q_B that on B.
„ w_A be the energy of discharge of A, and w_B that of B.

(i.) Then C_A = radius $= \frac{9}{2}$. And, by the formula of a spherical condenser, $C_B = 1 \div \left(\frac{1}{4\frac{1}{2}} - \frac{1}{5}\right) = 45$.

$$\left\{\begin{aligned} &\text{Hence } Q_A = \frac{\frac{9}{2}}{\frac{9}{2}+45} \text{ of } 33 = \frac{9}{99} \text{ of } 33 = 3 \text{ units,} \\ &\text{and } Q_B = \text{the remainder} \qquad\qquad = 30 \text{ units.} \end{aligned}\right.$$

(ii.) Again $V_A = V_B = \frac{Q_A}{C_A} = \frac{Q_B}{C_B} = \frac{2}{3}$.

$$\left\{\begin{aligned} &\text{Hence } w_A = \frac{1}{2} Q_A V_A = \frac{1}{2} \times 3 \times \frac{2}{3} = 1, \\ &\qquad\quad w_B = \frac{1}{2} Q_B V_B = \frac{1}{2} \times 30 \times \frac{2}{3} = 10. \end{aligned}\right.$$

§ 36. **General Consideration of Electrostatic Fields of Force.** It is probable that among all the important conceptions that are due in the first place to the genius and insight of Faraday, none has done more to place the physical theory of electrical and of magnetic phenomena upon a sound basis than his recognition of the part played by the medium across which the forces act. The former view of 'action at a distance,' and the purely geometric conception of lines of force, were essentially mathematical ideas; to the physicist they were both unreal and unsuggestive.

Taking our present case of electrostatic fields, we may explain the modern view, based upon Faraday's conception, somewhat as follows.

There is no such thing as 'a + charge' or 'a − charge' by itself; but on the contrary, wherever electrostatic phenomena occur there is an electrostatic field, on the two sides of which occur equal + and − charges respectively. The whole system consists of these equal and opposite charges separated by a dielectric which is the seat of the field of force. The lines of force have a real physical meaning. Whether they are lines along which the dielectric undergoes a kind of tension, or whether they are lines along which the molecules of the dielectrics are 'polarised' by a separation of + and − charges in them, is not yet known. But it seems certain that the electrostatic potential energy, implied by the existence of an electrostatic field, resides in the dielectric that separates the two sides of the field; somewhat as mechanical potential energy resides in a bent spring, a stretched piece of elastic, or a strained elastic solid.

Thus, when we speak of 'an isolated body charged with + Q

units to a potential V, whose energy of discharge is measured by $\frac{1}{2}$ Q V,' we really mean that there is on the body a charge + Q, on the walls or other surrounding surfaces an equal and opposite charge − Q, and that in the strained dielectric resides energy to the amount of $\frac{1}{2}$ Q V.

Dielectrics are bodies in which this electrostatic strain can be maintained; while *conductors* are those in which no such strain can be kept up, and in which therefore lines of force and fields of force cannot exist. In reality, bodies are not capable of sharp division into these two classes, since the yielding to electrostatic stress is only a matter of time. But in general we make use of bodies at the two extremes of the series; so that the foregoing definitions will serve very well for usual cases.

It is part of the above view to say that tubes of force are always terminated against surfaces so charged that at the two ends of each tube there are equal and opposite charges $+q$ and $-q$ respectively (*see* § 17).

It is also part of it to say that conductors act as *screens*; for lines of force cannot penetrate a conductor, and hence if an electrostatic field exist on each side of the screen, these two fields are independent of one another.

If lines of force were material, the phenomena of the electrostatic field would suggest that they act as stretched elastic threads, which moreover repel one another. Thus the two sides of a field are urged towards one another as if by tension of the lines of force connecting them.

CHAPTER XI.

THE PHENOMENA OF ELECTRIC CURRENTS. BATTERY CELLS AND BATTERIES.

§ 1. **Introductory.**—In Chapters IV.–X. we have dealt mainly with the phenomena connected with the actions between charged conductors, and with the strains of the intervening medium. We have, in fact, considered mainly *electrostatic fields of force*, or fields in which our test-charge of electricity is urged along lines of force.

We did, indeed, examine to some extent the *results* of discharge; but in general the readjustment of electrical equilibrium was so rapid that it was not easy to investigate the phenomena accompanying discharge. With such machines as the Holtz we might, however, have done so; but not so conveniently as with the help of other apparatus to be described in the present chapter.

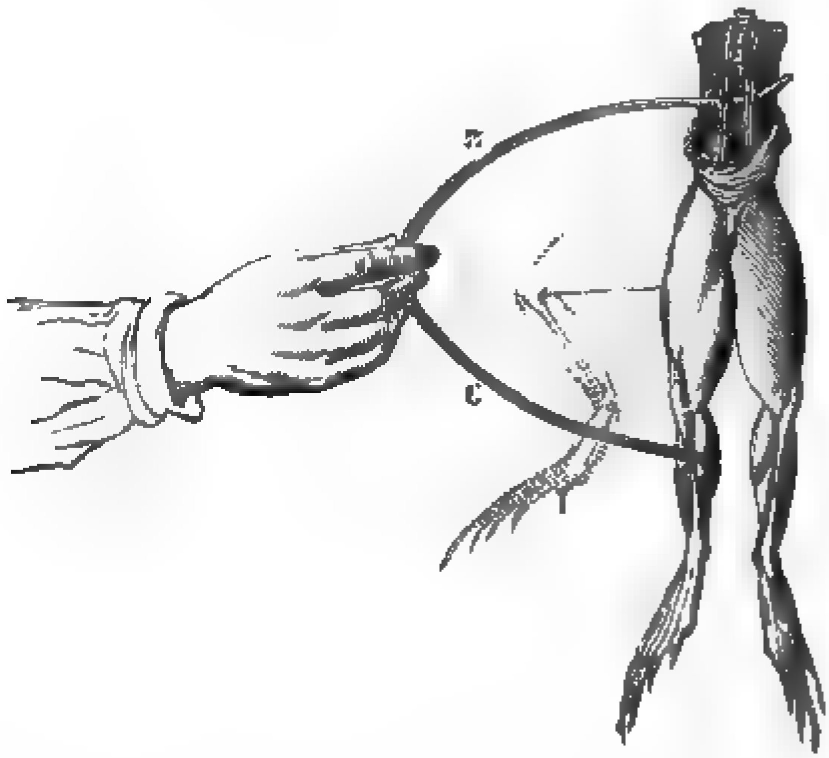
We shall find that, during discharge, new classes of phenomena arise. In particular there are observed chemical, and heat, phenomena; and a new field of force, viz. a magnetic field (or a field in which a magnetic pole is urged), springs into existence.

In our next division of the subject we enter into the consideration of the phenomena accompanying electric discharge. As a rule we shall have very small $\Delta V.s$ as compared with those hitherto employed, but very large quantities of electricity, and an even and continuous flow. Where this is not the case, attention will be drawn to the fact.

The above remarks will indicate that the popular terms 'statical electricity' and 'current electricity' must not be understood in their literal sense of two kinds of electricity; but must be considered to refer to two classes of phenomena that require different conditions of ΔV and of quantity for their investigation.

§ 2. **Galvani's Experiment.**—As stated in the preface, we shall not enter into any historical account of this or of any portion of our subject. But we must mention that experiment of Galvani's which, perhaps more than any other, started Volta on his very fruitful line of inquiry.

In the figure, Z C is a compound bar of zinc and copper, as indicated by the two letters employed. The zinc end is put into contact with the lumbar nerves of a frog's hinder quarters. It is found that whenever the copper end touches the muscles of the legs there is a sudden convulsion of the limb. This experiment



was first performed in 1786. Galvani considered that the metals acted merely as conductors to complete the circuit of 'animal electricity,' and followed out this idea in further investigations. As we do not intend to discuss in the present Course the relations of electricity to physiology, we shall not say more as to Galvani's views.

Volta, on the other hand, fixed his attention on the metals, and considered that they by their contact caused the current which the nerves and muscles of the frog's leg conducted. We shall have more to say as to his views, and as to modern modifications of them. For the present we will merely follow his experi-

ments and the explanations that he gave; using, however, some terms, such as 'potential,' which he did not use.

§ 3. **Volta's Experiments and Views.**—For clearness we will first *state* Volta's views, and will then give some of the experiments by which he supported them. The student can see how far each statement is supported or unsupported by the experiments given.

Volta's views.—(1) When two heterogeneous substances are in contact they are found to be at different potentials.

(2) As a rule any ΔV.*s* between liquid and liquid, or metal and liquid, are negligible as compared with the ΔV.*s* between metals and metals.

(3) *Metals* can be arranged in a certain series (such as *zinc, tin, lead, iron, copper, silver, platinum, graphite*), with respect to which the following facts hold: that any metal in contact with another occurring later in the list will have a higher potential than this latter; and that in a series of several metals in contact the ΔV between the first and last is the same as it would be were these metals directly in contact. Thus, if we make the potential of graphite *zero*, and find the potentials of all the other metals with respect to this body when in contact with it, then we can calculate the ΔV between any pair in contact by simple subtraction of their potentials with respect to graphite.

(4) This law, however, does not hold good with respect to a series in contact when composed partly of metals and partly of liquids.

Thus, whereas in the series *copper|zinc|gold|copper* there is no ΔV between the terminal metals, because it is the same as if the initial and final metals (viz. *copper|copper*) were directly in contact, yet in the series *copper|zinc|dilute acid|copper* there may be, and as a matter of fact there is, a ΔV between the terminal metals.

Experiments illustrating Volta's views.—In these experiments Volta employed the condensing gold-leaf electroscope, whose principle is explained in Chapter VI. § 14.

But far more satisfactory results can be obtained by use of any form of quadrant electrometer (say the 'Elliott' form of Chapter X. § 33), in which one pair of quadrants are to earth; such an instrument indicates quantitative results with an accuracy sufficient for lecture experiments.

(i.) The figure represents a simple experiment performed with the con-

densing gold-leaf electroscope and a compound bar composed of a piece of zinc and a piece of copper soldered end to end.

We may use the quadrant electrometer, putting one pair of quadrants to earth, and connecting the other pair with an insulated terminal.

If the zinc be held in the hand, while we touch the lower condensing plate of the gold-leaf electroscope (or terminal of the electrometer) with the copper, it will be found that the condensing plate (or insulated pair of quadrants) is now at a − potential.

(For use of electroscope and electrometer *see* Chapter X.)

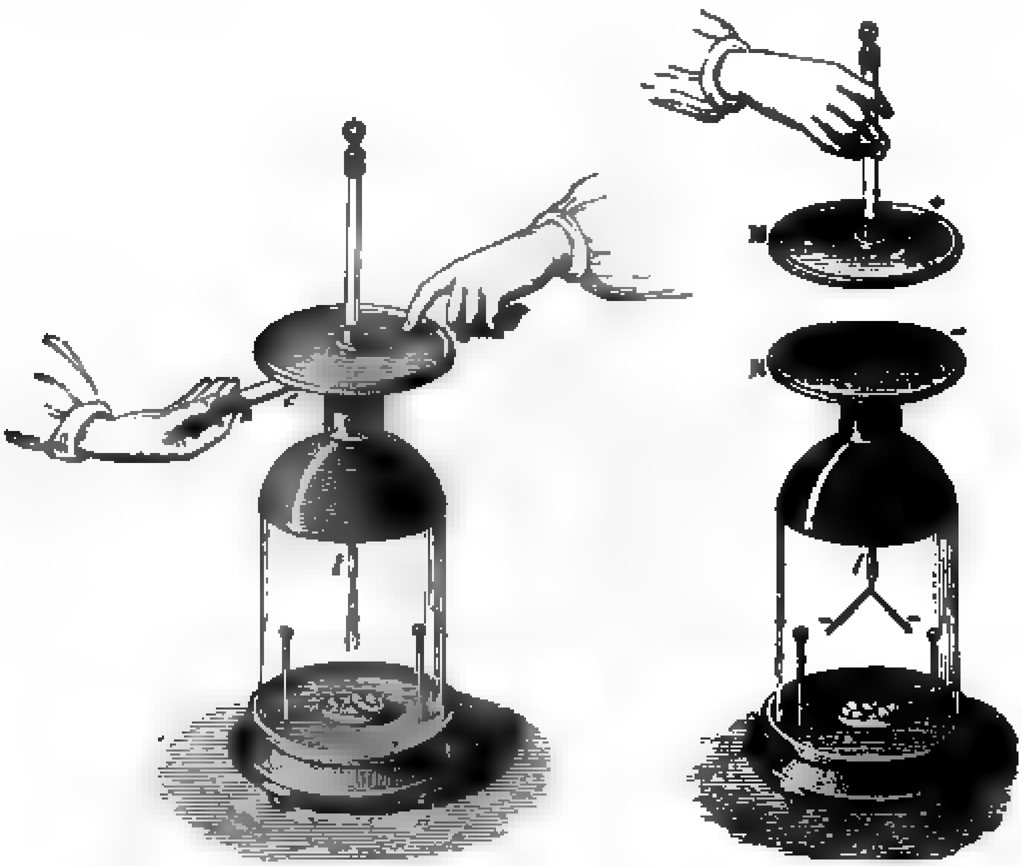


FIG. 1.

In this experiment *it is assumed* as sufficiently proved by various convergent pieces of evidence that the zinc, held in the moistened hand, is practically at the zero-V of the earth; and that the contact of the copper with the brass of the electroscope gives no Δ V.

Hence, it is argued, the electroscope or electrometer indicates the Δ V due to the contact of the zinc and copper alone, and shows that zinc is + to copper.

(ii.) An electroscope is provided with an upper condensing plate made of zinc. This is, as usual, provided on its under surface with an insulating layer of lac varnish. Both plates are as usual carefully discharged by means of a Bunsen's flame, until no movement of the leaves is observed on raising or lowering the upper plate.

Then a copper wire is made to connect the upper (unvarnished) surface of the zinc plate with the under surface of the copper plate.

It is found that the zinc and copper have now assumed a Δ V in which the zinc is + to the copper. This Δ V is of course (*see* Chapter X.) magnified, and so rendered evident, when the wire is removed and the upper condensing plate is lifted away. Here there is no doubtful contact of metal and wet hand.

(iii.) If in experiment (i.) we hold the copper end of the compound bar, and touch the plate of the gold-leaf electroscope with the zinc, we get no result. This supports Volta's conclusion (3).

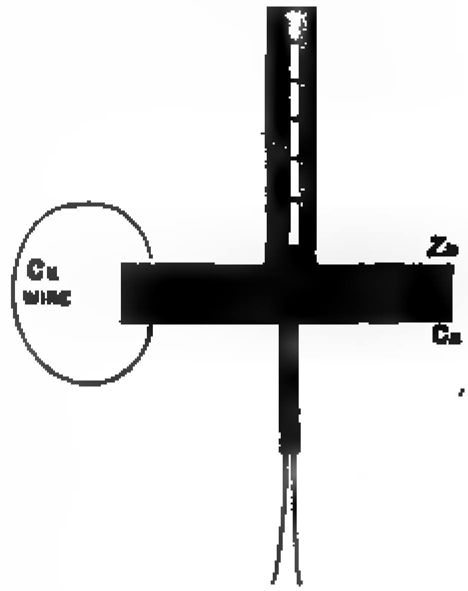
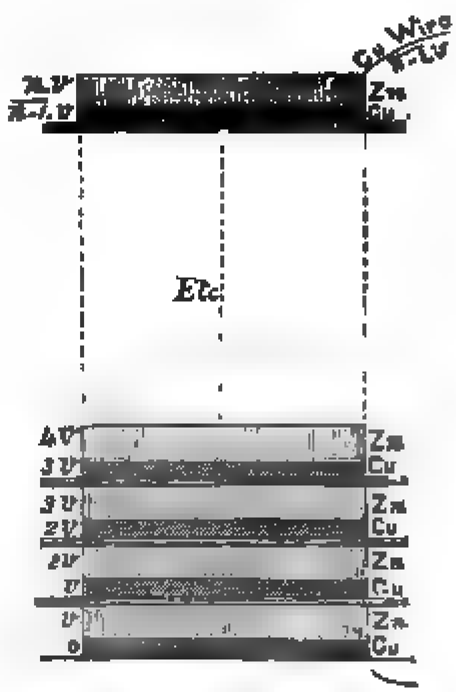


FIG. ii.

So if we make a bar of *zinc* | *gold* | *iron* | *copper*, and use it as in (i.), we get the same result as in (i.).

(iv.) If we hold the copper end of the bar and insert a piece of wet paper between the zinc and the gold-leaf electroscope, we shall find that, on removing the bar and raising the upper condensing plate, the lower plate is at a + V.

Volta's view would be that the copper held in the moistened hand is practically at zero-V; the zinc at a + V; the wet paper at practically the same V as the zinc; and the brass of the electroscope at practically the same V as the paper, and therefore at the + V of the zinc.



The reader should examine these experimental results in connection with the statements made above as to Volta's views.

§ 4. **Volta's Pile, from Volta's point of view.**—By the application of the method of experiment (iv.) above, it is easy to obtain Δ V.*s* high enough to give shocks and to produce small sparks.

We lay down a copper disc and put it to earth. On this we lay a zinc disc, which will be at some higher potential that we will call *v*. On this we place a somewhat larger disc of flannel, moistened with salt and water; this will also be

practically at v. On this another copper disc, which will be also at v.

Then a zinc disc on this will be at $2v$; and so on.

We may conveniently solder together each pair of copper and zinc discs. If there be n such pairs of plates, then we have the bottom copper at zero-V, and the top zinc at $n \times v$. If we solder a copper terminal to this top zinc, it is to be noticed that this wire will be at $\overline{n-1}\,.\,v$, or at the same potential as the n^{th} copper.

This arrangement is called 'Volta's pile.'

About fifteen or twenty such pairs will, when newly set up, give a perceptible shock. After a time the surfaces of the metals are altered from slow chemical action, and the pile must be taken down and cleaned in order to give the initial results.

Volta's view was that the action is due to the series of contacts; but that when, owing to oxidisation of the surfaces, the substances in contact are altered, we do not have, nor can we expect, the same results as initially.

Experiments.—(i.) We put the bottom copper, and one pair of quadrants of the electrometer, to earth. Then with an insulated platinum wire connected with the other pair of quadrants we touch the 2nd, 3rd, 4th, &c., copper plates in succession, putting the platinum wire to earth between each observation.

The deflexion of the needle will indicate the rise in potential up the pile.

(ii.) We can make a similar observation on the momentary currents sent through a delicate galvanometer of high resistance. But this assumes some previous knowledge of the galvanometer.

Other forms of Volta's pile.—There are other forms of Volta's pile used; of these we will mention one, viz. *Zamboni's.*

As a rule the *drier* the pile, the more lasting is its activity, but the more easily is it exhausted for the time.

In Zamboni's pile the arrangement answers to fixing together a zinc, wet paper, and copper disc, as one *element.* For each element of his pile consists of a disc of paper that is 'silvered' on one side (*i.e.* coated with *tin* or *lead*), and on the other side coated with manganese dioxide. Such a pile gives a very high ΔV for very little space occupied. It can be advantageously employed to give the requisite ΔV in the Bohnenberger's electroscope, where there is no completion of the circuit and therefore no running down of the pile.

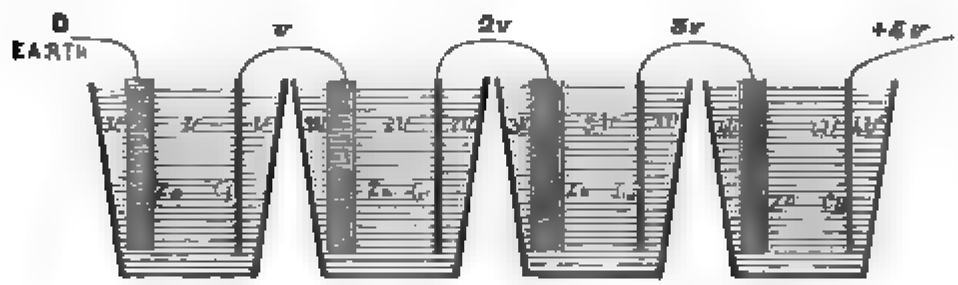
The Gymnotus.—In certain animals, notably in the Gymnotus 'electric eel,' there have been found organs that appear to be ılogous to a voltaic pile. Severe shocks can be received, curıts obtained, and other electric phenomena manifested. These nifestations seem to depend partly upon the actual condition the organ, partly upon the general health and vigour of the ımal, and partly upon its volition.

The study of the relation of these electrical phenomena to ıat we may call 'vital' phenomena is exceedingly interesting; t we do not intend to include in the present Course anything ıt belongs of right to the domain of physiology.

§ 5. **Volta's Cell, and the Couronne des Tasses, from Volta's int of view.**—In order to obviate the 'running down' of the wer of his pile, resulting from the alteration of the surfaces of e metals, Volta substituted dilute acid for wet paper.

His 'pile' then took the form of a number of cells or elements ıned end-on, forming what was called the *Couronne des tasses.* e here give a sketch in section of this arrangement.

The zincs and coppers are plates connected by copper wires; ey are immersed in vessels of dilute sulphuric acid. In the ;ure we have supposed the copper wire that is soldered to the ıc plate to be to earth and so at zero-V.



The rise of V through the arrangement is indicated. The .der will see that, *according to this view*, the Δ V of a single ele·nt is practically the same as that due to the first *copper* | *zinc* ıction. When the terminals are connected a *current* flows, and *·rgy* is given out in the circuit.

When pure water is used in the vessels we find the Δ V between : terminals to be approximately the same as if dilute acid had been :d. But if we complete the circuit for a few moments, and again :ak it, we find that this Δ V has almost or quite disappeared. The

battery has 'run down,' as did the pile. Examination will then show that the surface of the zinc has been oxidised, while the copper is covered with a film of hydrogen.

From Volta's point of view we should say that we now have a new series of bodies in contact; viz. *copper* | *zinc* | *zincic oxide* | *water* | *hydrogen* | *copper*. The cell is now very different from its original condition; we could not predict whether there would be a ΔV between the terminals, and in fact we find that there is none. This view of the matter is doubtless unsatisfactory; but it is of interest to notice that at least the 'running down of the battery' does not confute Volta's view.

But if we use dilute acid, the surfaces are continually renewed by chemical action; this action taking place far more freely in the present case than in the case of the pile.

§ 6. **The 'Contact' and 'Chemical' Theories.**—Volta fixed his attention mainly on the ΔV that, as it seemed, accompanied the contact of the dissimilar metals *zinc* and *copper*. His followers exaggerated a certain one-sidedness that existed in his views; and the *Contact school*, as they were called, considered that the chemical solution of the zinc played a subordinate part in the action of the cell, serving mainly to keep the surfaces clean and so to keep the same series of bodies in contact. In fact the word *contact* was the key-note to their theory of the Voltaic cell. They considered the ΔV between the terminals of the 'open' cell (*i.e.* of a cell in which the terminals were insulated) as the algebraic sum of the different ΔV. *s* due to the different contacts; of which, in the ordinary Volta's cell, the only one of importance was that where the copper wire was soldered to the zinc.

The 'Chemical school' of physicists considered the cell when the circuit was closed and a current was running. They pointed out how the strength of the current that flowed was proportional to the vigour with which the chemical action proceeded; and how the power of the cell depended on having one plate as much acted upon, and the other plate as little acted upon, as possible.

Faraday was the great exponent of this view. In modern phraseology, the 'Chemical school' insisted on the *chemical action* as the *source of the energy* of the cell.

They, in their turn, were for the most part too one-sided; and many denied that dissimilar metals in contact did exhibit a difference of potential at all without chemical action.

In the next section we shall attempt to show the position of

odern theory and of modern knowledge in this matter; and ill conclude by giving a view of the Volta's cell, taken as a ole, which can hardly involve any serious error.

But we should add that the whole question is still to a conerable extent unsettled.

§ 7. **Theory of the Simple Volta's Cell.**—As is usual in such ses there is truth in both the above extreme views; though the or of the extreme followers of the Volta school was perhaps the eater, inasmuch as they neglected the source of energy in the rrent.

There seems no doubt but that a difference of potential does general accompany the contact of dissimilar bodies; though it quite beyond our present knowledge to say that the contact *uses* the difference of potential.

But the phenomenon is not an easy one to observe; nor is e amount of Δ V as easy to measure as would at first appear.

When zinc and copper are in contact with each other, they are o in general in contact with the air or other medium surround- g them. It has been asserted that while the zinc and copper e nearly at the same V, the layers of (badly conducting) air or ier gas in contact with the two metals respectively will be found be at a considerable Δ V from each other; and that this Δ V ers both in magnitude and in sign with the nature of the gas.

It is said that in many electrostatic methods we really measure Δ V of these layers of gas, and not of the metals themselves. at while zinc really has under all circumstances at ordinary nperatures a very small $-$ V with respect to copper, yet when usual experiments are performed in air we find zinc to be parently very strongly $+$ to copper, and when performed in an nosphere of hydrogen sulphide we find zinc to be apparently y strongly $-$ to copper.

The Δ V.*s* between the metals and the gases are considered to ult fro n surface chemical action; and the minute Δ V.*s* between metals themselves to result from some molecular action prob- y unaccompanied by any chemical change. These latter true V.*s* between the metals have been measured by methods that l be discussed in the chapter on thermo-electricity.

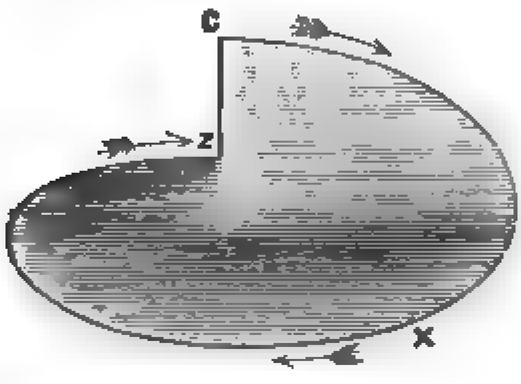
We have given here a view the difference of which from Volta's w shows how necessary it is at present to be very cautious in

making any dogmatic assertions as to the respective Δ V. *s* at the different places of contact; though at the same time we see that even this view allows the Voltaists to have been right in asserting that *some* Δ V does accompany mere contact.

The *Chemical school*, though they did not use modern expressions and had not fully grasped the laws of energy, at least understood that we cannot get work done without an equivalent disappearance of energy. They regarded the energy of the current, that ensues when the circuit is closed, as derived from the chemical consumption of *zinc* in the cell; and they saw that the energy of the current depended upon the intensity of the chemical affinities and on the amount of the chemical action.

Perhaps at present it is safest to regard the cell *as a whole*. Wherever it be that the Δ V. *s* occur, at any rate we may safely say that each cell tends to keep its two terminals or poles at a certain Δ V; and, if the circuit be completed and a current be allowed to pass, it is beyond dispute that the energy of the current is supplied by chemical action in the cell. We may regard the cell as a contrivance in which electricity is pumped up from a lower level to a higher by the expenditure of chemical energy; the electricity running down through the external circuit to the lower level again, and so on continuously.

In the simple diagram here given the cell is represented by the vertical line C Z. This indicates that in the cell the electricity is raised from a lower to a higher level by expenditure of chemical energy; and that then the electricity flows down from the higher level C to the lower level Z through the external circuit X.



We may here add that it is usual to call the zinc the + *plate*, and the copper the − *plate*; while the copper wire soldered to the zinc plate is the − *pole*, and that soldered to the copper plate is the + *pole*. In fact, that plate or pole is called '+' *from* which the current flows to the other plate or pole respectively. The current flows from zinc to copper through the liquid, and from copper to zinc through the external circuit.

The phenomena of the 'open' cell are, we see, electrostatic ; : ⅄ V of the terminals may be observed with a quadrant elec- meter. When the circuit is completed we get a flow of elec- :ity ; the phenomena accompanying this flow will be surveyed Chapter XII.

§ 8. **Digression on the Galvanometer.**—In Chapter XVII. e student will find discussed the construction and theory of the lvanometer. We shall, therefore, deal with it very briefly here.

The figures of Chapter XVII. indicate that when a current isses along a wire held parallel to a magnetic needle, above or :low it, the current produces a magnetic field that *tends* to set e needle in a position at right angles to the wire. As the needle deflected, the earth's couple, tending to restore it to its riginal position of rest in the plane of the magnetic meridian, ecomes greater ; while the disturbing couple due to the current ecomes less. Hence it will rest at some angle of deflexion from ie magnetic meridian ; and the magnitude of this angle can be nployed to calculate the strength of the current.

By '*strength of current*' we mean *quantity of electricity passing cross any cross-section of the wire in one second of time* ; as will be irther explained in Chapter XIII.

It is found that the galvanometer indicates the strength of irrent as just defined. It makes no difference in the indications hether this *quantity per second* flow quickly through a thin wire, : slowly through a thick one.

§ 9. **The Solution of Zinc in the Voltaic Cell.**—We will now :amine the nature of the chemical action that goes on in the oltaic cell, considering first the open cell, and then the closed rcuit.

I. In the open circuit.—If we immerse a plate of ordinary ommercial zinc in the dilute acid of the battery cell (we usually ix strong sulphuric acid with water in the proportion by weight *one* of the former to *ten* of the latter), we notice the usual dis- igagement of hydrogen from the surface of the zinc.

But if we immerse either very pure zinc, or ordinary zinc hose surface has been amalgamated with mercury, no such action kes place. Here the zinc and copper plates, unconnected with ich other, stand opposite to each other in the dilute acid, equally issive.

Probably the action of the mercury with which we have amalgamated the surface of the commercial zinc is to soften that surface, and to render the zinc superficially free from all strain; to give the zinc a mechanical homogeneity that apparently makes it behave like chemically pure zinc in being unacted upon by dilute sulphuric acid.

II. In the closed circuit.—If we repeat the above simple experiments, but now connect the copper and zinc plates, we get a current.

In the figures, A is the zinc plate, B is the copper plate, C and C′ are copper wires. The first figure represents the fact that the terminals C and C′ in the open cell are at different V . *s*; the

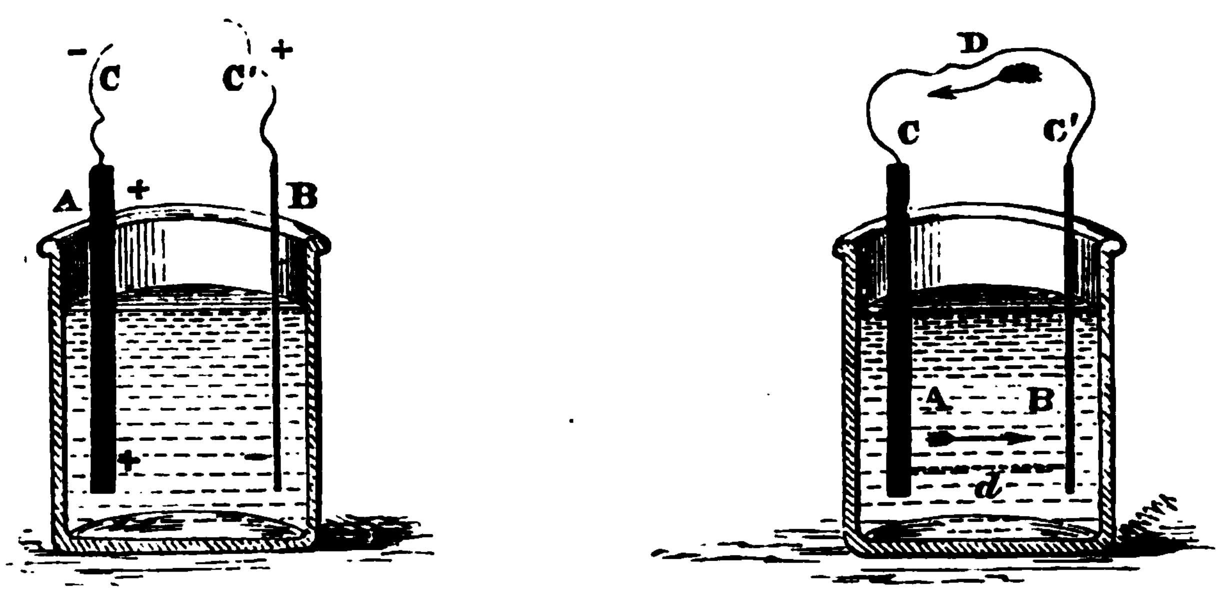


Fig. i. Fig ii.

second figure represents the direction of the current that ensues when the terminals are connected.

Now we shall find that, in the case of common zinc, when the circuit is completed immediately there is an increase in the evolution of hydrogen, the additional gas being given off the copper plate from the side that is turned towards the zinc plate.

In the case of pure, or amalgamated, zinc, we shall find that on completing the circuit chemical action at once commences; hydrogen is given off from the copper plate, while the zinc is dissolved. In both cases we find, on testing the liquid, that it is the zinc that is dissolved, the copper remaining unacted upon by the acid.

In the case of pure, or amalgamated, zinc, it would seem from this experiment that chemical action only occurs when a current passes, and that then, while it is the zinc that is acted upon, yet

hydrogen is set free from the surface of the copper plate. us the hydrogen seems to travel in some invisible manner *with* current, and to be set free off that surface by which the rent leaves the liquid.

In the case of impure zinc it seems likely that the solution, ich proceeded while the cell was open, still goes on when the cuit is closed, but that now there is additional solution of zinc, : hydrogen corresponding to this portion being set free off : copper plate.

Further investigations have made it almost certain that in no se will the zinc dissolve, saving step by step with the passage a current; but that in the case of impure zinc there are in- merable small currents circulating between portions of the pure zinc, that differ from one another in chemical or in echanical properties. These currents are called *local currents*, d the chemical action that occurs in the open cell is called *al action*. Amalgamation then is to prevent the occurrence *local action*, which latter contributes no energy to the main irrent.

§ 10. **Polarisation.**—In the simple Volta's cell in which we se a *copper* and a *zinc* plate immersed in dilute sulphuric acid, it found that the current falls off rapidly from its initial strength. 'e may show this by introducing a galvanometer into the circuit. hen a gradual decrease in the deflexion of the needle will be bserved (*see* § 8).

Note.—Mercury cups.—In all experiments where it is necessary to make d to unmake electrical connection between different conducting wires, it is ry convenient to use *mercury cups*. These may be simply small round ooden boxes filled with mercury, in the lids of which are bored holes. The re terminals of the battery, or of any piece of apparatus such as the galvano- ter, may be rapidly dipped into or removed from these cups, and, if the res be well amalgamated, the connections thus made are good.

On examining into the cause of the falling off in current, we d it to be due to the film of hydrogen that adheres to the pper plate at which, as we have seen, it is set free.

Experiment shows that this hydrogen acts to lessen the current two very different ways.

Firstly, it covers the copper plate with a layer that conducts e current very badly as compared with the dilute acid that

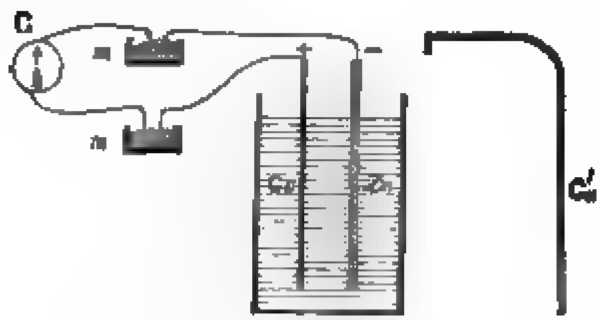
initially was in complete contact with the copper, thus lessening the current by passive *resistance*.

Secondly, owing to the presence of the hydrogen on the copper plate there is a tendency to a back current opposing the main current. This *back tendency* is called *polarisation*, and is of a very different nature from resistance. Indeed, if we now replace the zinc plate of the cell by a clean copper plate, and complete the circuit, we find that we actually get a reverse current; this continues until the hydrogen has combined with all the oxygen that is mechanically dissolved in the liquid, as we shall see later on (*see* Chapter XII. § 11).

It is pretty clear even at this stage that 'polarisation' can never actually reverse the original current, for this latter causes the polarisation. But the current can be nearly or quite brought to a standstill.

Experiment. The figure indicates in a simple diagrammatic manner one mode of showing this 'back tendency' or 'polarisation.'

G is a somewhat delicate galvanometer; *m m'* are mercury cups; Cu and Zn are the plates of the cell, in which we here use common water instead of dilute acid. C'u is a spare copper that may be readily substituted for the plate Zn.



(If we find the current too strong for the galvanometer, we may protect this latter, to a greater or less extent, by bridging over the cups *m m'* with a wire of less or greater resistance, as will be explained in Chapter XIII. § 10.)

We first complete the circuit, as shown in the diagram, and notice that the initial current soon almost, or entirely, ceases. We next replace the plate Zn by the 'passive plate' C'u, and a reverse current, almost as strong as the original current, will be observed.

If we use dilute acid in the cell, the current only falls off to some extent, and the reverse current is proportionally weaker.

§ 11. **Constant Batteries.**—The efforts of scientific men were soon directed towards the contrivance of battery-cells in which polarisation did not occur.

The hydrogen, which in the simple Volta's cell adheres to the copper plate, was to be got rid of. One method of doing this was

chanical method; viz., to use as a negative plate some plate hich the hydrogen could not stick in the form of a film. The r method was to give the hydrogen, as it appeared in its ent condition on the surface of the negative plate (*see* § 7), e chemical reducing work to do. We will now describe ral forms of so-called 'constant' battery-cells under the heads *uechanical* and *chemical* respectively.

Tote.—Electromotive force.—The poles of the open cell exhibit a certain the greater this is, the greater (*cæteris paribus*) will be the current when circuit is closed. When the current is running, that which urges it is d *electromotive force*; it can be measured in terms of the Δ V that would ar were the circuit cut. We shall hear more of electromotive force ılly called E.M.F.) when we come to Ohm's Law in Chapter XIII.

In Chapter XV. § 13 we shall see that when the hydrogen is rid of by chemical means we not only obviate polarisation, but gain a positive advantage in a greater E.M.F. (*see* preceding e).

I. *Mechanical.*—The earliest form of cell in ordinary use that ıes under this head is the *Smee's*. In this the copper plate is aced by a sheet of thick silver foil that is covered with very ly divided platinum (or 'platinum-black'). Off these points hydrogen rises to the top, and does not remain as a film ering the negative plate. The substitution of silver for copper :s, moreover, a slightly greater E.M.F.

II. *Chemical. Single-fluid cells.*—The most important single-d cell in which the hydrogen is used up in doing chemical ıction is the—

(i.) *Bichromate cell.*—Here the negative plate is of carbon (a cial kind) instead of copper; and the liquid is composed of a ıtion of potassium-bichromate in water, mixed with sulphuric l. The nascent hydrogen is employed in reducing the chromic l; so that chromic sulphate is formed. This battery has a high 1.F., yields no fumes, is simple in its arrangements, and there- : is adapted for general laboratory use. The zinc plates must removed from the liquid when not in use.

(ii.) *Latimer Clark's standard cell.*—For purposes of com-ison rather than for practical use, Latimer Clark proposed the owing cell.

As a negative plate, pure mercury. Over this a paste obtained

by boiling sulphate of mercury with a saturated solution of zinc sulphate.

As a positive plate, zinc; this resting on the sulphate paste. Insulated wires are connected with both plates.

This cell can be easily prepared. Its value consists in the fact that, when left with the circuit open, its E.M.F. is very constant, and can be used as a standard of comparison. When, however, the circuit is closed, and a current flows, the E.M.F. does not remain constant.

III. *Chemical. Two-fluid cells.*—There are many elements in which the negative plate is separated from the positive plate by a porous pot or partition. The positive plate is surrounded by a liquid that acts upon it when the circuit is closed, and the negative plate by a liquid containing something that the nascent hydrogen can reduce.

(i.) *Bunsen's cell.*—In this battery the + plate is zinc, surrounded by dilute sulphuric acid. The — plate is gas-carbon; it

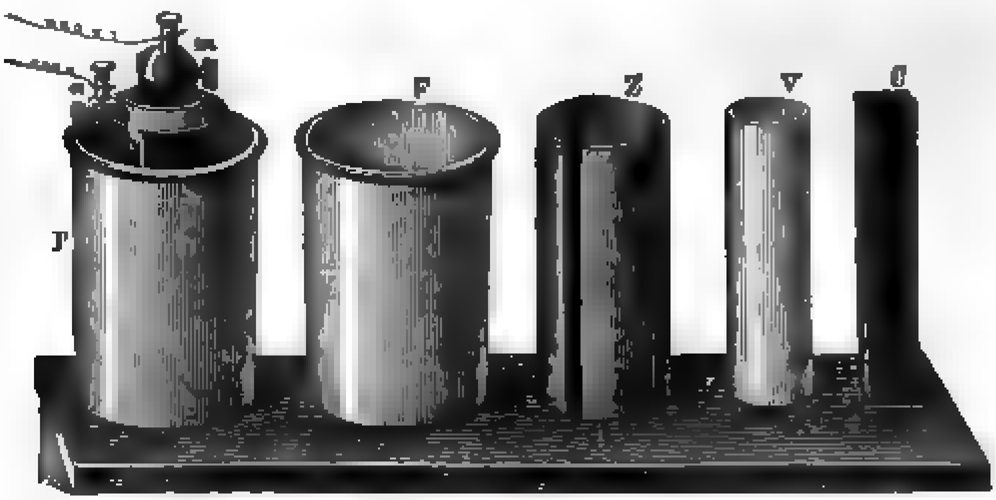


Fig. i.

stands in a porous pot, and is surrounded by nitric acid, the porous vessel thus preventing the two acids from mixing, while yet allowing the chain of chemical changes to pass with the current through its pores.

In the figure, P represents the entire cell; while F Z V C represent the outer vessel, zinc, porous pot, and carbon respectively.

The zinc dissolves in the dilute acid, forming zinc sulphate; while the corresponding hydrogen, set free against the carbon, reduces the nitric acid to lower oxides of nitrogen.

(ii.) *Grove's cell.*—This is an earlier form than the preceding. It differs from it only in having platinum instead of carbon. The shape of the porous pot is flat, to suit the flat plates of platinum-foil.

(iii.) *Daniell's cell.*—In this the only *essential* difference from the above is that we have copper in a saturated solution of copper sulphate, instead of platinum (or carbon) in nitric acid. But it is usually constructed having the zinc and acid in the porous pot, while very often the copper itself forms the outside vessel. The zinc may be surrounded by a semi-saturated solution of zinc sulphate, or of common salt, instead of by dilute sulphuric acid.

Here the hydrogen reduces the copper sulphate; sulphuric acid is formed, while copper is deposited on the copper plate. It is therefore necessary to keep up the strength of the copper sulphate solution by a supply of crystals of that salt.

In one very portable form we have but a single vessel; at the bottom is a plate of copper on which is a layer of crystals of sulphate of copper; over this is a layer of sawdust; and, resting on this, a plate of zinc immersed in dilute sulphuric acid or in a solution of sulphate of zinc. Insulated wires form connections with the plates.

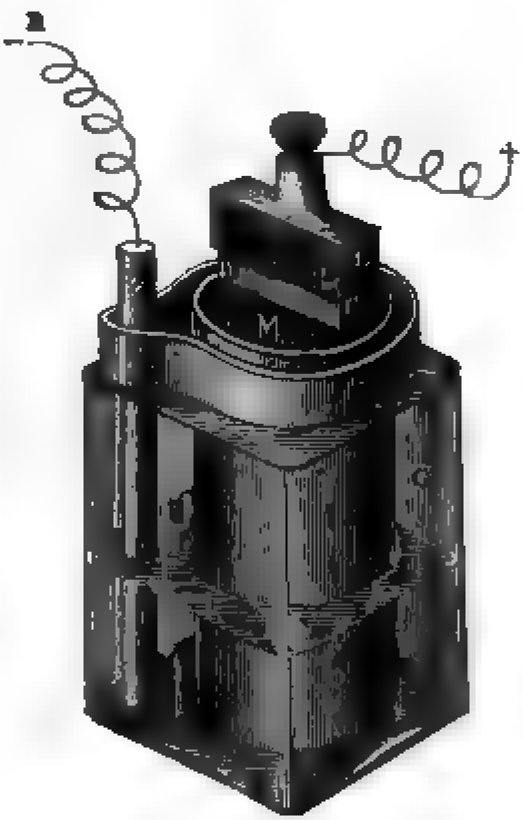


FIG. ii.

In '*gravity batteries*' the liquids are kept apart simply by the fact that the less dense liquid forms a layer above the more dense.

(iv.) *Leclanché's cell.*—In this the + plate is a zinc rod immersed in a strong solution of ammonium chloride. In the porous pot is a carbon rod, round which is tightly packed a mixture of manganese dioxide and of powdered carbon. The porous pot is closed at the top with pitch, a hole being left for the escape of gases.

The ammonium chloride solution soon soaks through and moistens the powder in the porous pot, and then the cell is ready for use.

Zinc chloride is formed in the outer vessel, the zinc displacing ammonium. In the inner vessel is set free ammonia, while the remaining hydrogen of the ammonium reduces the manganese dioxide.

§ 12. **Remarks on Cells and on Batteries.**—We have now mentioned the chief cells that are of interest to the general student. A few remarks will be made in conclusion.

Chemical action in the cell.—For a discussion of the manner in which the chemical action takes place, in what way the hydrogen displaced by the zinc 'travels with the current' and appears at the negative plate, and what relation the amount of chemical action bears to the strength of the current, the student is referred to Chapter XII.

Efficiency of different cells.—We have stated that different cells give a different Δ V at their terminals when the circuit is open, and a different E.M.F. (which, when the other conditions are the same, will cause different currents) when the circuit is completed. For a further discussion of this matter we refer the reader to the sections on 'Electromotive force' in Chapters XIII. and XV.

Again, cells differ from one another in offering more or less resistance to the passage of the current through them. This matter will be discussed in Chapter XIII.

Coupling cells together.—If we connect n cells as we connected the elements in the *Couronne des tasses* we get a Δ V in the open circuit, or an E.M.F. in the closed circuit, which is n times that of a single cell.

Again, if we connect all the positive plates together and all the negative plates together respectively, we have what amounts to one large cell whose plates are n times the size of those of a single cell. Such an arrangement has the E.M.F. of only one cell.

The advantages of the above two methods of coupling in different circumstances respectively will be discussed in Chapter XIII., and the proper terms will be there given.

Amalgamating zinc.—If the zinc be wetted with dilute acid it is readily amalgamated with mercury.

Or, again, if a little sodium be added to the mercury it enables one to amalgamate zinc and other metals with greater ease.

Use of carbons.—In order to obviate the tendency of the car-
ons to 'soak up' the liquids in which they are immersed—a result
ery unpleasant and very injurious to the binding screws attached
the carbons—it is usual to soak the upper part of the carbons
melted paraffin wax. The surface must then be scraped at the
laces where the binding screws make contact.

CHAPTER XII.

THE CHEMICAL PHENOMENA ACCOMPANYING THE PASSAGE OF THE CURRENT.

§ 1. **Introductory.**—In this chapter we shall consider the chemical phenomena that accompany the passage of an electric current. We shall assume that the student is acquainted with the elements of theoretical chemistry.

Of the other classes of phenomena, the *heat* effects will be only briefly mentioned here, a fuller treatment being found in Chapters XV. and XVI.; the *magnetic* effects will be discussed in Chapters XVII. and XVIII., &c.; and the *induction* phenomena form a large portion of our subject and will occupy our attention in Chapters XXI.-XXIV.

§ 2. **Heating Effects; a Brief Account.**—When a current passes through a conductor it is found that the said conductor is heated. The stronger the current, and the greater the resistance of the wire, the greater is the quantity of heat evolved. The exact law relating to this matter will be given in Chapter XV. A battery of from three to six large bichromate cells will render white-hot, or even melt, fine platinum wire, and will illumine small 'glow' lamps of ten candle-power.

Now, whence does this heat-energy come? Our source of energy is the battery-cell. If in this a certain amount of zinc be dissolved, there will be a certain amount of heat evolved. If the circuit of action be contained within the cell, then all the heat due to the quantity of chemical action will appear in the cell. But if the cell drive a current and this current heat a wire external to the cell, so much the less heat will appear in the cell. The total amount of heat evolved during the solution of a given mass of zinc will be constant; but, by making the external circuit of

some substance that offers a very high resistance to the current, we may cause nearly all the heat, due to the amount of chemical action in the cell, to be evolved outside the cell. We lose chemical-potential-energy, and we gain equivalent heat-energy (*see* Chapter X. § 4).

§ 3. **Chemical Effects; General View.**—Bodies may be roughly divided into three classes with regard to their behaviour as to allowing a current to pass through them.

(i.) *Conductors.*—All those solids and liquids that allow a current to pass, while themselves undergoing no change saving a rise in temperature, are called *conductors.* Such are *metals* whether solid or molten, *carbon*, the *bodies of animals*, &c.

(ii.) *Insulators.*—Bodies that do not allow any appreciable current to pass are termed *insulators.* A few examples of insulators are *glass*, *ebonite*, *paraffin oil*, *dry vapours*, &c.

These two classes merge the one into the other. The reader should refer to Chapter XIV. § 14, for tables of resistances.

(iii.) *Electrolytes.*—A great many bodies allow a current to pass, but themselves suffer a chemical decomposition that proceeds step by step with the current; not allowing any appreciable current to pass without this chemical decomposition occurring.

Such bodies are termed *electrolytes.* This class consists almost entirely of compound liquids or solutions of salts, or of molten salts; the word 'salt' being understood in its widest sense. A few examples are *aqueous solutions of acids*, *of metallic salts*, or *of alkalis*; also such liquids as *melted potash or soda*, &c. The action, as we shall see hereafter, requires that the molecules of the bodies shall move with freedom; hence all electrolytes are liquids or pastes.

Explanation of terms used.—Referring to fig. (ii.), § 4, we see that usually the electrolyte is introduced into a glass vessel, while the current enters by a metal plate A and leaves by another plate B. This cell is called an *electrolytic cell*; A is called the *anode*, B is called the *kathode.* These plates are usually of platinum, this being unacted upon by most liquids. In what follows it is understood that they are of this metal, unless the contrary is stated.

It is found that the electrolyte is invariably split up into two molecular (or atomic) groups, the metallic radicle and non-metallic

radicle respectively. Thus H_2SO_4 splits into H_2 and SO_4, *not* into H_2O and SO_3; NH_4Cl into NH_4 and Cl, *not* into NH_3 and HCl; $CuSO_4$ into Cu and SO_4; and so on.

These groups are set free at the two *electrodes*; the metallic groups (as H_2, Cu, $(NH_4)_2$, &c.) at the *kathode*; the non-metallic groups (as O, SO_4, Cl_2, &c.) at the *anode.*

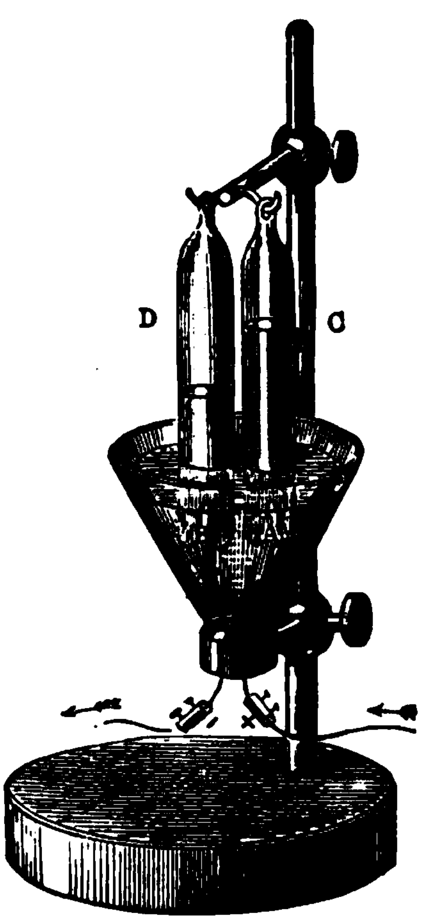
These groups are, from their 'travelling,' called *ions*; the metallic groups are called *kations* or *electropositive*, from being set free at the kathode or negative plate; the non-metallic groups are called *anions* or *electronegative*, from a similar reason.

The process of chemical decomposition through the agency of a current is called *electrolysis.*

Note.—A slight acquaintance with Greek will enable the reader to discover for himself the derivation of the terms *anode*, *kathode*, *electrode*, *ions*, &c.

Experiments in electrolysis.—Experiments to illustrate electrolysis may be devised almost *ad infinitum.* We here describe a few typical cases.

(i.) *Decomposition of water.*—It is not certain that *pure* water can be electrolysed. Certainly as we approach purity the water becomes almost an 'insulator'; it being remarkable that mere traces of acids or salts in solution have a very great influence in destroying the insulating power and in rendering the water an electrolyte.



When we wish to decompose water we usually mix it with about one-tenth its volume of strong H_2SO_4. What part the H_2SO_4 plays is not known for certain. Here we will assume that it only renders the water capable of electrolysis.

The figure represents the usual arrangement, or one form of it. The vessel contains the acidulated water; A and B are the anode and kathode respectively; C and D are glass vessels at first filled with the liquid and inverted over the electrodes in order to collect the gases.

When the current passes, the H_2 and the O of the H_2O are set free in chemical equivalents at the kathode and anode respectively. Errors in the volumes collected occur from the greater solubility of oxygen, from part of the oxygen being set free in the form of ozone, and from hydrogen being 'occluded' by the platinum electrode to a greater extent than is oxygen. The first and third errors can be nearly eliminated by allowing the action to proceed for some time before collecting the gases; the second error by heating the tube, or by having electrodes of such area that the current is not too dense.

In several of the following experiments it is very convenient to ›e a V-shaped tube ; the electrodes occupying the two arms of : tube respectively. Or we may have an ordinary cell in which : two electrodes are separated by a porous earthen diaphragm, two vessels connected by wet cotton wick. Each of these ›thods enables us to examine at leisure the condition of the uid about the two electrodes after the action has proceeded for me time ; the mixing of these two portions of the liquid being a greater or less extent prevented.

(ii.) *Electrolysis of $CuSO_4$.*—If we employ a solution of $CuSO_4$, we find ι set free at the platinum kathode ; while from the platinum anode is set free , the liquid about this electrode at the same time losing colour and showing e presence of free H_2SO_4.

If we employ copper electrodes we find fresh Cu coating the kathode, hile the anode is dissolved with the formation of $CuSO_4$.

We shall in § 5 argue that in both cases there is *primarily* set free Cu at ıe kathode and SO_4 at the anode.

(iii.) *Electrolysis of Na_2SO_4.*—In this case (using platinum electrodes) we ınd H_2 and $2NaHO$ appearing at the kathode ; from the anode is set free O, ›hile the liquid about it shows signs of free H_2SO_4. We shall in § 5 show hat this is equivalent to a setting free of Na_2 at the kathode, and of SO_4 at ıe anode.

If some extract of red cabbage be mixed with the solution, and a drop or ' of dilute acid be added (if necessary) until the whole is of a dull purple lour, the alkali and acid will be indicated by green and red colourations out the kathode and anode respectively.

(iv.) *Electrolysis of $NH_4.Cl$.*—In this case we get Cl at the anode, and I_3 together with H at the kathode ($NH_4Cl = NH_4 + Cl$). If, however, kathode be of *mercury*, this latter swells up and forms what is by some sidered to be an amalgam of mercury and the metal NH_4.

The solution of $NH_4.Cl$ must be weak and cold ; otherwise we may get the y unstable and dangerous 'chloride of nitrogen' formed.

(v.) *Electrolysis of KHO.*—Some potassium hydrate is fused and is placed a piece of platinum foil, which forms the anode. In a cavity on the upper face of the salt is placed a drop of mercury which is made to form the hode.

The salt will have absorbed from the air, when it cooled after fusion, ›ugh water to render it an electrolyte ; it will, in fact, be a very stiff 'paste.' er passing a current from a battery (say of four bichromate cells) for a few ments, we drop the globule of mercury into water. It is seen to give off drogen, while KHO is found in solution.

This indicates that the KHO ($2KHO = K_2O + H_2O$) was decomposed, being set free at the kathode and there forming an amalgam with the mercry, while O was set free at the anode.

(vi.) *Electrolysis of* $Pb.\bar{A}_2$.—We here use a fairly strong solution of plumbic acetate, and a pair of lead electrodes.

The Pb is set free at the kathode, there forming a beautiful 'lead tree.' With a small cell this can be projected on the screen by means of a lantern. A battery of four bichromates causes a very rapid growth of the tree; any battery-cell will give the result in time.

The anode will at the same time be dissolved, giving $Pb.\bar{A}_2$.

§ 4. **Grothüss's Hypothesis. Nature of Electrolysis.**—In a later section we shall see that, as we should have expected, the *ions* set free at the two electrodes are always *chemical equivalents* of one another. Thus (we assume the reader to be acquainted with the exact meaning of chemical symbols) for H_2 and $2NaHO$ at the kathode we get O and H_2SO_4 at the anode; we could not get O_2, since that is equivalent to $2H_2$ or to $4NaHO$.

It may be noticed further that no signs of decomposition can be detected saving at the surfaces of the electrodes.

We give here a view of electrolysis that is consistent with the above facts; it is the view of Grothüss, or *Grothüss's hypothesis*, slightly modified to suit the modern theory of compound liquids or of salts in solution.

It is believed that, in a compound liquid, not only are the molecules in continual 'slipping' motion, but the atoms themselves are being continually dissociated and recombined. Thus, in water, neighbouring molecules of H_2O are continually exchanging partners, the H_2 of one molecule taking the O of the other, and reciprocally. This cannot be observed, because the interchange is molecular only, and the average constitution of the liquid remains the same; for much the same reason, indeed, we cannot observe the molecular motion called 'Heat.'

It is considered that the presence of the electrodes, connected with the + and − poles of the battery respectively, has the effect of directing this interchange; or, when one electrode is made + and the other −, then the interchange of partners is such that on the whole the H_2.*s* in their changes move towards the — electrode, and the O.*s* towards the + electrode.

Probably the 'current' is thus, and thus only, conveyed; passing by *convection* in an electrolyte. In a metallic conductor it passes by *conduction*.

We thus have the H_2.s arriving at the kathode and the O.s at the anode; while the liquid between, being in the same condition as to average constitution as it was before we tried to pass a current, appears to be unaffected.

This *directed travelling* of the groups has received the name of the *migration of the ions*.

Experiment illustrating Grothüss's hypothesis.—The following experiment illustrates how the electro + and electro − groups (or kathions and anions) travel from one electrode to the other, while the intervening liquid shows no signs of change.

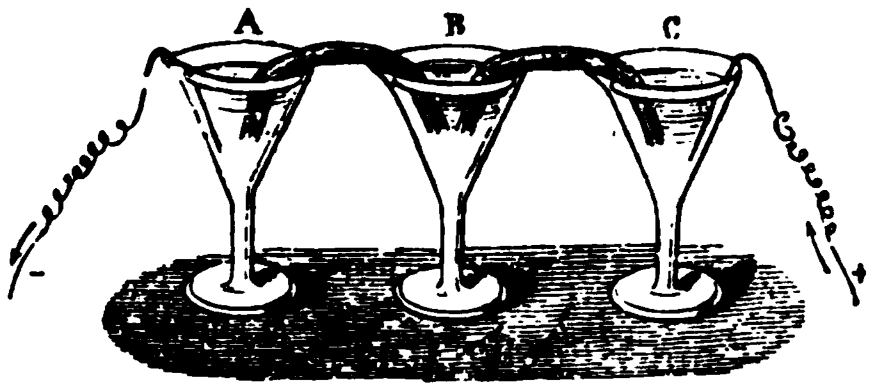


FIG. i.

A, B, and C are three vessels containing a solution of Na_2SO_4 coloured to a dull purple with extract of red cabbage. They are connected by moistened lamp-wick, and in A is the kathode, and in C is the anode. If two Leclanché cells be employed, and the whole be left for a day or so, it will be found that A has become alkaline, and C is acid; while B——through which have passed both the alkaline and acid groups——is still unaltered.

We will now represent in chemical symbols the interchanges that are continually taking place in an electrolyte when the current is passing, choosing a few typical cases. The upper brackets represent the condition of the chain of molecules before interchange; the lower brackets give the new grouping after interchange, with the *ions* set free at the two electrodes respectively.

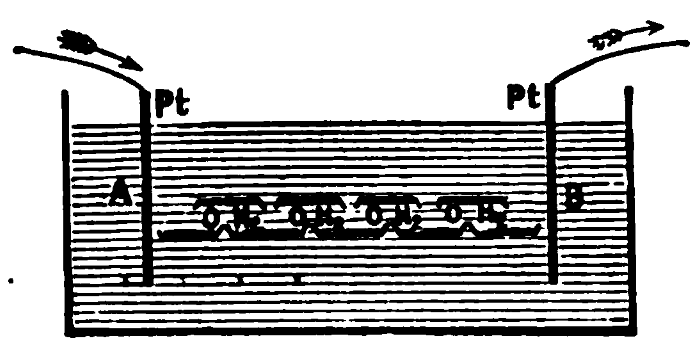


FIG. ii.

I. *The case of water.*—Here we see that initially we have groups of OH_2 only, while finally we have an odd H_2 at the kathode, an odd O at the anode, with groups of OH_2 between, as before.

II. *The Daniell's cell.*—Here we will consider the case of a battery-cell itself, since all we have said of electrolysis applies to the battery-cell equally with the electrolytic-cell, to chemical combination equally with chemical decomposition. In the battery-

cell the copper plate is the kathode, and the zinc plate is the anode.

The 'Zn' against the zinc plate represents an atom of the originally undissolved zinc. The arrangement of brackets represents that when the circuit is closed this Zn takes SO_4 from the nearest H_2SO_4; this leaves H_2 to take SO_4 from the next H_2SO_4, and so on; H_2 then passing through the porous cell takes SO_4 from the next $CuSO_4$, and so on, until finally Cu is deposited on the copper plate (the kathode).

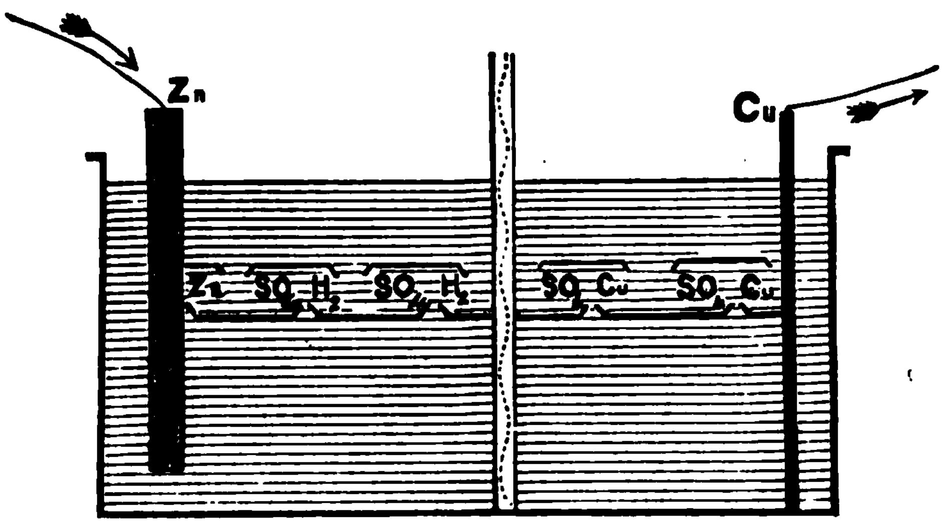


FIG. iii.

As explained earlier, no change due to this interchange will be observable saving at the surfaces of the battery plates.

§ 5. **Primary and Secondary Decompositions.**—We will now complete the theory of electrolysis, explaining the results of decomposition noticed in § 3, (ii.), (iii.), (v.), &c.

There is every reason for, and no reason against, the view that in electrolysis we have *primarily* the metallic, and the non-metallic, groups set free against the kathode and anode respectively.

But for reasons of 'chemical affinity' such a condition of things may not be stable. Thus if Na_2SO_4 be decomposed into Na_2 and SO_4, the Na_2 will decompose water and give $2NaHO+H_2$, while the SO_4 with water will give H_2SO_4+O.

Thus we have *by primary decomposition*,

at anode SO_4 . . . . Na_2 *at kathode*;

and then *by secondary decomposition*,

at anode $O + H_2SO_4$ . . . . $2NaHO + H_2$ *at kathode.*

The reader will notice that, in both cases, what we have stated as to metallic groups moving with, and non-metallic groups moving against, the current, still holds good.

In the case of the decomposition of $CuSO_4$, the Cu is de-

posited at the kathode whether that be of copper or of platinum, Cu being stable. But if the anode be platinum, the SO_4 gives H_2SO_4+O; while if it be of copper it will act on this and give simply $CuSO_4$.

So in the case of other salts; the general principle of electrolysis determines the *primary decomposition*, and then a knowledge of chemistry enables us to predict the nature of any *secondary action* that may occur.

§ 6. **Simultaneous Decompositions.**—When the strength of current per unit area of the electrode is great, we find that we get in many cases a simultaneous decomposition of the salt and of the water in which it is dissolved.

Thus, if we use very small electrodes of Cu in a solution of $CuSO_4$, and drive a large current through, we get H_2 set free with Cu at the kathode, and O set free as well as Cu dissolved at the anode.

Such is a case of what is called *simultaneous decomposition.*

§ 7. **Faraday's Laws of Electrolysis.**—The laws of electrolysis were experimentally investigated, and enunciated, by Faraday. We have somewhat forestalled them in what has preceded; as it was impossible to discuss the action in cells and the nature of electrolysis in a purely historical order.

We have already (Chapter XI. § 8) stated that in the galvanometer we have a means of measuring current-strength, or quantity of electricity passing per second. Hence we have a means of investigating the relation between the amount of chemical action per second, and the strength of the current.

Faraday found the following two laws to hold.

I. '*The amount of chemical action per second is directly proportional to the current-strength.*'

The physical meaning of this law would seem to be twofold; 1*st*, that the current does not pass through an electrolyte partly by electrolysis and partly by ordinary conduction, but only by electrolysis; 2*nd*, that an *ion* can only carry with it a fixed quantity of +, or of −, electricity, so that additional current requires a proportionally additional number of *ions* to carry it. The above law involves also the statement '*that a fixed current liberates per second a fixed mass of each ion respectively*' (*see* § 9).

II. '*If the same or equal currents pass through several electrolytic cells, the weights of ions* (which may be either atoms or molecular groups) *set free at the several electrodes will be in the proportion of the chemical equivalents of these ions.*'

An example will make this clear, our modern system of chemical notation rendering explanation an easy matter. Let the same current pass in succession through four cells containing *copper sulphate*, *ammonium chloride*, *sodium sulphate* solutions, and fused *silver chloride*, respectively. Then, taking secondary actions into account and considering the kathodes only, we get set free at the successive kathodes (i.) *copper*, (ii.) *ammonia and hydrogen* (which can be regarded as *ammonium*), (iii.) *hydrogen*. and (iv.) *silver* respectively. Now these are, when in chemical equivalents, represented by Cu, $2NH_4$, H_2, and Ag_2, respectively ; giving us as the numbers, to which the masses set free are proportional, (i.) 63 of *copper*, (ii.) 36 of *ammonium*, (iii.) 2 of *hydrogen*, and (iv.) 216 of *silver*.

The physical meaning of this law seems to be somewhat as follows. Each equivalent atom or group such as Cu, H_2, Ag_2, SO_4, $2(NO_3)$, &c., carries with it the same unalterable quantity of electricity as it migrates to that electrode at which it is 'set free.' We have, in fact, a new significance attached to the expression 'chemical equivalent,' viz., that those groups or atoms which are chemical equivalents have also the same power of carrying electricity.

We may throw Law II. into the form . . .

When the same or equal currents pass through several cells, there are the same number per second of equivalent atoms or groups set free against each electrode.

§ 8. **Further on Faraday's Laws of Electrolysis.**

The case of simultaneous decompositions.—In this case (*see* § 6) the form of the law last given is the more useful.

Now if we have P grammes of copper and P grammes of silver, then the numbers of atoms in each are proportional to $\frac{P}{63}$ and $\frac{P}{108}$ respectively. But, since copper is dyad and silver is monad, the numbers of *equivalent* atoms or molecules are proportional to

and $\frac{P}{216}$ respectively. If in successive cells we have set free at
: kathodes P grammes of copper, Q grammes of silver, and R
ımmes of hydrogen, respectively, then the second form of the
v gives us the result that $\frac{P}{63} = \frac{Q}{216} = \frac{R}{2}$.

Now if in one cell we get simultaneous decompositions, it is
ɔt hard to see what relation must hold. If, *e.g.*, in the first cell
: get both copper deposited, and water also decomposed setting
ee S grammes of hydrogen at the kathode, then we must have

$$\frac{S}{2} + \frac{P}{63} = \frac{Q}{216} = \frac{R}{2}.$$

But the proportion of S to P cannot be predicted ; it depends
pon the density of current and upon other conditions.

§ 9. **Electro-chemical Equivalents.**—We shall in Chapter XIV.
: explain in what units we measure current-strength. We shall
here find that the generally-employed unit of current is the
'mpère ; this being a current in which a definite quantity of elec-
ricity called a *coulomb* passes across any section of the conducting
ircuit in one second of time.

The quantity, one *coulomb*, of electricity performs a definite
mount of electrolysis in its passage through an electrolyte ; it
berates a definite mass of each *ion*. If we express this mass Z
1 *grammes*, the number is called the *electro-chemical equivalent* of
hat *ion*. We usually, however, give Z in milligrammes.

From Faraday's laws it is clear that if we know this number
or H, we know it for all *ions*. Now, the chemical equivalent of
ilver has been of late experimentally determined with great care.
nd it seems certain that if we take 1·118 milligrammes of silver
ɔ be set free by 1 ampère in 1 second, we shall be right to
ithin $\frac{1}{10}$ part in 100 parts. In these experiments the strength
f the current was of course measured independently of chemical
ction.

§ 10. **Electro-plating.**—One case of electrolysis forms the
asis of an important industry. It is the case where the electro-
yte is a solution of some metallic salt, the anode is a plate of the
ame metal, and the kathode is a conducting surface that we
esire to coat with this metal.

Name of the body	Atomic weight	Chemical equivalent	Electro-chemical equivalent Z, in milligramms per ampère per second
Hydrogen	1·0	1·0	0·0104
Potassium	39·1	39·1	0·405
Sodium	23·0	23·0	0·238
Gold	196·6	65·5	0·678
Silver	108·0	108·0	1·118
Copper, ic-salts . . .	63·0	31·5	0·326
,, ous-salts . . .	63·0	63·0	0·652
Mercury, ic-salts . . .	200·0	100·0	1·035
,, ous-salts . . .	200·0	200·0	2·070
Tin, ic-salts	118·0	29·5	0·305
,, ous-salts	118·0	59·0	0·611
Iron, ic-salts	56·0	18·3	0·189
,, ous-salts	56·0	28·0	0·290
Nickel	59·0	29·5	0·305
Zinc.	65·0	32·5	0·336
Lead	207·0	103·5	1·072

This table belongs to § 9.

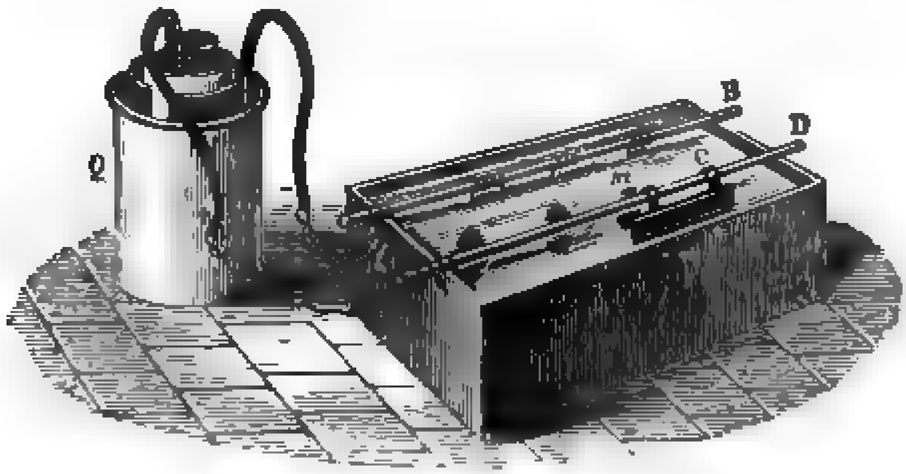
Sometimes it is desired to coat the body (as a baser met with a permanent adherent layer (as of gold or silver) ; sometin with a non-adherent layer that, when detached, gives us a ' verse' impression of the surface on which it was deposited. T processes of *electro-gilding* and *electro-silvering* come under t first head ; those of *reproduction of engravings*, *casts of coins*, & come under the second head. Sometimes also we employ process to etch designs on the anode.

In § 11 we shall see that under the above conditions, *i.e.* wl the solution remains unaltered in strength while there is a trans ence of metal from the anode to the kathode, any single however weak can effect the electrolysis (*see also* Chapter XV.)

As a general rule a single battery-cell with large electro gives a compact film of metallic appearance, while a powe battery, with electrodes too small for the current, gives a spo non-metallic-looking coating. In a word, for good results current must not be too dense.

I. *Copper-plating.*—In this case we wish to coat metals with opper. In more technical books (such as Hospitalier's 'Formuiire Pratique de l'Electricien' and Sprague's 'Electricity') the :udent will find recipes for copper solutions suitable for different urposes, and directions as to current-density, &c., such as will isure the best results. In the present Course we shall give little ut the principles of the different methods, and shall therefore ive but one recipe for a copper solution. 'A quantity of water is nixed slowly with from one-tenth to one-twelfth its volume of strong H_2SO_4, and the mixture when cold is saturated with $CuSO_4$.'

As a battery we may use one Daniell's cell of sufficiently large ize. This is connected up with two metal rods B and D as



iown, so that conducting bodies hung from B form the kathode, hile a copper plate hung from D forms the anode. All wires, &c., hich it is desired to protect from action must be coated with ax, or with caoutchouc, or other varnish. If the body is a non-onductor it should be covered with some conducting substance, id care must be taken, by encircling it with a copper wire or by ome other means, to make the action uniform over the surface. he bodies to be coated may, more simply, be made to form the egative plate in a Daniell's cell. In this case the substitution f salt and water for battery acid, as the solution acting on the nc, will make the action slower and the copper-film more com-act. If the action is sufficiently slow, the deposit of a film of $_7$ mm. should take from twelve to twenty-four hours (Hospitalier), iough a quicker rate of deposit *may* give good results.

II. *Reproduction of engravings, &c.*—If the object to be re-oduced be a delicate one, we may proceed as follows. We

will suppose, *e.g.*, that it is required to get a facsimile of one side of a rare coin. First, we take a wax cast (in reverse) of the coin. This wax reverse is then coated with plumbago, surrounded with a thin copper wire so as to distribute the current more evenly, and made to form the kathode in an electrolytic cell. The copper coating will, when removed, reproduce the surface of the coin in question. Or we may make complete casts of bodies and obtain electro-copper copies or reverses, according as the cast is a reverse or not.

III. *Electro-etching.*—By covering a copper plate with wax, tracing characters through the wax, and using the plate as an anode, we may etch out designs to any required depth. It is, however, not very easy to obtain any delicate results in this way.

IV. *Obtaining designs in relief.*—If we use the plate, prepared as in III., as the kathode in the cell, we shall obtain the same designs in relief.

V. *Silver-plating.*—For this purpose we must employ a silver solution, and a silver plate as anode. One recipe for the silver solution is as follows (Hospitalier). 'Dissolve 150 grammes of $AgNO_3$ in ten litres of distilled water ; add 250 grammes of pure KCN ; stir until there is complete solution ; then filter.'

In silver-plating good results are more difficult to obtain than in copper-plating, and more technical books than the present Course should be consulted if the student intends more than mere illustrative experiment.

§ 11. **Polarisation of the Electrodes.**—In simple electrolysis, *i.e.* where no simultaneous decompositions (*see* § 6) occur, there are two cases to be considered.

I. *Where there is no polarisation.*—In the cases of electro-plating considered above, the condition of the plates and of the solution remains constant. The fact that the kathode increases, and the anode decreases, in thickness during the action obviously does not affect the question of whether there will be a 'back tendency.' Hence, there is no more tendency for the chemical changes to take place in a reverse order after electrolysis has proceeded than there was at first. There is, in fact, no chemical-potential-energy stored up by the electrolytic action ; and no tendency, therefore, for the electrolytic cell to drive a reverse current (*see* Chapter XI. § 10 and Chapter XV. §§ 8 and 9).

Note.—There may be a *very slight* amount of polarisation owing to the fact that the deposited metal is not of exactly the same quality as the anode plate.

Experiment.—We may pass a current through a Cu | $CuSO_4$ | Cu cell for some time, and then prove the absence of polarisation by connecting the terminals of the cell with a galvanometer, when no current will be observed. Care must be taken that the original current be not so dense as to give simultaneous decomposition of water.

There being no back E.M.F., we can perform such electrolyses with any cell, however weak. (For further on this matter, *see* Chapter XV. § 11.)

II. *Where there is polarisation.*—But in all such cases as that of the decomposition of water, *i.e.* cases where the condition of the cell changes after electrolysis, we have a decomposition effected by the current against chemical affinities. We have, set free at the electrodes, *ions* that have a chemical tendency to reunite ; and this recomposition can take place most readily by means of a reverse current and a reverse chain of molecular interchanges. We have, in such cases, done chemical work ; we have gained chemical-potential-energy. The cell is now like a battery-cell ; and, like a battery-cell, it will tend to drive a current. The cell will have a certain 'electromotive force' depending on the energy of the chemical affinities of the *ions* ; and this E.M.F. must obviously be opposed to that of the original current by which the electrolysis was effected. (For E.M.F. *see* Chapter XI. § 11, note.)

Such cells are in fact battery-cells ; but, as a rule, they will not act as such for a long time, as the supply of *ions* ready to reunite is limited.

Experiments.—(i.) *Grove's gas-battery.*—The figure represents several 'cells' for the decomposition of water. It will be noticed that the electrolysed gases are collected separately, and that the platinum electrodes are so long that their upper extremities are always immersed in the gas, however little of this latter there may be.

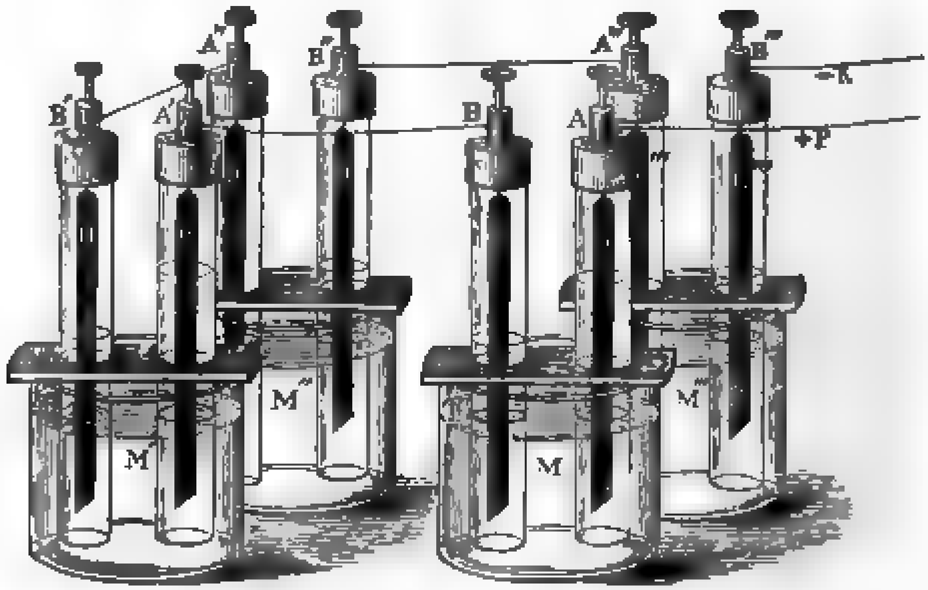
First, a current is passed through each cell separately until a considerable amount of gas has been set free in each ; this amount must be the same in all.

Then these cells are in reality battery-cells. They can be coupled up 'for small internal resistance,' all the kathodes together and all the anodes together ; or 'for E.M.F.,' anode to kathode and so on.

The hydrogen in the one tube and the oxygen in the other tube of each tend to reunite ; and it is found that when the external circuit is completed this reunion will take place step by step with the passage of a current, the

action being merely the molecular interchange, explained in § 4, reversed in direction. The current will be in the reverse direction to that by which the electrolysis was in the first instance effected.

Here each cell answers to a Volta's cell in which the zinc plate has been replaced by hydrogen in contact with platinum; the copper plate by oxygen in contact with platinum.



Notes.—(i.) The recombination can occur only when the circuit is closed; and, further, only when there is a *platinum* plate that is in contact both with the gas and the liquid. This action of the platinum will be familiar to those who have studied elementary chemistry.

(ii.) There will be a current if we replace the platinum-oxygen plate by plain platinum. This is due to the fact that there is always some oxygen dissolved in the dilute acid.

§ 12. **Secondary, or 'Storage,' Cells.**—Any arrangement by means of which chemical-potential-energy may be used up in such a manner as to give us the equivalent energy of an electric current is called a '*battery-cell*,' or often more shortly '*a cell*.' Where the cell is put together out of certain materials and then *does not* require the passage of an electric current before it is fit for use, it is called a '*primary-cell*.' All the cells described in Chapter XI. § 11, were of this class.

But where the cell requires the passage of a current before it is ready for use, *i.e.* where the chemical-potential-energy is the result of electrolysis, in such cases the term '*secondary-cell*' or '*storage cell*,' is employed. Such a cell, *e.g.*, is the Grove's gas-cell. The latter term was intended to imply that we 'stored up' elec-

tricity in the cell, since we could use a primary current to effect the original electrolysis, and then could recover a current after a greater or less interval of time. So understood, the term is a misnomer. We use up some of the electrical energy of the primary current, and get equivalent chemical-potential-energy stored up in the secondary-cell ; later, when we complete the circuit of this cell, we lose this chemical-potential-energy and get again the electrical energy of a current. But we did not store up electricity ; a condenser does this, not a secondary-cell.

Analogy.—We might use up some wind-energy to turn a fan, and this might *wind* up a weight from a lower to a higher level. We might, later on, let the weight again descend, and so obtain a wind driven in the reverse direction by the fan. We should have stored up *mechanical-potential-energy*, not *wind.*

The problem of inventing a good secondary-cell is of a two-fold nature.

(i.) How can we retain a large amount of the *ions* in contact with the electrodes, ready to drive a reverse current?

(ii.) How can we prevent action occurring while the cell is lying idle with open circuit?

In our next two sections we shall describe one form of cell devised to satisfy these two conditions.

§ 13. **Planté's Secondary-Cell.**—In the original form of Planté's cell there are initially two lead plates immersed in dilute H_2SO_4.

I. *Formation of the cell.*—A somewhat lengthy process is needed in the first instance in order to get the cell ready to act as a battery.

(i.) A current is passed through it ; this resulting in hydrogen being given off at the kathode, while the surface of the anode is oxidised into the condition of PbO_2.

(ii.) The current is reversed, and is continued until the PbO_2 is all reduced to spongy lead, while the other plate is in its turn per-oxidised.

(iii.) This process of sending currents in alternate directions is repeated until the lead has been acted upon to some depth. Thus, the plate that served last as anode is left coated deeply with PbO_2, that which served last as kathode is deeply coated with spongy lead. This process is called '*Formation of the cell,*' and it

is said to be left '*charged*'; this is represented in fig. i. Subsequent chargings will need only a single passage of the current.

II. *Discharging the cell.*

In our discussion of the cell we will at present consider only the degree of oxidisation of the lead, and will only touch on the part played by the dilute acid. It is clear that if we have PbO, the form-

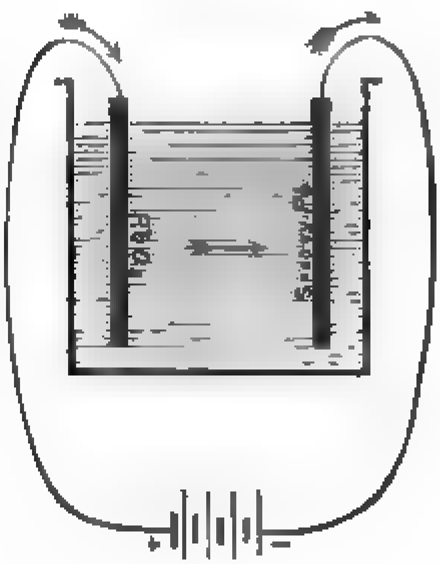


FIG. i.

FIG. ii.

ation of $PbSO_4$ from this ($PbO + H_2SO_4 = PbSO_4 + H_2O$) can be regarded as a secondary action, and has no direct bearing on the electrolytical theory of the cell.

If we connect the terminals of the cell, a current is set up in a reverse direction to the current that was last passed through the cell in its formation. One atom of oxygen from each molecule of PbO_2 passes back to the other plate, giving PbO on each plate, instead of PbO_2 on the one and Pb on the other. This action from the one plate across the liquid to the other plate takes place only step by step with the current; and the current will continue as long as there is PbO_2 left. When the cell has 'run down,' each plate will be coated with PbO (or, by action with the dilute acid, with $Pb.SO_4$, which is equivalent to PbO as regards degree of oxidisation).

III. *For the cell to be recharged*, a current must be again passed through the cell. If we consider that there is PbO on both sides, the action is simple; by an electrolytic passage of H_2 to

the kathode, and of O to the anode, we get again Pb on the former and PbO_2 on the latter.

If there be $Pb.SO_4$ on both plates we *may* represent the action as follows. [See fig. (iii.)]

For convenience we have written $Pb.SO_4 + H_2O$ (there is plenty of available water in the dilute acid) in the two equivalent forms $H_2SO_4.PbO$ and $OPb.H_2SO_4$.

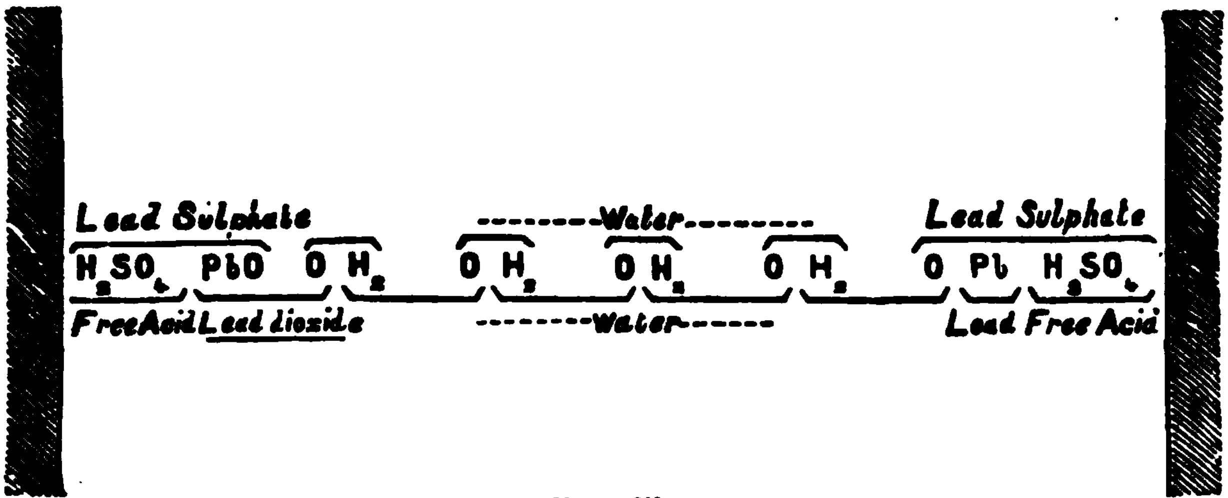


FIG. iii.

This indicates two main facts.

(i.) That the electrolytic action of 'charging' the cells is the same, whether there be PbO or $Pb.SO_4$ on the two plates of the exhausted cell.

(ii.) That the recharging of the cell is practically a transference of O from the kathode to the anode.

IV. *Local action, and waste of energy, &c.*—It may naturally occur to the reader that there seems no reason why the whole action should not take place between the PbO_2 and the lead plate upon which it rests, instead of between the PbO_2 and the other lead plate that is separated from it by the dilute acid; so that after a short period of lying idle with open circuit, the cell would have 'run down' and be useless.

No doubt there is some such action at first. But it seems that the insoluble layer of $PbSO_4$, which is thus formed between the PbO_2 and the lead plate on which it rests, hinders further local action; while the surface turned towards the other lead plate, being a free surface, remains always more open and porous. This appears to be a very important part played by the insoluble $PbSO_4$.

As regards the energy wasted in the charging of the cell, this will be more fully discussed in Chapter XV. We will here only

remark that we waste more energy in *heat* when the back E.M.F. of the secondary-cell is small as compared with that of the primary battery, or when the charging current is large.

It is of interest to remark that by charging cells arranged 'in parallel,' and then coupling them 'end-on,' we can obtain a secondary battery of as great an E.M.F. as we please; one that could drive a current back through the primary battery.

§ 14. **Faure's Accumulator.**—In order to obviate the necessity for the lengthy and energy-wasting process of '*formation*,' Faure devised the following important modifications in Planté's cell.

The two lead plates were coated with *minium*, this being Pb_2O_3, or $PbO.PbO_2$. One passage of the current then sufficed, by the electrolytic setting free of H_2 at the one plate and of O at the other, to convert the one layer into spongy lead, and the other into lead peroxide.

Since, however, Pb_2O_3 requires but O in order to per-oxidise it, while it requires $3H_2$ to reduce it, it is clear that we must have three times the amount of minium on the anode (where the electrolytic oxygen is set free) as on the kathode (where the hydrogen is set free). Thus the electrolysis of $3H_2O$ will reduce Pb_2O_3 on the kathode, and will per-oxidise $3(Pb_2O_3)$ on the anode.

In all other respects the theory of the '*Faure's accumulator*,' as it is called, is the same as that of the Planté's cell. The question of *energy* will be discussed more fully later on.

CHAPTER XIII.

OHM'S LAW.

§ 1. **General Ideas as to the Scope of Ohm's Law.**—Up to the present point we have, in our treatment of the battery-cell, of the current driven by it. and of the circuit through which the current flows, used terms in a vague and qualitative, rather than in an exact and quantitative sense.

But in the present chapter we propose to discuss at some length the conditions which determine the magnitude of an electric current in any particular case ; and to state and explain the law which makes the calculation of the current a matter of simple arithmetic. The law referred to is that known as *Ohm's law* ; it was enunciated by Dr. G. S. Ohm in the year 1827. This law—which must be accepted as confirmed by countless direct and indirect experiments and refuted by none—states in the first place that when the circuit of a cell is completed we are concerned with three quantities only; and in the second place that these three quantities are connected by a very simple relation, viz., that given in § 2.

These three quantities are . . . .

I. *Electromotive force.*—Each battery-cell possesses a certain power of driving a current, which is directly proportional to, and can be measured by, the ΔV that is found to exist between the poles when the circuit is broken. Since that which moves *matter* is called *force*, so by analogy that which moves '*electricity*' was called '*electromotive force.*'

The reader must remember that this term is an inexact one, as the '*electromotive force*' is not a *force* at all in the scientific sense defined in Chapter II. § 4. To prevent confusion we shall henceforth use the letters E.M.F. instead; so that the actual word *force* will never be used except in the strict sense of Chapter II. § 4.

We may give an analogy from hydrostatics to make the relation between E.M.F. and ΔV somewhat clearer.

Let us imagine two reservoirs, the levels of water in the two being L_1 and L_2 respectively, connected by a pipe filled with coarse sand.

A current of water will be urged through the pipe, and its magnitude will depend, *cæteris paribus*, on the pressure due to the difference of level. The pressure can be *measured* by the difference of level $(L_1 - L_2)$; we might, indeed, if our system of units is suitably fixed, say that the pressure '*is*' the difference of level; this would be scientifically inaccurate, but—with our fixed system of units—would lead to no error in calculations.

So we can measure the E.M.F. of a cell by the ΔV that appears at the poles when the circuit is broken; or by the greatest ΔV that we discover between two points in the circuit when the current is flowing. And we may sometimes, somewhat inaccurately perhaps, use ΔV as synonymous with E.M.F.

This E.M.F. of a cell depends solely upon the nature of the cell. As we include the coatings of gases on the plates, &c., in the term '*nature of the cell*,' the reader will see that we have taken into account the phenomenon of 'Polarisation' (*see* Chapter XI. § 10), which diminishes the E.M.F. of the cell from its initial value.

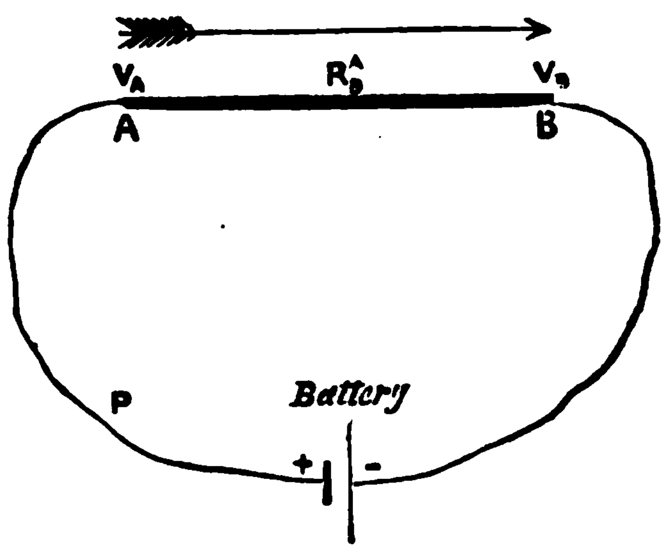
However large a current be flowing, the E.M.F. remains unaltered save by polarisation.

II. *Resistance.*—Somewhat as, in the hydrostatic analogy given above, the pipe will offer more or less resistance to the current of water according to its length, its section, and the closeness with which it is packed with gravel or sand, so a conductor in the electric circuit will offer more or less resistance according to its length, section, material, and temperature.

Resistance is that which hinders the passage of a current and limits its magnitude without producing any tendency to a back-current; it is purely passive.

III. *Current.*—This term has been explained earlier. It simply means *quantity of electricity passing across any section of the circuit in one second of time.* We *may* measure this quantity as in Chapter V. § 1; but in dealing with electric currents another system of units (explained in Chapters XIV. § 1, and XVIII. §§ 3 and 4) will be employed.

§ 2. **Exact Statement of Ohm's Law.**—Let A B be a conductor including no source of difference of potential. And let there be means of maintaining the points A and B at various Δ V.*s*, and of measuring these Δ V.*s* by electrometer or other methods; and also of measuring by electrolysis or galvanometer methods the current that then flows in the conductor A B.



Then the experimental law known as 'Ohm's Law' is that

'*So long as the conductor A B remains unaltered in temperature and in all other physical respects, the current flowing in it is directly proportional to the difference of potential maintained between its extremities A and B.*'

We may in symbols express this by saying that

$$C \propto (V_A - V_B).$$

On trying how the current varies when the conductor is varied, the Δ V between the extremities being maintained constant, our observations lead us to follow the hydrostatic analogy and to say that the conductor 'offers a greater or less *resistance*' to the current according to the dimensions, material, temperature, &c. Symbolising this resistance by R^A_B, we *define* R^A_B by the relation

$$C = \frac{V_A - V_B}{R^A_B}.$$

It occurs to us to inquire next whether 'Ohm's Law' applies to the entire circuit, including both connecting wires and battery. The difficulty now is to know what replaces the '$V_A - V_B$' of the experimental law given above. Let E represent the electrostatic Δ V between the terminals of the battery when the circuit is broken. Let B and r represent respectively the resistances of battery cell and of connecting wires (*see* §§ 3 and 4, and Chapter XIV.) Then a great mass of evidence has convinced investigators that *when the current is not dense enough to cause polarisation or other alteration in the battery, we may state that*

$$C = \frac{E}{B + r}.$$

And if there are many cells in the circuit we have

$$C = \frac{E_1 + E_2 + E_3 + \&c.}{B_1 + B_2 + B_3 + \&c. + r}$$

where the E.M.F.s are to be reckoned as negative if they oppose the resulting current.

Note.—It is in fact believed that Ohm's law always holds, but that if the current is too dense then E and B have some new values not known from our previous measurements.

§ 3. **Resistance further Discussed.**—We will now discuss further the nature of resistance, and the physical meaning of Ohm's law.

Ohm's law connects the three quantities C, E, and R; of which, up to the present point, only the two quantities C and E have received an exact meaning.

We might therefore be inclined to think that the law simply *defines* R as such that '$C = \frac{E}{R}$;' and therefore is no discovery at all, but a mere truism.

If this were the case the law would be of little use to us, since R might not be a fixed property of the conductor, one to be found once for all, but might depend upon E, so that the law would not then lend itself to the most important problem, viz., that of determining the current from a knowledge of the E.M.F.s and of the nature and dimensions of the conductor.

Thus it *might* have been the case that when once a current was started with a certain E.M.F. E, the resistance of the conductor was once for all broken down; so that an E.M.F. of 2E with the same circuit might give more than twice the current.

But direct experiment gives us two important results.

(i.) That R is a definite quantity for each conductor, and capable of measurement as such. Hence we can predict the current which any given E.M.F. will drive through any given conductor.

(ii.) That R depends in a very simple way on the length, cross section, temperature, and material of the conductor.

§ 4. **The Exact Conditions on which Resistance Depends.**—If the resistance of such bodies as *wires* be examined experimentally, it is found that R is directly proportional to the length l of the

wire, inversely proportional to its sectional area A, and depends also on its 'specific resistance' c, and on its temperature.

We will define later the exact meaning of 'specific resistance,' and we will for the present omit the factor expressing the dependence of the resistance on temperature. Thus we may state that

$$R = k \cdot \frac{c \cdot l}{A}.$$

Here k is a constant depending on the unit of resistance employed.

The nature of the experimental methods of proving the above is indicated in what follows.

Experiments.—(i.) *External resistance.*—If we wish to examine most simply the law of resistance r of such external conductors as wires, we must get rid of the internal resistance B of the battery (*see* § 2, equation (ii.)).

We can practically make $B = o$ by employing, instead of an ordinary battery, a *thermo-cell.* This can be constructed of metallic bars of such thickness as to make their resistance relatively negligible; and a constant E.M.F. can be insured by keeping the two sets of junctions at two fixed temperatures (*see* Chapter XVI.).

With such a battery the formula of Ohm's law becomes $C = \dfrac{E}{r}$.

We can measure C by means of a galvanometer, as will be explained fully in Chapter XVII. Then, by varying the length, material, and cross section of the wire, we are led to the result that $r = k\,\dfrac{c \cdot l}{A}$.

It is to be noticed that this result proves that the current flows through the whole body of a wire equally, and not along its surface. Provided that the temperature of the wire is constant, it makes no difference whether a cross section of (*e.g.*) 1 *sq. cm.* be circular, square, or of the form of a flat rectangle. Here is a notable difference between conductors as used for electrostatic purposes, and as used for conveying a current, respectively.

(ii.) *Internal resistance of a battery.*—The internal resistance of a battery is a far more difficult matter to investigate. Indeed the conditions are so complex, that it is invariably the rule to *find experimentally* the resistance of a cell or battery, instead of to *calculate* it—as we do in the case of a wire—from a knowledge of the materials and their dimensions.

But still it is possible to show that in all probability the same general laws hold; or that $B = k \cdot \dfrac{c \cdot l}{A}$.

To investigate internal resistance we get rid of the external resistance r by employing very thick copper conductors, of no appreciable resistance, outside

the cell. The cell itself is constructed with plates of various sizes (so as to vary A), whose distance apart can be altered (so as to vary l). The same general results are found to hold; though, since we cannot confine the current to that portion of the liquid which lies directly between the opposed plates, we cannot arrive at very exact results.

It is important to note that—

(i.) n cells in series offer to the current n times the resistance of one cell, since, in the former, the length l of liquid traversed is n times that in the latter.

(ii.) If n cells be coupled zincs to zincs and coppers to coppers, or 'in parallel,' we practically make one large cell of plates n times the area of those of one cell. Hence the area A of the column of liquid traversed is increased n-fold; and the resistance is $\frac{1}{n}$th of that of one cell, and $\frac{1}{n^2}$ of that offered by n cells in series.

§ 5. **Conductivity.**—Since $C = E \cdot \frac{1}{R}$, we may call $\frac{1}{R}$ by the name 'conductivity,' and say that the current is directly proportional to the conductivity, instead of inversely proportional to the resistance, of the circuit.

In § 9, fig. iii., we represent two points A and B in a circuit joined by several wires of resistances $r_1, r_2, r_3, r_4 \ldots$ respectively. We may call their conductivities c_1, c_2, c_3, and $c_4 \ldots$ respectively; where $c_1 = \frac{1}{r_1}$, &c.

Now it is possible to replace these wires by *one* wire of resistance R′, such that the current is unaltered. When this is the case we call R′ the *equivalent resistance*; and if $c' = \frac{1}{R'}$, then c' will be the equivalent conductivity.

Now it hardly seems to require proof that

$$c' = c_1 + c_2 + c_3 + c_4 + \ldots\ldots$$

If we admit this, then it follows that

$$\frac{1}{R'} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} + \ldots\ldots;$$

which enables us to determine that resistance which is *equivalent* to, and might replace, the many branches.

Note.—In case the above statement seem rather 'likely' than convincing, [w]e here give a proof.

Let V_A be the potential of A, V_B that of B; so that $V_A - V_B$, which we [m]ay write as E_B^A, is the E.M.F. between A and B.

Let C be the total current passing, and let C_1, C_2, C_3, &c., be the portions [of] C that traverse r_1, r_2, r_3, &c., respectively.

Then by Ohm's law we have . . . .

$$C_1 = \frac{E_B^A}{r_1};\ C_2 = \frac{E_B^A}{r_2};\ \text{\&c.}$$

$$\text{Whence } C_1 r_1 = C_2 r_2 = C_3 r_3 = \text{\&c.} = E_B^A,$$

$$\text{or } \frac{C_1}{\frac{1}{r_1}} = \frac{C_2}{\frac{1}{r_2}} = \frac{C_3}{\frac{1}{r_3}} = \text{\&c.} = E_B^A.$$

By ordinary algebra, each of these terms

$$= \frac{C_1 + C_2 + C_3 + \text{\&c.}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \text{\&c.}}.$$

But $C_1 + C_2 + C_3 + \text{\&c.} = C$, since the latter is the total current.

$$\text{Hence } E_B^A = \frac{C}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \text{\&c.}}.$$

But if R′ be the *equivalent resistance*, then $C = \frac{E_B^A}{R'}$; or $E_B^A = \frac{C}{\frac{1}{R'}}$.

$$\text{Whence } \frac{1}{R'} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \text{\&c.};$$

or the (*equivalent conductivity*) = (*sum of the several conductivities*).

§ 6. **Application of Ohm's Law in a Simple Case.**—Suppose that we have a battery of *n* cells, and wish to know the best way of coupling them up in the two extreme cases of

(i.) Practically zero internal resistance.

(ii.) Practically zero external resistance.

Let B be the resistance of each cell, *r* the external resistance, E the E.M.F. of each cell, and C the current. (Read again § 4, end.)

(I.) *Let B, and even n B, be negligible as compared with r.*

(i.) The current from a single cell will be . . . .

$$C_0 = \frac{E}{B + r} = \frac{E}{r} \text{ (approx.).}$$

(ii.) The current from the n cells in parallel (*see* § 4) will be

$$C_1 = \frac{E}{\frac{B}{n} + r} = \frac{E}{r} = C_0 \text{ (approx.).}$$

(iii.) The current from the n cells in series (*see* § 4) will be

$$C_2 = \frac{nE}{nB + r} = \frac{nE}{r} = nC_0 \text{ (approx.).}$$

Hence we see that with arrangement (ii.) we have no advantage over a single cell; while with arrangement (iii.) we get n times the current that we get with a single cell.

(II.) *Let r be negligible with respect to B, and even with respect to $\frac{1}{n}B$.*

(i.) With a single cell . . . .

$$C_0 = \frac{E}{B + r} = \frac{E}{B} \text{ (approx.).}$$

(ii.) With n cells in parallel . . . .

$$C_1 = \frac{E}{\frac{B}{n} + r} = \frac{E}{\frac{B}{n}} = \frac{nE}{B} = n \,.\, C_0 \text{ (approx.).}$$

(iii.) With n cells in series . . . .

$$C_2 = \frac{nE}{nB + r} = \frac{nE}{nB} = \frac{E}{B} = C_0 \text{ (approx.).}$$

The reader should compare this result with that in (I.), and should try to understand the physical meaning of these results; it is not enough to follow merely the algebraic reasoning.

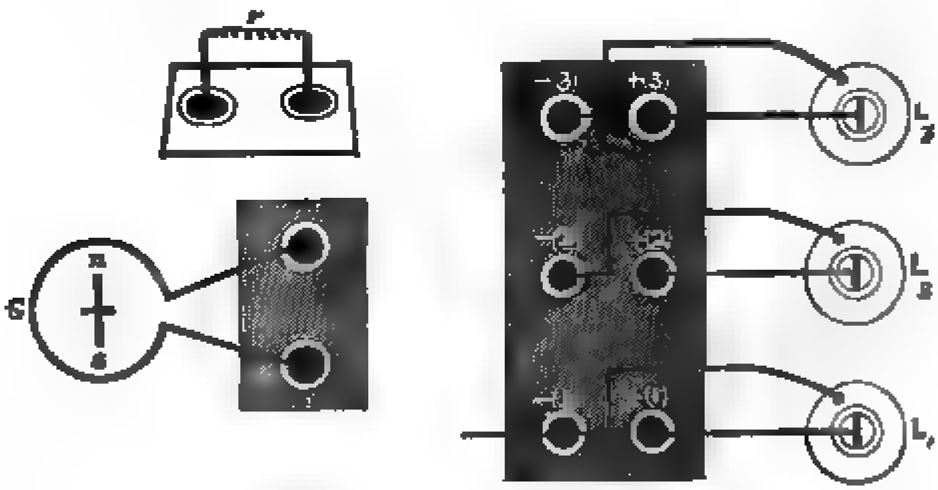
Experiments.—The figure represents diagrammatically three Leclanché cells whose + poles are connected with three mercury cups, +(1), +(2), and +(3), respectively, the − poles being connected with the mercury cups −(1), −(2), and −(3), respectively. By means of thick copper pieces we can connect these cells either in parallel or in series.

By means of other mercury cups, and of other thick copper pieces, we can throw into the circuit of one or more cells the resistance-box r (*see* Chapter XIV.), and the galvanometer G. This latter may either be a common *tangent galvanometer* of no appreciable resistance, suitable for measurements of large currents; or may be a more delicate instrument by means of which much smaller currents may be measured (*see* Chapter XVII.).

The mercury cups and thick copper pieces give us *connections* of practically zero resistance.

(i.) Let us put into the circuit a very large external resistance r of (say) 000 ohms; and let us also put in our more sensitive galvanometer, as the rrent will be small.

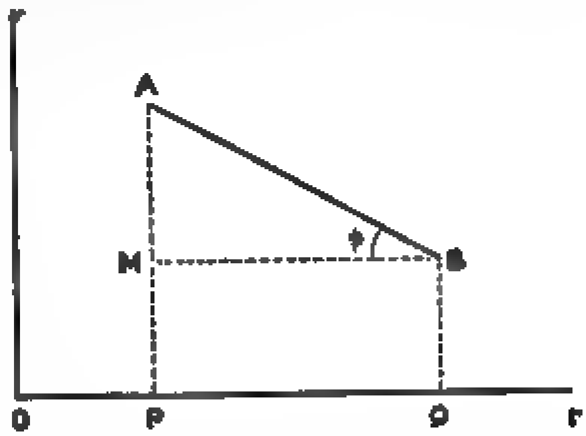
We can then, by coupling up our three cells in the different ways indicated (I.) above, verify the results there deduced from Ohm's law.



(ii.) Next we may remove r, and have in the circuit only a tangent galvanometer of practically no resistance.

The results of (II.) above may thus be verified.

§ 7. **Graphic Representation of Ohm's Law.**—By a graphic method, familiar to those who are acquainted with the elements of co-ordinate geometry, and capable of being understood also by those of less mathematical knowledge, we may represent to the eye in one view (i.) the difference of potential between two points A and B in a circuit; (ii.) the resistance between A and B; (iii.) the current which flows through this resistance on account of this difference of potential.



Let distances measured long the line O r represent y their length the magnitude of the resistances. If he wire be uniform, then qual lengths of wire will be represented by equal lengths along) r. If the conductor be not uniform, this will not be the case. . mètre of a very thick good conductor may be practically a

point on our resistance-line O r, while a millimètre of bad conductor may be represented by a considerable length on O r.

Let distances along O v, drawn at right angles to O r, represent potentials. If the reader will bear in mind that we are concerned only with *differences of potential*, he will see that it does not matter what potential we choose as our zero-potential, *i.e.* as the potential of our starting-point or 'origin' O. We can, in fact, take the point O to represent any fixed point in the circuit that we choose, and can measure off the potentials of other points as so much above or below the potential of this point taken as our arbitrary zero; this will make no difference in the magnitude of the line representing the Δ V of two points in the circuit.

Thus if A and B represent two points in the circuit, and if lines A P and B Q be drawn perpendicular to O r, and if B M be drawn parallel to O r, then the line A P represents by its magnitude the potential of the point A above our arbitrary zero, the line B Q that of the point B, and A M the Δ V between A and B; while O P and O Q represent the resistances between our starting-point and the points A and B respectively, and P Q represents the resistance between A and B.

Now Ohm's law gives us that $C = \dfrac{E_B^A}{R_B^A}$ (*see* § 2 (iii.)); whence $C = \dfrac{AM}{PQ}$, if we have drawn our figure to proper scale; or $C = k \cdot \dfrac{AM}{PQ}$, where k is some constant, if we have taken our scale of drawing at random. Let us suppose that $C = \dfrac{AM}{PQ}$.

But if ϕ be the angle that the line A B makes with the axis O r, we have $\dfrac{AM}{PQ} = \tan \phi$; whence we have

$$C = \tan \phi.$$

Thus the 'slope' of the line A B, as measured by $\tan \phi$, indicates to us the current strength. Of course, since the current C is the same throughout the circuit, the line A B has the same slope whatever points in this circuit A and B may be. If we take in the whole circuit we have

$$\tan \phi = \frac{\text{ordinate representing total E.M.F. in circuit}}{\text{abscissa representing total resistance in circuit}}$$

This last result, if known, obviously enables us to find the position of the point A in the diagram when we know that of B, and when we know also the resistance Q P between B and A; just as we can *algebraically* find the potential of A when we know the potential of B, the resistance between A and B, and the current flowing in the circuit (*see* § 11).

§ 8. Applications of the Graphic Method.

We can thus represent to the eye each result arrived at algebraically from Ohm's law. We will indicate in a few cases how this may be done.

(1) Referring to § 6 (I.). To represent *case* (i.) the reader should mark off from O v a distance O A representing the E.M.F. of one cell; and from O r a distance O a representing the external resistance r, B being negligible. The current is then represented by tan ϕ_0, where ϕ_0 is the angle which A a makes with O r.

To represent *case* (ii.) we do not (appreciably) alter the resistance O a, and the *E.M.F.* O A is unaltered. We have, therefore, the same line A a making an angle $\phi_1 = \phi_0$; so that the current is unaltered.

In *case* (iii.) the resistance O a is practically unaltered, but we mark off on O v a distance O N $= n \times$ O A, and we join N a. This line N a makes with O r an angle ϕ_2 such that tan $\phi_2 = n$. tan ϕ_0, or the current is n times greater than in *case* (i.).

(2) The reader can easily represent the three cases of (II.).

(3) It will be a useful exercise for the reader to draw the rises and falls of potential through a circuit in which there are abrupt changes at different points (due, let us suppose, to contacts of different metals at those points), as well as a general fall following Ohm's law. The method will be as follows. From a knowledge of the total E.M.F. and the total resistance in the circuit we can find C, or can find *tan* ϕ; thus we know the *slope* of the line between points separated by homogeneous conductors. The abrupt changes of potential must be given. It is easy then to trace the slope of, and abrupt rises and falls of, potential from any point in the circuit round to that point again.

(4) *Note.*—For ordinary purposes it is sufficient to mark off from O v a length representing the total algebraic sum of the E.M.F.s in the circuit, as if this E.M.F. were an abrupt rise of potential occurring at one point in the circuit and nowhere else. The slope (*i.e.* tan ϕ) of the line is not affected by this convenient assumption.

§ 9. Divided Circuits.—We will now discuss more fully than in § 5 the case of a divided circuit. The notation employed is

sufficiently explained by the diagrams and by what has preceded in § 5. (Read § 5 again.) We will take several cases.

I. *Two equal branches.*—Here we have for the equivalent resistance . . . .

$$R' = \frac{r}{2}. \qquad \frac{1}{R'} = \frac{1}{r} + \frac{1}{r} = \frac{2}{r}.$$

Hence, if R be the resistance in the rest of the circuit and in the battery together, and if E be the E.M.F. of the battery, we have. . . . . . .

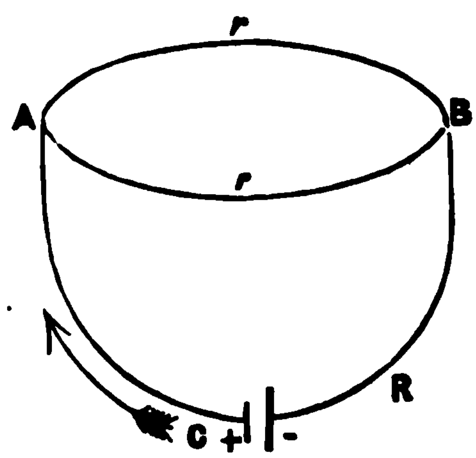


FIG i.

$$\text{Total current } C = \frac{E}{R' + R}.$$

And since $E_B^A = C_1 r = C_2 r$, it follows that $C_1 = C_2 = \frac{C}{2}$; as is otherwise evident.

II. *N equal branches.*—Here in a similar manner we can show that . . .

$$C = \frac{E}{R' + R}; \; R' = \frac{r}{n};$$

$$C_1 = C_2 = C_3 = \&c. = \frac{C}{n}.$$

III. *Two unequal branches.*—Here we have . . . . . .

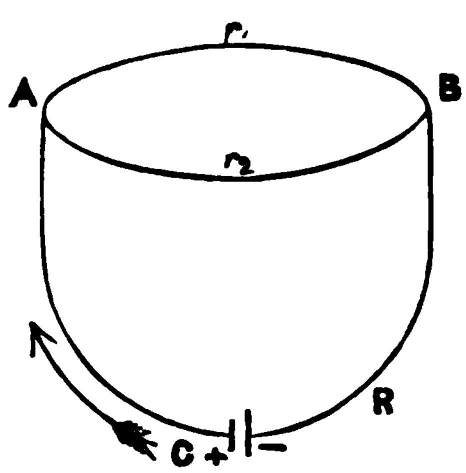


FIG. ii.

$$\frac{1}{R'} = \frac{1}{r_1} + \frac{1}{r_2};$$

$$\text{or } R' = \frac{r_1 r_2}{r_1 + r_2} \quad . \; . \; . \; . \quad \text{(i.)}$$

$$\text{Also, } \quad C = \frac{E}{R' + R} \quad . \; . \; . \; . \quad \text{(ii.)}$$

And since $E_B^A = C_1 r_1 = C_2 r_2 = CR'$, we get the results that . . . .

$$\frac{C_1}{C_2} = \frac{r_2}{r_1} \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(iii.)}$$

$$\left.\begin{aligned} C_1 &= \frac{R'}{r_1}.C = \frac{r_2}{r_1 + r_2}.C \quad . \quad . \\ C_2 &= \frac{R'}{r_2}.C = \frac{r_1}{r_1 + r_2}.C \quad . \quad . \end{aligned}\right\} \quad . \quad . \quad . \quad (iv.)$$

IV. *N unequal branches.*—In this most general case we have

$$\left.\begin{aligned} \frac{1}{R'} &= \frac{1}{r_1} + \frac{1}{r_2} + . \quad . \quad . \&c. + \frac{1}{r_n} \quad . \quad . \\ &\text{or,} \\ R' &= \frac{r_1 . r_2 . r_3 . \quad . \quad . \quad r_n}{r_2 r_3 \ldots r_n + r_1 r_3 \ldots r_n + \&c.} \end{aligned}\right\} \quad (i)$$

And by a similar reasoning to above we find that . . . .

$$C = \frac{E}{R' + R} \quad . \quad . \quad . \quad . \quad . \quad (ii.)$$

$$\left.\begin{aligned} C_1 &= \frac{R'}{r_1} C \\ C_2 &= \frac{R'}{r_2} C \\ &. \quad . \quad . \quad . \\ C_n &= \frac{R'}{r_n} C \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (iii.)$$

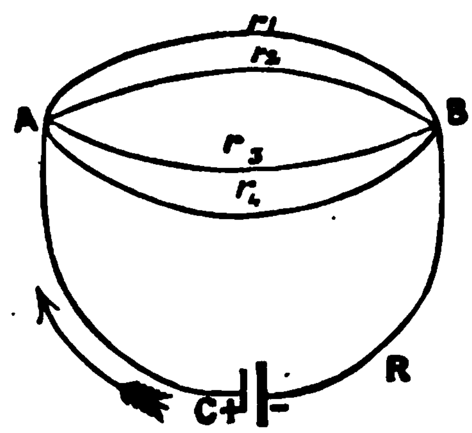


FIG. iii.

Example in divided circuits.—'Two wires, A D B and A E C B, of uniform section, &c., and of equal resistance, connect the points A and B. A third wire, A F C, of equal resistance, connects the point A with the middle point C of one wire. When a current flows from A to B, find what fraction of it passes through the wire A F C.'

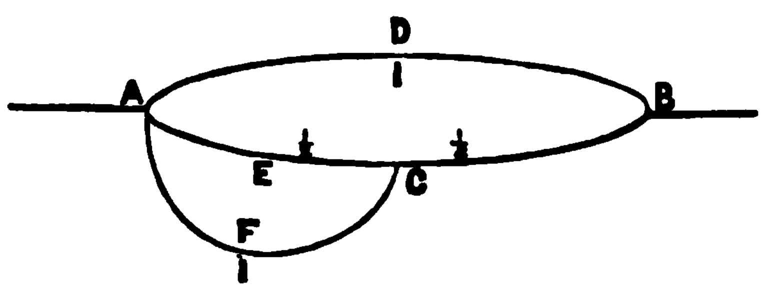


FIG. iv.

In the figure we represent the arrangement intended, and we have called the resistances of A D B and A F C each 1, while those of A E C and of C B are each $\frac{1}{2}$. We will *first* find the equivalent resistance of the paths from A through C to B, and so find the fraction of the current that passes through A D B; and *next* we will find what fraction of the remainder takes the route A F C. Now the resistance of A D B = 1, or $\frac{6}{6}$. And the equivalent resistance

R′ between A and C is given by $\frac{1}{R'}=\frac{1}{\frac{1}{2}}+\frac{1}{1}=3$; whence $R'=\frac{1}{3}$. Hence the total resistance of the route by C is $\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$. Therefore the resistances by the routes D and C are in the ratio of 6 : 5 respectively. Therefore $\frac{5}{11}$ of the current passes by D, and $\frac{6}{11}$ passes from A through C.

Again, this $\frac{6}{11}$ will be distributed between the branches A E C and A F C in the inverse ratio of their resistances; that is, in the direct ratio of 2 : 1. Hence $\frac{1}{3}$ of $\frac{6}{11}$ will pass by the route A F C; or the answer required is $\frac{2}{11}$.

§ 10. '**Shunts.**'—The case of *two unequal* branches has a very important application. It is often necessary (i.) to protect a sensitive galvanometer against a strong current by allowing only a part of the current to pass through the galvanometer, and yet (ii.) to measure the current. This is done by connecting the terminals of the galvanometer by a wire of greater or less resistance, and thus leaving a greater or smaller fraction of the current to pass through the galvanometer. If this fraction be known, and its magnitude be measured, we can easily calculate the total current. We shall see, however, that the introduction of this 'short cut,' or '*shunt*,' has the effect of increasing the total current by decreasing the total resistance; and hence the fraction measured is a certain fraction of the *new* total current, not of the *original* current. Let R be the resistance of the battery and rest of the circuit combined; let G be the resistance of the galvanometer which here takes the place of r_1 in § 9, case (III.); let S be the resistance of the *shunt* which here takes the place of r_2; and let R′ be the equivalent resistance of G and S. This R′ is of course less than either G or S. Let C be the total current; C_G the current passing through the galvanometer; and C_S the current through the shunt. Let C_0 be the original current passing before the shunt was used. Then we have . . . .

$$\begin{cases} C_0 = \dfrac{E}{G+R}\,;\ C = \dfrac{E}{R'+R}\,;\ \dfrac{1}{R'} = \dfrac{1}{G} + \dfrac{1}{S}\,; \\ C_G = \dfrac{S}{G+S}\cdot C\,;\ C_S = \dfrac{G}{G+S}\cdot C. \end{cases}$$

We will now proceed to consider several points.

(i.) If we wish to allow $\frac{1}{10}$th, $\frac{1}{100}$th, $\frac{1}{1000}$th, &c., of the total current to pass through the galvanometer, then $\frac{S}{G+S}$ must be $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1000}$ respectively; or S must be $\frac{G}{9}$, $\frac{G}{99}$, $\frac{G}{999}$, &c., respectively.

In general, if $S = \frac{1}{n-1} \cdot G$, then will $\frac{1}{n}$th of C pass through the galvanometer.

(ii.) If G be negligible as compared with R, then will $C = C_0$, whatever shunt we use. This follows from the above formulæ, since we there may neglect G, and therefore *a fortiori* may neglect R′, which is less than G.

(iii.) *If G, and $\frac{1}{n}$ G, be so great that R may be neglected*, then we find $C = nC_0$; and as a consequence we should have passing through the galvanometer a current $= \frac{1}{n}C = \frac{1}{n} \cdot nC_0 = C_0$; thus exposing the galvanometer to a current as strong as the whole original current.

By having in the circuit an adjustable resistance, we can introduce a compensating resistance r, and so maintain C equal to C_0.

Note.—To keep $C = C_0$, we must have the total resistance of circuit the same; or the compensating resistance r must be such that $G + R = \frac{GS}{G+S} + R + r$; or $r = G - \frac{GS}{G+S} = \frac{G^2}{G+S}$. Thus if $S = \frac{1}{n-1}G$, so as to give $C_G = \frac{1}{n} \cdot C$, we have $r = \frac{n-1}{n}G$. In this case $C_0 = C$; and $C_G = \frac{1}{n}C_0$, as was desired.

§ 11. **Fall of Potential through the Circuit.**—The following view of a circuit, a view indicated by the figure to Chapter XI. § 7, is convenient, and, though not accurate in detail, it can for general purposes be substituted for a more accurate view, and can lead to no error in such considerations as follow.

In the figure referred to, it is supposed that in the cell the E.M.F. occurs as an abrupt rise in potential; in fact, that the energy of the cell is spent in, as it were, pumping the electricity straight up from the lower to the higher level. Thus, the line C Z

represents, in the graphic method, the total *E.M.F.* E of the circuit. The potential falls round the circuit from the higher to the lower level again; the total fall being E.

Ohm's law, as expressed by the formulæ given in § 2, means that this fall proceeds proportionally to the resistance. Hence, if R′ be the resistance of any portion of the circuit, and if R be the total resistance of the whole circuit, then the fall of potential between the beginning and end of R′ will be $\frac{R'}{R}$. E. We deduce the following.

(*a*) In the case of negligible external resistance the whole fall of potential will (approx.) take place in the cell itself. We find (approx.) no ΔV between the beginning and the end of the external circuit; or the poles of the battery will have (approx.) the same potential.

(*b*) In the case of negligible internal resistance the whole fall of potential will (approx.) take place externally to the cell. We shall find (approx.) between the beginning and the end of the external circuit a ΔV equal to the whole E.M.F. of the battery; or the poles of the battery will have (approx.) the same ΔV as if the circuit were broken.

(*c*) Where internal resistance equals external, we shall have the fall of potential equally divided. The poles of the battery will exhibit a ΔV equal to $\frac{1}{2}$ E.

(*d*) Where there is a multiple arc between two points A and B, and we wish to find the ΔV E_B^A between A and B, we have merely to calculate the equivalent resistance R′. We then have

$$E_B^A = \frac{R'}{\text{Total resistance}} \cdot E.$$

Note.—All the above cases are readily exhibited graphically (*see* § 7).

§ 12. **Kirchhoff's Two Laws.**—By a careful application of Ohm's law, and of the principle that *when the V of a point is constant there must be as much electricity flowing away from it as flows to it in each second*, it is possible to investigate the distribution of current and potential in very complicated cases; cases where we have any number of cells connected by a net-work of conductors in any way whatever.

Kirchhoff has enunciated in the form of two 'Laws' the principles that must guide us in such an investigation.

Law I.—If any number of conductors meet at a point, and if all currents flowing to the point be considered +, and all currents flowing from the point be considered —, and if the condition of things be steady, or the potential at the point be not altering, then the algebraic sum of the currents meeting at the point must be zero. Or . . . .

$$\Sigma . C = 0.$$

Law II.—Let us suppose there to be such a net-work of conductors as that imagined above; there being cells of various E.M.F.s, and turned various ways, in this net-work.

If we imagine ourselves to start from any point in this net-work and to make a circuit through the conductors back to our starting-point, we shall have passed through conductors of various resistances, shall have passed through various cells whose E.M.F.s are directed so as to drive a current against or with us, and shall have found various currents, some with us and some against us.

As long as we are passing along a single conductor r_1, to which there are no outlets, the current has some fixed value C_1; but on passing a point where two or more conductors meet we may find a different current C_2, which will remain constant over the next piece of resistance r_2 up to the next place where two or more branches meet. We can thus divide our circuit into portions of resistances r_1, r_2, r_3, &c., respectively; along each of which will be a current C_1, C_2, C_3, &c., respectively. If we call these currents + or —, according as they flow with us or against us respectively; and if we call the E.M.F.s that we encounter + or — according as they tend to drive a current with us or against us respectively; then it can be shown, by successive applications of Ohm's law to these different portions of our circuit, that *in the complete circuit* we must have . . . .

$$C_1 r_1 + C_2 r_2 + C_3 r_3 + \ldots = e + e' + e'' + \ldots$$

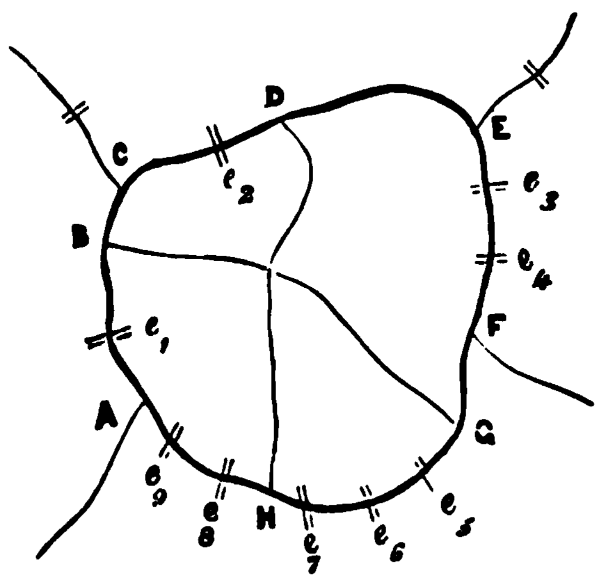
where e, e', &c., are the various E.M.F.s that we pass; attention being paid to signs both of the C . s and of the e . s.

These E.M.F.s e, e', e'', may occur in any way in the circuit, and do not necessarily (or indeed usually) occur in the pieces of circuit r_1, r_2, &c., respectively.

This law is usually expressed thus.

'*In any complete circuit*, $\Sigma . CR = \Sigma . e$.'

Proof of Kirchhoff's second law.—We will here indicate the manner in which Kirchhoff's second law may be proved. The figure represents part of a net-work of conductors, in which are introduced any number of battery-cells, e_1, e_2, e_3, &c. Between any two consecutive junctions, as A and B, the current has some constant value C_{AB}; and between B and the next junction C there will be a constant current C_{BC}, which will in general be different from C_{AB}.



The total resistance of conductor and cells between A and B is represented by r_{AB}.

The symbols e_1, e_2, &c., represent the numerical values of the various E.M.F.s, and we will suppose the *signs* of these E.M.F.s to be also included in these symbols.

The potentials of the points A, B, C, &c., will be represented by V_A, V_B, &c. Then, by successive applications of Ohm's law, we have

$$\begin{cases} C_{AB} \times r_{AB} = V_A - V_B + e_1. \\ C_{BC} \times r_{BC} = V_B - V_C. \\ C_{CD} \times r_{CD} = V_C - V_D + e_2. \\ C_{DE} \times r_{DE} = V_D - V_E. \\ C_{EF} \times r_{EF} = V_E - V_F + e_3 + e_4. \\ C_{FG} \times r_{FG} = V_F - V_G. \\ C_{GH} \times r_{GH} = V_G - V_H + e_5 + e_6 + e_7. \\ C_{HA} \times r_{HA} = V_H - V_A + e_8 + e_9. \end{cases}$$

Whence, by addition, we have . . . .

$$\Sigma . CR = \Sigma . e.$$

These two laws may be applied in any complicated system of E.M.F.s and of conductors, in such a way as to determine the currents in all the branches, giving us always n independent equations to determine n unknowns.

As a simple illustration we may take the case of fig. iii. § 9, where there is but one *E.M.F.* E. Let us suppose that E, R, r_1, r_2, &c., are known, and that it is required to find C, C_1, C_2, &c.

Applying *Law I.* to the point A (or B) we have . . . .

$$C = C_1 + C_2 + C_3 + C_4 \quad . \quad . \quad . \quad . \quad . \quad \text{(i.)}$$

Applying *Law II.* successively to the circuits R A r_1 B R, A r_1 B r_2 A, r_1 B r_3 A, A r_1 B r_4 A, we get the equations . . . .

$$C\ R + C_1 r_1 = E \qquad \text{(ii.)}$$
$$C_1 r_1 - C_2 r_2 = O \qquad \text{(iii.)}$$
$$C_1 r_1 - C_3 r_3 = O \qquad \text{(iv.)}$$
$$C_2 r_2 - C_4 r_4 = O \qquad \text{(v.)}$$

These are five independent equations, and will enable us to determine the five unknown currents.

However, the method employed in § 9 was in this case simpler to pursue.

§ 13. **Maximum Current with a given Battery.**—If we have a battery of n cells, each of *E.M.F.* E, and of internal resistance B, it is of great importance to arrange them so as to drive the largest possible current through a given external resistance r.

If n has factors m and l, we may couple the cells so as to have m in series and l parallel. We thus get a battery of E.M.F. $= m$ E, and of internal resistance $= \frac{m\,B}{l}$. The current obtained will be

$$C = \frac{m\,E}{\frac{m\,B}{l} + r}.$$

If we investigate either algebraically, or by means of the calculus, the arrangement that will give a maximum value of C, we find that this occurs when . . . .

$$\frac{m}{l}\,B = r,$$

Or, *we obtain the maximum current with a given battery when it is so arranged as to make the internal resistance as nearly as possible equal to the external resistance.* (For proof, *see* note.)

It is therefore convenient to have a battery in which the number of cells is a number having several pairs of factors.

Example.—We will here assume current, E.M.F., and resistance to be expressed in the units, *ampères*, *volts*, and *ohms* respectively; these are units which will be spoken of later.

Let us have $n = 24$, E = 2 *volts*, B = 2 *ohms*, and $r = 3$ *ohms*.

Then we have the following arrangements possible.

(i.) 24 *cells end-on.*

$$\text{Here } C_1 = \frac{24 \times 2}{24 \times 2 + 3} = \frac{48}{51} = \frac{16}{17}\ \textit{ampères}.$$

(ii.) 12 *cells end-on*, 2 *parallel*.

$$\text{Here } C_2 = \frac{12 \times 2}{\frac{12}{2} \times 2 + 3} = \frac{24}{15} = \frac{8}{5} \textit{ ampères.}$$

(iii.) 8 *cells end-on*, 3 *parallel*.

$$\text{Here } C_3 = \frac{8 \times 2}{\frac{8}{3} \times 2 + 3} = \frac{16}{\frac{25}{3}} = \frac{48}{25} \textit{ ampères.}$$

(iv.) 6 *cells end-on*, 4 *parallel*.

$$\text{Here } C_4 = \frac{6 \times 2}{\frac{6}{4} \times 2 + 3} = \frac{12}{6} = 2 \textit{ ampères.}$$

(v.) 4 *cells end-on*, 6 *parallel*.

$$\text{Here } C_5 = \frac{4 \times 2}{\frac{4}{6} \times 2 + 3} = \frac{8}{\frac{13}{3}} = \frac{24}{13} \textit{ ampères.}$$

It is needless to continue, for it is clear that (iv.) gives us the greatest current. And case (iv.) is that in which the internal resistance of the battery is 3 *ohms*, or equals the external resistance. Of course we cannot in general so subdivide our battery as to make the equality exact.

In such extreme cases as those of § 6 we approximate most nearly when the internal resistance is made as great as possible in case (I.), and as small as possible in case (II.).

Note.—Proof of the above rule.—Let the number of cells be n, and let $n = m\,l$. We have $C = \dfrac{m\,E}{\frac{m\,B}{l} + r}$. This may be written also in the form.

$$C^2 = \frac{m^2 l^2 E^2}{(m\,B + l\,r)^2} = \frac{n^2 E^2}{(m\,B - l\,r)^2 + 4\,m\,l\,r\,B}. = \frac{n^2 E^2}{(m\,B - l\,r)^2 + 4\,n\,r\,B}.$$

Hence C is greatest when the denominator is least, since we cannot alter the value of the numerator by varying the arrangement.

This will be the case when the term $(m\,B - l\,r)^2$, which must be positive, disappears; and this again will be the case when $m\,B = l\,r$, or when $r = \dfrac{m\,B}{l}$. But r is the external, and $\dfrac{m\,B}{l}$ is the internal, resistance; hence our theorem is proved.

CHAPTER XIV.

MEASUREMENT OF RESISTANCES AND OF E.M.F.S.

§ 1. **Preliminary, on the Units Employed.**—In Chapter V. we explained the unit of quantity of electricity that is employed in electrostatics ; and in Chapter X. we explained the unit employed in measuring differences of potential. Both these were based upon the absolute C.G.S. system of units, and started from the electrostatic repulsion between two very small charged spheres. We might build upon these two units a further system of derived units for *current* or *quantity per second*, *E.M.F.*, *resistance*, &c. But such a system would only have a theoretical interest, and would not be the most convenient system to employ in dealing with currents and with the phenomena accompanying currents ; for in considering these phenomena we are rarely concerned with electrostatic actions.

The most important class of phenomena, accompanying electric currents, are the *magnetic actions* of a current. Hence, in 'current electricity' we employ a system of units, based on the C.G.S. system, and starting from the action of a current on a magnetic pole.

This system will be fully explained in Chapter XVII. Here we will merely remark that the absolute units of E.M.F. and of R in this system are so small that they are inconvenient for practical purposes. We therefore use generally . . . .

(i.) As *unit of E.M.F.*, the *volt*. This is almost exactly the E.M.F. of a cell consisting of a Cu and a Zn plate immersed in a solution of $ZnSO_4$. This unit is 100,000,000 (or 10^8) times the absolute unit of E.M.F. In somewhat the same way we use a *kilomètre*, instead of the centimètre, in large measurements of length.

(ii.) As *unit of resistance*, the *ohm*. This is the resistance of

a column of mercury at 0° C. of 1 sq. mm. section and nearly 105 cm. in length. It is 1,000,000,000 (or 10^9) times the absolute unit. The ohm is usually designated by the symbol ω.

Note.—The *ohm* here given (called the 'B.A' ohm) is now found to be a little too small; *i.e.* it is not quite 10^9 times the absolute unit.

(iii.) As *unit of current*, the *ampère*. This is the current given by 1 *volt* E.M.F. when the total resistance is 1 *ohm*. From Ohm's law it is clear that the ampère must be $\frac{10^8}{10^9}$, or $\frac{1}{10}$th, the absolute unit of current. We have then, generally, ...

$$Ampères = \frac{volts}{ohms}.$$

§ 2. **Resistance Coils, and Resistance Boxes.**—In electrical measurements we need, as standards of reference, resistances of known magnitude; just as in chemical investigations we need to use 'weights' of known magnitude. We must therefore have at our disposal convenient multiples and sub-multiples of the *ohm*. Now a column of mercury is obviously a very unmanageable unit; whereas coils of wire are very convenient, and should be durable also, if carefully made. While, therefore, we can keep our mercury column as the unit to which to refer, we can for purposes of use make copies of it in wire. Coils are made some having 1 *ohm* resistance, others having 2, 3, 4 . . . 10 . . . 50 . . . 100 . . . 1,000, &c., *ohms* resistance, and such standards of resistance are very durable and occupy but little space.

For the making of coils we choose some wire which fulfils as far as possible the following conditions: (i.) it should not corrode; (ii.) its resistance should not alter with time or with a moderate amount of bending; (iii.) it should not alter much with temperature; (iv.) it should not be too expensive. The wire which is best for the purpose is an alloy of copper, zinc, and nickel; or of silver and platinum.

The alteration with *time*, &c., can only be checked by periodic testing. As to *temperature*, either the coils are arranged so that they can be immersed in water at some standard temperature, or there may be used a 'correction formula' which will approximately allow for the rise in resistance with rise in temperature.

The wire is covered with an insulating material, and is wound

in a particular way, so as to obviate as far as possible induction effects; the wire being doubled on itself before winding, so that all through the coil there are equal currents in opposite directions lying side by side. When wound it is soaked in melted paraffin wax. Resistance coils are usually arranged in *resistance boxes*, in such a way that we can readily throw into the circuit any number of *ohms* desired.

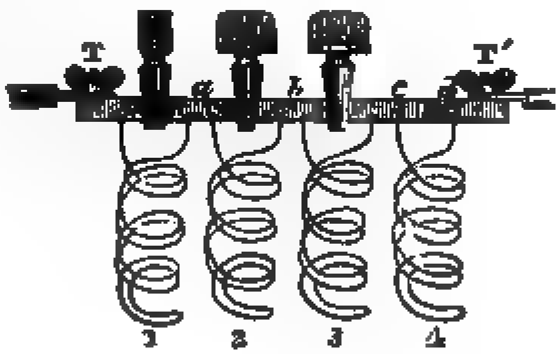


FIG. i.

The first figure indicates diagrammatically the arrangement in a resistance box. Into the circuit we introduce the thick brass piece T T′. This is not continuous, but is divided into portions connected by the resistance coils. The gaps, however, can be filled up by well-fitting brass plugs, and when all these plugs are in their places, there is between T and T′ a continuous thick brass conductor of practically no resistance. The removal

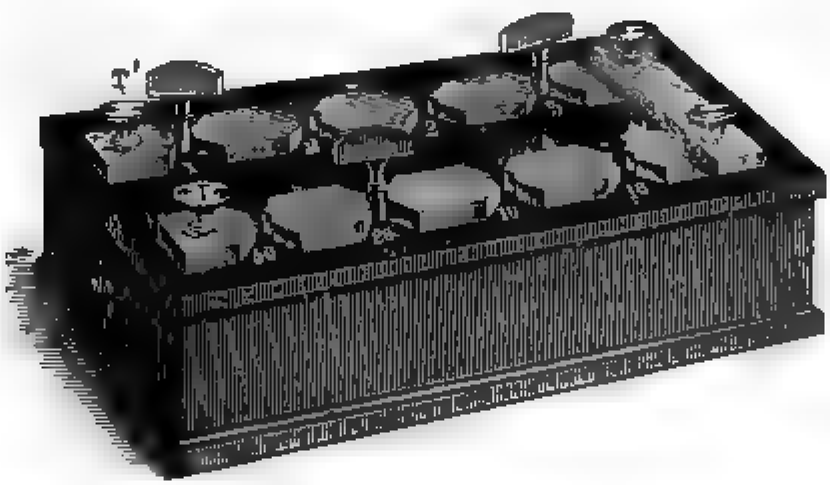


FIG. ii.

of any plug throws into the circuit that resistance coil which in the figure lies below that plug. Hence, by a suitable removal of plugs we can throw into the circuit any resistance that is contained in the box.

The figure indicates that the coils are so wound as to obviate errors due to induction (*see* Chapter XXI.).

§ 3. **Wheatstone's Rheostat.**—We will now mention an instrument *intended* for the measurement of resistances continuously, the resistance box above described being evidently discontinuous.

Note.—We may here remark that on account of certain defects of contact, &c., that appear to be inherent in the very nature of a Wheatstone's rheostat, this instrument cannot be used in any but very rough measurements.

Its chief use is as a continuously adjustable resistance, in cases where it is not desired to *measure* the resistance thus introduced.

A and B are two parallel cylinders, A being of brass and B of wood. A fine uniform wire can be wound from A on to B, or *vice versâ*. The wire that is on A is in good contact with the brass of A, and thus forms one conductor with the cylinder. The wire that is on B is wound in a spiral groove cut in the wood, and is thus insulated. In the figure the wire is represented as partly on that portion of B which is nearest to the spectator, and partly on the further portion of A. One end of the wire is attached to a brass ring on B, this brass ring being in connection with the terminal *o* by means of a brass spring that presses against it. The other end of the wire is attached to the far end of the brass cylinder A, and then is in connection, by means of the cylinder itself and a spring that presses against it, with the other terminal *n*. By turning the handle *d* we can wind more or less of the wire on to the cylinder B where it is insulated, and so throw more or less of it into the circuit. That portion of the wire which is on A gives no appreciable resistance, being then practically part of a very thick brass conductor.

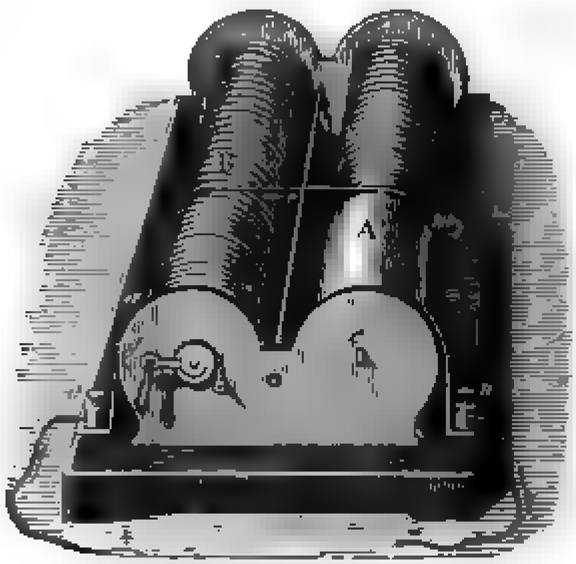


FIG. 1.

By noticing against what mark on the graduated scale (seen in the figure between the two cylinders) the wire stands, we can read off the integral number of turns of the handle *d*, the fractions of one turn being read from a graduated circle round which the handle *d* turns.

We can use this instrument in two main ways.

I. *Method of substitution.*—In the figure, R is the rheostat; $m\,m'$ are two mercury cups between which either the unknown resistance x, or a thick copper piece of no resistance, can be inserted; and G is a galvanometer

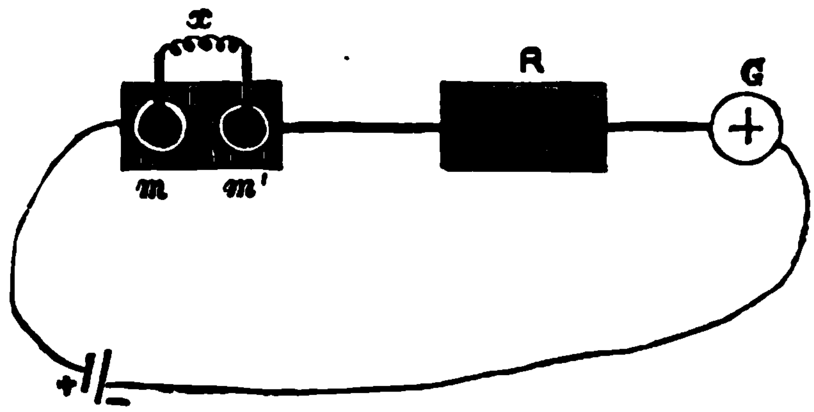


FIG. ii.

We first measure the deflexion α when x is in the circuit. We then replace x by a thick copper piece, and wind in a measured length of rheostat wire of resistance l until we have the same deflexion α as before.

Then, if the E.M.F. E has remained constant, we can easily show by Ohm's law that

$$x = l.$$

II. *Method of comparison of deflexions.*—Here we use a measuring galvanometer; *e.g.* a tangent galvanometer in which, if α be the angle of deflexion, the current-strength C is given by the relation $C = k \,.\, \tan \alpha$, where k is some constant. Let E be the E.M.F. of the battery, x the unknown resistance, R the resistance of rest of the circuit.

Let α_1 be the deflexion when x is not in the circuit. Then

$$C_1 = k \,.\, \tan \alpha_1. \quad \text{Also } C_1 = \frac{E}{R}.$$

$$\text{Whence} \quad R = \frac{E}{k} \,.\, \cot \alpha_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i.)}$$

Now introduce the resistance x, and let the new deflexion be α_2.

$$\text{Then} \quad R + x = \frac{E}{k} \,.\, \cot \alpha_2 \quad . \quad . \quad . \quad . \quad \text{(ii.)}$$

Next remove x, and introduce some measured resistance l, and let the new deflexion be α_3. Then we have

$$R + l = \frac{E}{k} \,.\, \cot \alpha_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iii.)}$$

By subtracting (i.) from (ii.) and from (iii.) respectively, and by

dividing the one result by the other, we finally eliminate both R and $\frac{E}{k}$; the final result being

$$\frac{x}{l} = \frac{\cot a_2 - \cot a_1}{\cot a_3 - \cot a_1},$$

which gives us x in terms of known quantities.

Both the above methods assume that E is constant. Now E varies with time, rendering method (I.) inaccurate; and in method (II.) E varies not only with time, but with the strength of the current flowing.

The *Wheatstone's bridge* method, next to be described, is free from the above and other objections.

§ 4. **Wheatstone's Bridge, General Principle.**—Fig. i. represents diagrammatically the principle of Wheatstone's bridge.

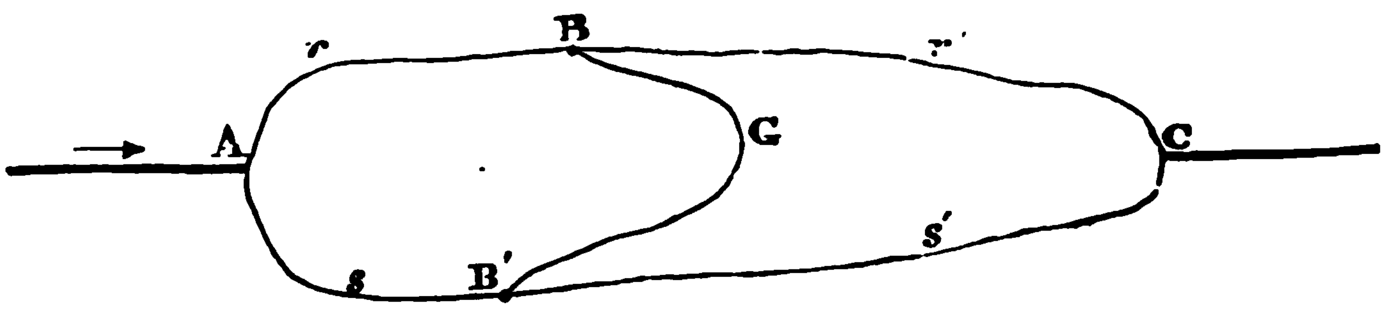


Fig. i.

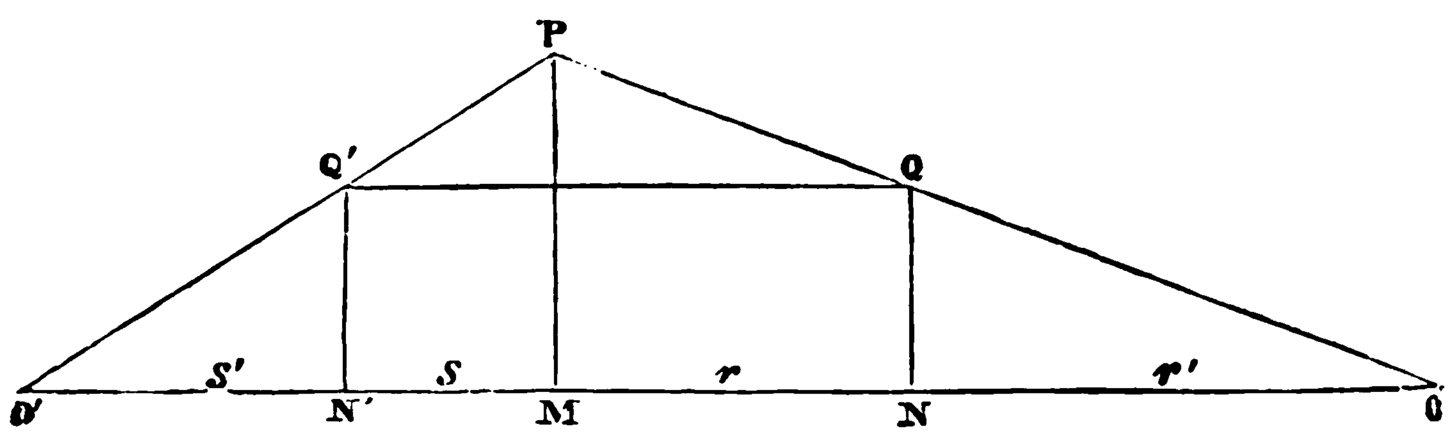


Fig. ii.

Let the current from A to C divide into two branches A B C and A B′ C; and let the resistance A B be measured by r, B C by r', A B′ by s, and B′ C by s'.

Then A is at one potential V_A, and C at a lower potential V_C; and along both branches the potential falls from V_A to V_C proportionally with the resistance. If $(r + r')$ be great as compared with $(s + s')$, then the fall through A B C is gradual as compared with the 'steeper' fall through A B′ C. But whatever proportion

$r + r'$) bear to $(s + s')$, there will be on the branch A B′ C points of the same potentials as those of any points on A B C respectively.

Now if we connect two points B and B′ by a 'bridge' B G B′ containing a galvanometer G, then according as we see indicated a current from B to B′, no current, or a current from B′ to B, we shall conclude that B is above B′ in potential, B and B′ are at the same potential, or B′ is above B in potential, respectively.

The case of no current through G is the most important; in this case B and B′ must be at the same potential.

Now in fig. ii. M P represents the potential V_A; and we have taken V_C as our zero potential, so that the point C is represented by O or by O′. The fall of potential down the one circuit is given by the line P Q O, the fall down the other circuit by P Q′ O. The points Q and Q′ represent B and B′ respectively; when these latter are at the same potential, as proved by the absence of current in G. The rest of the figure is clear after what we have said in Chapter XIII. § 7.

Then when B and B′ are at the same potential, or when $QN = Q'N'$, we have, by Euclid, Book VI. . . . .

$$\frac{r'}{r + r'} = \frac{NQ}{MP}; \text{ and } \frac{s'}{s + s'} = \frac{N'Q'}{MP};$$

$$\text{and therefore } \frac{r'}{r + r'} = \frac{s'}{s + s'}$$

$$\text{or } \frac{r}{r'} = \frac{s}{s'} \qquad \text{. . . . . (i.)}$$

Hence one of the resistances is determined by the other three.

Algebraic proof of the same.—We will now use a notation that will be readily understood with reference to fig. i. of § 4, the notation being similar to that employed in Chapter XIII. §§ 9 and 10.

Since there is no current through G, we have $C_r = C_{r'}$; and $C_s = C_{s'}$.

Applying Kirchhoff's second law to the circuit A B G B′ A, we have . .

$$C_r \,.\, r - C_s \,.\, s = o; \text{ or } C_r \,.\, r = C_s \,.\, s.$$

Applying the same law to the circuit B C B′ G B, and remembering that $C_r = C_{r'}$, and $C_s = C_{s'}$, we have also in like manner . . . .

$$C_r \,.\, r' - C_s \,.\, s' = o; \text{ or } C_r \,.\, r' = C_s \,.\, s'.$$

Whence we readily obtain the result that

$$\frac{r}{r'} = \frac{s}{s'},$$

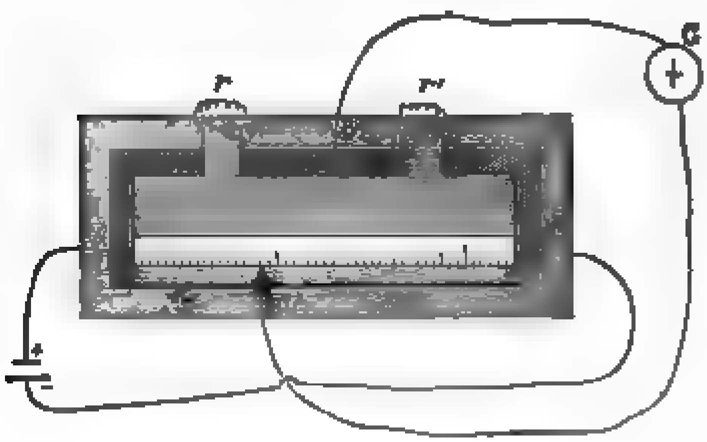
It is clear that this formula remains true, however the E.M of the battery may alter. In this fact lies the main superior of this method, over the previous methods, of measuring r sistance.

§ 5. **Slide-form of Wheatstone's Bridge.**—The figure her given represents roughly the simplest and least expensive form o Wheatstone's bridge.

On a board are fixed three bands, A, B, and C, of stout copper the pieces A and C being elbow-shaped.

Between A and B, and B and C, are gaps; and here can be inserted, most conveniently by means of mercury cups, any resistances we please.

The other ends of the pieces A and C are joined by a thin *uniform* wire of high specific resistance; and parallel to this runs a scale that should have its initial and final points exactly opposite the points where the wire is soldered to the thick copper pieces. This scale is graduated to read both ways; and, on the supposition that the wire is of uniform resistance,



and that the scale corresponds properly to the beginning and end of the wire, we can read off the ratio of the two resistances s and s' into which any point such as B′ divides the wire.

Over the wire slides a spring key B′; by pressing the button of this key we make contact with the wire by means of a metallic edge or fine wire. The point where contact is thus made should be *exactly* opposite to the point on the scale at which the little index carried by the key is then pointing. The 'bridge' B G B′ connects the piece B and the key B′; the battery terminals are connected with A and C.

Referring to fig. i. of § 4, we see that we have complete correspondence; only the *points* A, B, and C of that figure have been replaced by the metal *pieces* A, B, and C, which are, from a resistance point of view, equivalent to points. Further, we see that

the point B′ is moveable, so that we can make the ratio $s : s'$ anything that we please.

In using the instrument we insert our unknown resistance at r, and a resistance box at r'. We then adjust the position of the key B′ until the making of contact with the wire at B′ gives no current through the galvanometer in the bridge.

When this is the case, B and B′ must be at the same potential; and we then have . . . .

$$r = \frac{s}{s'} . r'$$

which determines r.

Note.—There is often some want of exact correspondence between the ends of the scale and the ends of the wire. Such a defect produces less error in the result if we have s and s' not very unequal. We can contrive this by having a resistance box at r', and by removing the plugs (*i.e.* by throwing in resistance) until we get our zero deflexion of G when B′ is not far from the centre of the wire; in other words we make r' not very unequal to r.

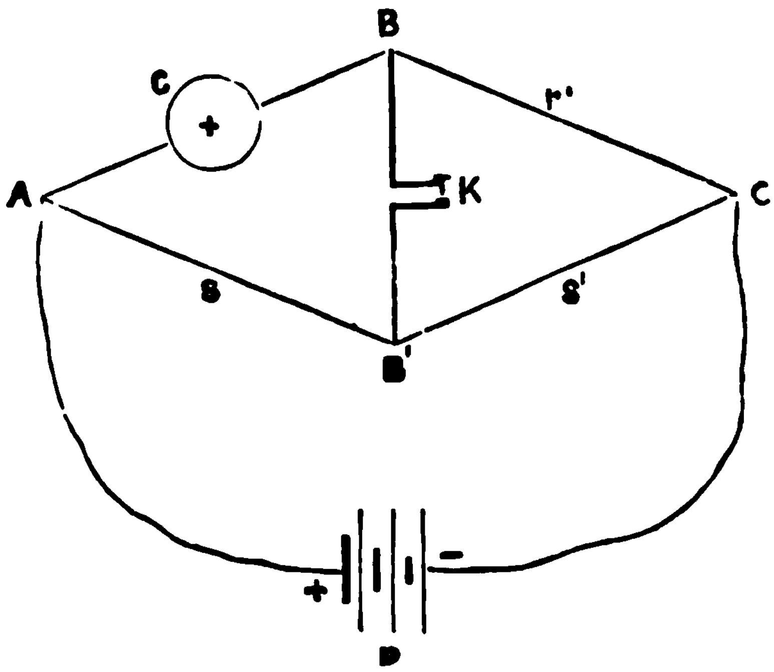
§ 6. **Wheatstone's Bridge; Resistance Box Form.**—The above described instrument is liable to error. The wire, by much use, loses its uniformity; so that equal lengths no longer correspond to equal resistances. Moreover, there is, even in the best instruments, some uncertainty as to the exact correspondence between the point of contact B′ and the point on the scale against which the index rests. (This second defect can, however, be eliminated.) In the instrument used for technical purposes B′ is fixed, and s and s' are resistance boxes containing coils of (*e.g.*) 1, 10, 100, and 1,000 ohms. For the known resistance r' we have another resistance box. We can thus, by means of the two boxes s and s', measure from $\frac{1}{1000}$th to 1,000 times any resistance in the box r'; this is simply a matter of removing the suitable plugs. We can thus with certainty measure to $\frac{1}{1000}$th of 1 ohm, which is better than measuring to smaller fractions with uncertainty.

§ 7. **Resistance of a Galvanometer.**—This can be measured as any other resistance; but a very simple method is as follows.

Sir Wm. Thomson's method.—Here we put the galvanometer G, whose resistance is required, in the place of one of the resistances r; and, in the place of the bridge with its galvanometer, we

have a key K, by means of which we can make or break connection between B and B′ at will.

The galvanometer G will have a deflexion due to the current through A B; the battery power and resistance in the external circuit A P C must be so adjusted as to give a reasonable deflexion of G.



We adjust the resistances s and s' until the making of contact with the key K produces no alteration in the deflexion of G. When this is the case, it can be proved that no current then flows through the bridge; and that as before, ...

$G = \frac{s}{s'} \cdot r'$, where G is the galvanometer resistance.

§ 8. **Resistance of a Battery-Cell.**—The measurement of the resistance of a battery-cell is not at all a simple matter.

For whereas in the above measurements we had only one constant unknown, viz. the constant resistance x that we desired to measure, in the case of the battery-cell we have two unknowns, one of which is variable from moment to moment; we have, in fact, the resistance x, and also the E.M.F. y, which, owing to varying degrees of polarisation, is not constant.

We here give two methods for measuring x.

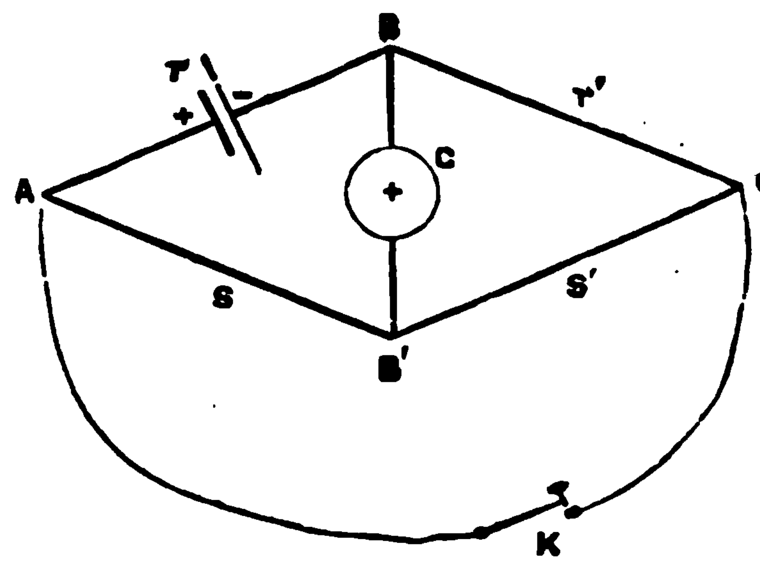
I. *The E.M.F. reduced to zero.*—If we have any even number $2m$ of similar cells, and if we set m against m, the total E.M.F. will be zero. When the current, employed in measuring the resistance, passes through these $2m$ cells, the polarisation produced should still be zero in total amount. With cells thus matched m against m, we can measure the resistance of the whole $2m$ cells as a 'dead' resistance; that of one cell is found by dividing by $2m$.

Against this method it may be stated that, as a rule, we require the resistance of the cell or battery as it is at any given time; and

we cannot generally provide another cell or battery, equal to this in E.M.F., to oppose to it.

We may call this method the '*Method of opposition.*'

II. *Mance's method.*—Let us arrange a Wheatstone's bridge as in § 5, but with the cell or battery, whose resistance is to be determined, in the place of the resistance r, and with a simple contact key in the place of the usual battery. The cell at r will drive a current; and no arrangement of resistances can prevent there being a current through the bridge and the galvanometer G. Hence, G will be deflected.



It can be shown that if the resistances be so adjusted that G maintains the same deflexion whether the key K be opened or closed, then we have the old relation . . . .

$$r = \frac{s}{s'} r'.$$

The proof is not very simple. By application of Kirchhoff's laws we obtain equations by means of which the unknowns can be determined. The current through G will be found to involve the external resistance of A K C in such a way that it is only independent of it when $\frac{r}{r'} = \frac{s}{s'}$. Whence it follows that, if this current is the same whether the key is open (giving an infinite resistance) or closed (giving a finite resistance), this relation must hold.

(For further discussion of this method see 'Phil. Mag.,' Series V., Vol. 3, 1877, Professor O. J. Lodge; and also in Glazebrook and Shaw's 'Practical Physics.')

§ 9. **Measurement of E.M.F.**—Let us take our standard cell, whose E.M.F. we suppose (for simplicity) to be just 1 *volt*, and let us measure the ΔV between its terminals when the circuit is

open (*see* Chapter X. § 34). Let us measure in the same absolute potential measure the ΔV between the terminals of any other cell or battery, the circuit being open. Using the ΔV of the standard cell as unit, we thus can express the ΔV of the latter in *volts*. We thus get a clear idea of what is the *initial* E.M.F. of any battery, expressed in *volts*. But when the circuit is closed, polarisation sets in ; and we find that the same cell has a different E.M.F. for each strength of current passing through it.

'The E.M.F. of a cell' is, therefore, a term whose meaning is undefined unless it be stated what current is running. As a rule, therefore, the *initial E.M.F.*, or the ΔV between the poles of the open cell, is given ; and it is then stated with more or less exactness to what extent this falls off with currents of various magnitude.

The E.M.F. when the current is *zero*, or the statical ΔV (expressed in *volts*) between the poles of the open circuit, is therefore what we record as 'the E.M.F. of a cell.'

To find the E.M.F. *with any given current* is a very difficult matter. For, if we try to determine the E.M.F. and the internal resistance of a battery by means of two equations derived from Ohm's law, we must use two different currents ; and in this case neither the E.M.F. nor the resistance of the battery will be the same in the two equations.

§ 10. **Electrometer Methods; Open Circuit.**—(i.) If we have an absolute quadrant electrometer we can measure the E.M.F. of the open cell by measuring the ΔV between its poles. Dividing this absolute value by that obtained for 1 *volt*, we express the E.M.F. of the open circuit in *volts*.

(ii.) If we have an ungraduated electrometer we may proceed as follows. We take m of our cells, and n standard cells, arranged end-on. We put one terminal (say the $-$ terminal) of each battery to earth ; and connect the $+$ terminals of each battery with one pair of quadrants respectively.

We arrange m and n until either we have no deflexion, or until with n standard cells we get the deflexion of the electrometer needle one way, and with $\overline{n+1}$ cells we get the deflexion the other way.

Then, if e be the E.M.F. of the cell required, and if e_0 be that of the standard cell, we have . . . .

her $$e = \frac{n}{m} e_0,$$

e lies between $\frac{n}{m} e_0$ and $\frac{n+1}{m} e_0$; thus obtaining a measure-

) within $\frac{1}{m}$th of e_0.

1. **Volt-meter Galvanometers.**—Let us suppose that we have nts A and B in a circuit; and that it is required to measure in *volts* between them, or the *E.M.F.* E_B^A. We have above ed the electrostatic method of doing this. A current l is also possible. For if we have a galvanometer G of a gh resistance as compared with the resistance between A the current that passes through G when its terminals are ted with A and B will be, by Ohm's law, directly propor- o E_B^A; and at the same time, owing to the relatively very sistance of G, the *Δ V* E_B^A will not be appreciably altered necting the points through G. Thus . . . .

If we connect the terminals of an open cell through G, we appreciably alter the E.M.F. of the open cell, the resistance eing relatively so great (*see* Chapter XIII. § 11 (*b*)).

If we connect G with two points in the circuit, we do not, oducing a branch circuit of so high a resistance, practically e Δ V between A and B.

nce, if we have a galvanometer of a high known resist- , and which *measures* currents accurately, we can from it the Δ V between the points A and B with which its ils are in contact, by means of the equation $C = \frac{E_B^A}{G}$; and l be practically the same ΔV as existed before we made ion with the galvanometer.

W. Thomson's *graded volt-meter galvanometer* is such an ient.

Chapter XVII. we shall give some detailed description of shall indicate how it can be used to measure both large all E.M.F.s. Here we shall say little more about it

m the formula $C = \frac{E^A}{G}$ the instrument can be experiment-

ally or theoretically graduated, so that we can read off at
volts the ΔV E_B^A between the terminals A and B of the
ment, answering to any particular current C.

§ 12. **Method of Opposition.**—Here, as in § 10, we ha
current; but we use a galvanometer instead of an electr

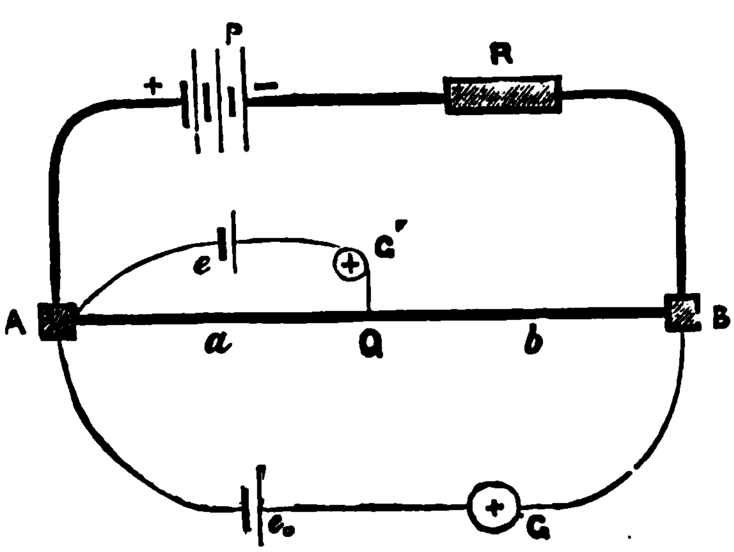
If m of the cells, whose E.M.F. we require, be oppos
standard cells, and if m and n be so adjusted as to give
zero-deflexion of a galvanometer in the circuit, then, as
we can measure to within $\frac{1}{m}$th of the E.M.F. of a single
cell.

§ 13. **Latimer-Clark's Potentiometer.**—In this instru
method of opposition is employed.

P is a battery; R is a Wheatstone's rheostat; A B is a
graduated wire of high resistance, fixed between two ter
and B; e_0 is a standard cell; e is a cell of a smaller E.
we desire to measure; G and G′ are delicate galvano
indicate when there are zero currents. Contact keys are r
sented.

Note.—As an example we may state that in a certain instrume
A B has a resistance of about 53 ohms. It is about 53 mètres lo
coiled in a screw-groove cut round an ebonite cylinder.

Now the total fall of V due to the battery P will be d
between the portions A R B and A Q B of the circuit, acc



their respective re
Hence, by manipu
we can arrange th
tween A and B unti
the E.M.F. e_0 of the
cell; this will be
by the fact that whe
then no current
through G, since
equal and opposite
are opposed. Sinc
no current through this cell, and so no polarisation, th
remains constant, and therefore E_B^A is also constant.

E.M.F. of P gradually alters by polarisation, we can so diminish the resistance of R as to maintain G undeflected, or E_B^A constant.

All the above may be regarded as a contrivance for keeping the two points A and B at a constant ΔV equal to the E.M.F. e_0 of the standard cell. And the uniform wire A B gives us by its graduations any fractional parts of e_0 that we desire; since, by Ohm's law, the ΔV, e_0, falls uniformly from A to B.

Now, if we arrange the cell e as shown, and if we get zero-deflexion of G′ when we make contact at Q, where A Q $= a$ and B Q $= b$, it follows that then $e = E_Q$, or that . . . .

$$e = \frac{a}{a+b} e_0,$$

since evidently $E_Q^A = \frac{AQ}{AB} \cdot E_B^A$.

Thus we can measure e accurately when less than e_0.

Note.—It may make matters clearer if we point out that the positive poles of the battery P, and of the cells e_0 and e, are all connected with the same end A of the wire A B; and their negative poles are all connected with the other end B.

If e be greater than e_0 we have merely to interchange the positions of e_0 and e, and the same experimental method gives us that

$$e = \frac{a+b}{a} e_0.$$

§ 14. **Table of Resistances in Ohms (or ω).**—We shall now give some information as to the actual resistances of various substances; and, in order that the statements may be scientifically exact, we shall explain certain terms that will be used.

Microhms.—A microhm is the $\frac{1}{1000000}$th of an ohm; or 1 *microhm* $= \omega \times 10^{-6}$.

Megohms.—A megohm = 1,000,000 times an ohm; or 1 *megohm* $= \omega \times 10^6$.

Specific resistance ρ.—The resistance offered by a conductor of 1 *centimètre* length and 1 *square centimètre* section to a current passing through it from one face to the opposite face is called the '*specific resistance* ρ' of the substance of which the conductor is made. We here use the word *conductor* to include all substances that do to any measurable degree conduct.

The results given are derived from the results collected Hospitalier's small book, and from Lupton's tables of consta (*see* below). We do not, in this Course, consider changes due temperature; nor do we consider the slight difference between *B A unit* and the *ohm* (*see* Chapter XIV. § 1 (ii.) note).

I. *Specific resistances of metals in microhms.*

Silver, annealed	1·521	($\omega \times 10^{-6}$).
Silver, hard drawn	1·609	,,
Copper, annealed	1·616	,,
Copper, hard drawn	1·642	,,
Gold, hard drawn	2·154	,,
Aluminium, annealed	2·946	,,
Zinc, pressed	5·690	,,
Platinum, annealed	9·158	,,
Iron, soft	9·827	,,
Tin, pressed	13·360	,,
Lead, pressed	19·847	,,
Mercury, liquid	96·146 to 99·740	,,
German silver	21·170	,,
Brass	5·800	,,

For further facts as to resistances, such as resistances of differ masses of wire of known gauge, we refer the reader to Hospitali 'Formulaire Pratique de l'Électricien' (Masson, Paris), and to Lupto 'Numerical Tables and Constants' (Macmillan).

II. *Specific resistances of liquids in ohms.*
(Taken from Lupton's book.)

Water at ·75° C.	$1·188 \times 10^{6}$
Water at 4° C.	$9·100 \times 10$
Water at 11° C.	$3·400 \times 10$
Dilute hydrogen sulphate at 18° C.; 5 per cent. acid	4·8
Dilute hydrogen sulphate at 18° C.; 30 per cent. acid	1·3
Hydrogen nitrate at 18° C., density 1·32	1·6
Saturated solution of copper sulphate at 10° C.	29·
Saturated solution of zincic sulphate at 14° C.	21·
Hydrogen chloride, 20 per cent. acid, at 18° C.	1·3
Ammonium chloride, 25 per cent. salt	2·5
Sodium chloride saturated, at 13° C.	5·3

The above show the very great difference that exists between water and solutions of salts, with respect to the conduction current.

III. *Specific resistances of bad conductors in megohms.*

Ice at $-12·4°$ C.	2240 ($\omega \times 10^6$).
Glass; (soda-lime, density 2·54) at 20° C.	$9·1 \times 10^7$,,
Glass; (crystal, density 2·94) below 0° C.	*practically infinite.*
Glass; (crystal, density 2·94) at 105° C.	$1·16 \times 10^7$ ($\omega \times 10^6$).
Shellac; at 20° C.	$9·00 \times 10^9$,,
Paraffin; at 46° C.	$3·4 \times 10^{10}$,,
Ebonite; at 46° C.	$2·8 \times 10^{10}$,,
Air; usual pressure	*practically infinite.*
True vacuum	*practically infinite.*

Very dense gases, it would seem, do conduct (or 'convect') to ›me extent.

In very rare gases, or 'imperfect vacua,' there appears to be con- ıction; but it seems certain that in this latter case, at all events, the ıscharge is disruptive, and that there is no true conduction.

§ 15. **Table of E.M.F.s in Volts.**—We here give some results s to the E.M.F.s of certain cells. The reader must, however, bear ı mind that the E.M.F. of a cell depends (i.) upon its general ıake; (ii.) upon details which are different in different cells pro- ɜssedly of the same E.M.F.; (iii.) upon the current. The ɪ.M.F.s here are therefore only approximately correct for cells in eneral. They are the *initial* E.M.F.s; and so we have some nomalous results, as, *e.g.*, that the E.M.F. of Volta's cell is greater ıan that of a Smee's—a result certainly not true when there is ıuch current running.

Volta's cell; Zn | *dilute* H_2SO_4 | Cu; about	1·00 volt.
Smee's; Zn | *dil.* H_2SO_4 | *platinised silver*; from	0·50 to 1·00 volt.
Daniell's; Zn | *dil.* H_2SO_4 | $CuSO_4$ *solution* | Cu; about	1·12 volt.
Grove; Zn | *dil.* H_2SO_4 | HNO_3 | Pt; nearly	2·00 volt.
Bunsen; Zn | *dil.* H_2SO_4 | HNO_3 | C;	1·75 to 1·94 volt.
Clark's standard cell; Hg | $Hg_2.SO_4$ | Zn; at 15°C.	1·438 ,,
Leclanché; Zn | NH_4Cl *solution* | $\left.\begin{matrix} MnO_2 \\ and\ C \end{matrix}\right\}$	1·42 ,,
De la Rue; Zn | NH_4Cl | $AgCl$ | Ag	1·04 ,,
Marie-Davy; Zn | *dil.* H_2SO_4 | $HgSO_4$ | C	1·52 ,,
Bichromate; *initially, solution new*	2·00 ,,
Planté secondary cell . . . about	1·80 to 2·50 ,,

These results are for the most part those given in Lupton's book ove referred to.

CHAPTER XV.

JOULE'S LAW, AND CONSERVATION OF ENERGY.

§ 1. **General Survey.**—We have already pointed out how (i.) all forms of energy may be measured in work-units (*i.e.* in *ergs*); . . . .

(ii.) energy is indestructible, or in any self-contained system we have transformation of energy from one form to another, but no loss or gain of energy.

In addition to these statements we may now add the law, which seems to apply to all *inanimate* nature at any rate, 'that all energy tends to run down to the form of uniformly diffused heat.' This law is called the law of '*Degradation of energy.*'

Now in the battery-cell our store of energy is in that form which is called *chemical-potential-energy.* In any particular cell the total energy at our disposal is proportional to the total mass of zinc (or other metal) to be dissolved. This energy also depends upon the other chemical changes that take place; and thus it is that in different cells, in which the rest of the chemical action is different, we have different amounts of energy answering to the same mass of zinc dissolved.

This energy may pass (apparently) direct into the form of heat energy, as when we simply dissolve unamalgamated zinc in acid.

Or we may obtain electrical energy as an intermediate form; and part of this electrical energy may be converted again into chemical-potential-energy if it be caused to decompose an electrolyte, or it may be transformed into mechanical energy if it be caused to work an electro-motor (*see* Chapter XXIV.), or it may be allowed to be entirely converted into heat-energy if it be left simply to heat the circuit.

But in every case we have simply the energy answering to the

e of, and amount of, the chemical changes that have taken in our sole source of energy, the cell or battery.

2. **Units of Heat, Work, and Activity.**

Unit of work.—In the absolute system this is the *erg*, dy discussed in Chapter X.

I. *Unit of heat.*—The calorimetric unit of heat is that ınt of heat which will raise one gramme of water from 0° C. to ; or we may with sufficient accuracy measure the number of imetric units of heat by the product of the (*number of grammes ıter*) into the (*number of degrees Centigrade through which they r fall in temperature*). This unit is called the *calorie.*

'he *calorie* is not a C.G.S. unit, as the Centigrade scale of erature is independent of the C.G.S. fundamental units. We , therefore, to connect this unit with the C.G.S. system by t experiment, measuring the heat, as we can measure all forms ıergy, in *ergs.*

Now it has been shown experimentally that . . . .

$$1 \textit{ calorie} = 4{\cdot}175 \times 10^7 \textit{ ergs} = 42 \times 10^6 \textit{ ergs} \text{ [nearly]},$$

at one gramme-degree unit of heat would, if totally used up oing mechanical work, do 41,750,000 ergs of work ; or the would raise 424 grammes through one mètre against gravity re g is taken as 981).

t was *Joule* who first established the definite measurement of in units of work. The factor which reduces *calories* to *ergs* be called J, so that $J = 4{\cdot}175 \times 10^7$. We may, for ordinary oses, take J to be equal to 42×10^6.

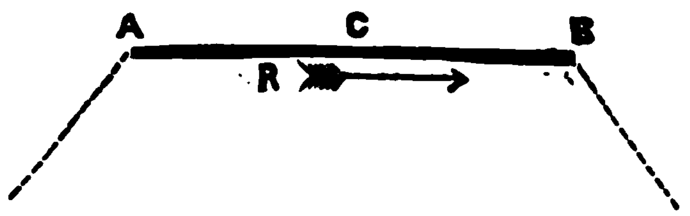
Note.—We use the *true calorie*, the *gramme-degree-Centigrade* unit. her calorie is sometimes used, viz. the kilogramme-degree-Centigrade ie ; but its use will probably soon be entirely discontinued.

III. *Unity of activity, or rate of work, or power.*—It is ı very important to consider the rate at which work is done, or *work per second.* It is convenient to give the name of '*activity*' *ıte of work.*

The C.G.S. unit of *activity* is the activity of *one erg per second.* English engineering practice the *horse-power* is much used ı convenient unit of activity when work is being done on a e scale.) All *energy*, then, can be measured in *ergs* or units of k, and all *energy per second* in *units of activity.*

§ 3. **Energy of the Electric Current.**—Consider a current C running steadily between the points A and B of a circuit, where A is at the potential V_A, and B is at some lower potential V_B.

Each unit of electricity at A is at a higher potential than when it comes to B ; in fact, there is a continual *discharge* from the constant potential V_A to the constant potential V_B. Each unit, therefore, loses potential energy to the amount represented by $V_A - V_B$ (*see* Chapter X. § 28) in its passage from A to B ; and, if the current be C, or if C units pass per second, there will be a loss of electrical-potential-energy between A and B to the amount of $C \times (V_A - V_B)$ per second. We may write this as $C \times E_B^A$. If C and V be measured in suitably chosen units, the above product will give us the electrical energy lost in *ergs per second*, or in absolute C.G.S. *units of activity*. Now the absolute electro-magnetic system of units, referred to in Chapter XIV. § 1, and explained in Chapter XVIII., is such that (*current*) × (*E.M.F.*) does give us this *activity* in absolute C.G.S. units. If, however, we use *ampères* to measure C, and *volts* to measure E M.F. or ΔV, then the product $C \times E$ gives us the activity in a new unit which we call a *watt.* Since 1 *ampère* = $\frac{1}{10}$th absolute unit, and 1 *volt* = 10^8 absolute unit, it follows that . . . .



$$1 \textit{ watt} = 10^7 \textit{ (ergs per second)}.$$

To repeat ; when a current of C *ampères* runs between two points whose difference of potential is measured by E *volts*, then electrical energy disappears between these points at the rate of. . . . .

$$C \times E \textit{ watts} \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(i)}$$
$$\text{or } C \times E \times 10^7 \textit{ ergs-per-second.}$$

We may add that

One English horse-power = 746 *watts* [nearly].

If there be no source of E.M.F. between the points A and B, then by Ohm's law we have $C = \frac{E_B^A}{R}$, where R is the resistance between A and B. In this case, which is the case where A B is a

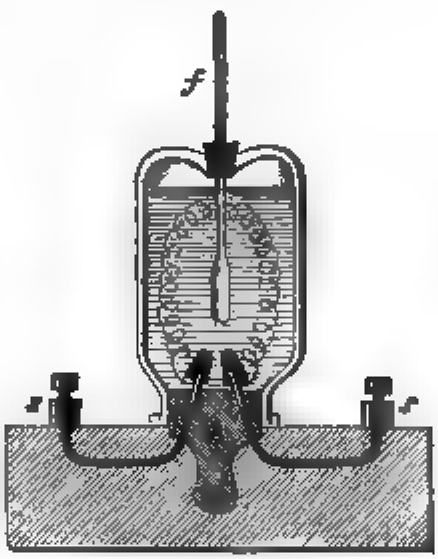
simple conductor, and includes no form of pile or electrolytic-cell or thermo-cell, we have that . . . .

$$Activity = C^2 R \; watts \quad . \; . \; . \; . \; . \; . \quad \text{(ii.)}$$

But if we simply know E^A_B as the ΔV between A and B, and if we do not know whether or no there be any source of E.M.F. between A and B, then we cannot write C R instead of E^A_B. For if there be such an E.M.F., *e*, between A and B, then by Kirchhoff's extension of Ohm's law, we have that C R does *not* equal E^A_B, but equals $(E^A_B \pm e)$. Hence the expression (i.) given above is of the more general application.

In the case of a battery of *E.M.F.* E, through which is passing a current C, the above considerations will show us that the rate at which chemical-potential-energy is used up in giving electrical energy must be represented by an activity of C E, since electricity to the amount of C per second is being raised through a ΔV represented by E. In fact, the rate at which chemical energy is used up in 'pumping up' electricity in the cell from the lower level to the higher (the difference of level being E *volts*) is represented by the *activity* of CE *watts*.

Inside the cell, then, we are using up chemical or other energy in raising electricity from a lower to a higher level, *i.e.* in gaining electrical energy; while *partly inside and partly outside the cell* this electrical energy is being again lost—giving us, as we shall see later on, other forms of energy.



§ 4. **Joule's Law.**—The same Joule who first clearly showed that heat was a form of energy, and could therefore be measured in work-units, was also the first to point out the exact connection between the electrical energy lost in any portion of a circuit and the heat evolved in that portion of the circuit.

He showed experimentally that if a current C be flowing through a conductor of resistance R, then the heat evolved per second is proportional to the product C^2 R;

and further investigation has shown the exact relation that the amount of heat evolved bears to the electrical energy lost in this portion of the circuit.

The nature of his experiments is indicated by the accompanying figure.

A wire of known resistance is immersed in a known mass of water, a current measured by a galvanometer is passed through the wire for a measured time, and the rise in temperature of the water is noted. We thus measure C, R, and the heat evolved. From a comparison of many experiments in which C and R were varied, the law above given was deduced.

Further experiment showed that the same law held with respect to the battery-cell itself. It is then true that in the whole circuit or in any part of it, the . . . .

(*Heat evolved per second*) is proportional to (*the product* $C^2 R$).

If we express our heat, current, and resistance, in suitable units, then this proportionality becomes equality, and we have. .

$$(\text{H } \textit{per second}) = C^2 R.$$

We shall now for convenience introduce a new symbol $\mathfrak{E}$ having a very simple meaning. Let $\mathfrak{E}$ be the algebraic sum of the E.M.F.s in any portion of the circuit considered; so that, by Ohm's law or by Kirchhoff's extension of it, we have the relation $C = \dfrac{\mathfrak{E}}{R}$; or $C R = \mathfrak{E}$. (Thus if we consider the portion of the circuit A B, and if no source of E.M.F. lie between A and B, we have $\mathfrak{E}$ the same as E_B^A or $(V_A - V_B)$; but if there be a direct E.M.F. e, or an opposed E.M.F. $-e$, between A and B, then $\mathfrak{E} = E_B^A \pm e$). We can then write $C\,\mathfrak{E}$ instead of $C^2 R$; and we have, when suitable units are employed . . . .

$$H\ (\textit{per second}) = C^2 R = C\,\mathfrak{E}.$$

If C, R, and $\mathfrak{E}$ be measured in *ampères*, *ohms*, and *volts*, then the product $C^2 R$ or $C\,\mathfrak{E}$ gives us the *heat-activity* in *watts*.

If *watts* and *calories per second* be compared, by referring both to *ergs per second*, it can be shown by the data given earlier that .

$$1 \textit{ watt} = \cdot 24 \textit{ calories per second} \text{ (approx.)}.$$

This relation may be verified directly by experiment.

We may then express Joule's law as follows. *If in any circuit or portion of a circuit C be the current in ampères, R the resistance in ohms, and 𝕰 be the algebraic sum of the E.M.F.s* (in such a sense that by Ohm's law $C\,R = 𝕰$) *measured in volts, then there will be evolved in that circuit or portion of circuit heat to the amount of* $C^2 R \times \cdot 24$, *or* $C\,𝕰 \times \cdot 24$, *calories per second.*

We have thus two formulæ for the heat evolved in calories per second; the formula $C^2\,R \times \cdot 24$ is always true; and $C\,𝕰 \times \cdot 24$ is true if 𝕰 be such that $C = \frac{𝕰}{R}$.

§ 5. **The Heating of Uniform Wires.**—The most important practical application of Joule's law is the application to the case of conducting wires.

We will use formula (i.) of the last section.

(I.) If a wire have resistance R, and there be flowing through it a current C, then there is evolved . . . .

$$C^2\,R \times \cdot 24 \textit{ calories per second.}$$

With a given current we can thus calculate the *calories per second* evolved if we know the length, section, and specific resistance ρ, of the wire. For, if ρ be given in *ohms*, or be reduced to *ohms*, we have that . . . .

$$R = \rho \,.\, \frac{l}{\text{cross section}} \textit{ ohms.}$$

If r be the radius of a wire of circular section (the most usual make of wire) then . . . .

$$R = \rho \,.\, \frac{l}{\pi\, r^2} \textit{ ohms.}$$

(II.) *The temperature to which a wire is raised.*—If the heat were not dissipated by convection, conduction, and radiation, no limit (saving that imposed by the fusion of the wire) could be fixed to the temperature to which a wire would rise. As a matter of fact, however, there soon obtains a state of equilibrium between the heat evolved per second and the heat dissipated per second. Hence the temperature will be, with more or less exactness, proportional to C^2 and to R; or is higher as the current is larger, and as the wire offers a greater resistance.

Experiment.—Put in the same circuit lengths of Pt and of Ag wire of the same radius. The platinum is the worse conductor of the two; and therefore

for it the resistance is greater, while the current is the same for both. It therefore is seen to attain a white-heat, while the silver remains much cooler; in the final condition, the greater dissipation due to the higher temperature compensates for the greater amount of heat evolved per second.

Note.—If in the same circuit we have (were it possible to find such) two wires of same radius, same conductivity, same surface power of dissipating heat, but of different masses and specific heats, then it is sometimes stated that the temperatures will depend upon the masses and on the specific heats, varying inversely as both. This, however, is not the case. That wire whose water-equivalent (or *mass* × *specific-heat*) is the least, will first attain the final temperature. But, when this is once attained, only the resistance and the dissipating-power come into the question; and, since by hypothesis this is the same for both, it follows that the final temperatures will be the same.

III. *Temperature as dependent on radius.*—Let us now consider wires of different radii, but of the same material, included in the same circuit, so that the same current necessarily passes through each. And let θ represent the number of degrees by which the temperature of the wire exceeds the so-called 'temperature of the air,' when a steady condition of temperature has been arrived at.

We have then, for each unit length of the wire . . . .

$$\left[\textit{Heat produced by current per second}\right] = \left[\textit{Heat lost by radiation, \&c., per second}\right]$$

If we assume—[but it is not at all accurate to do so]—that the resistance of the wire does not depend upon its temperature, we may under the conditions assumed say that . . . .

$$(\textit{Heat produced per second}) \propto \frac{1}{r^2}$$

where r is the radius of the wire.

Again, if we assume—[but at such high temperatures this is not accurate]—that the rate of loss of heat is simply proportional to θ and to the area of surface exposed, we have . . . .

$$(\textit{Heat lost per second}) \propto 2\pi r \,.\, \theta.$$

With these assumptions, then, we have that for wires of same material in the same circuit . . . .

$$(\textit{The rise in temperature}, \theta) \propto \frac{1}{(\text{radius})^3}.$$

§ 6. **Distribution of Heat in the Circuit.**—Let the current in a circuit be C; the resistances of battery and of various parts of the

ircuit be B, R_1, R_2, R_3, &c., respectively ; the total resistance e R ; the total heat evolved per second be H *calories* ; the several ortions evolved in the above different portions of the circuit be H_B, H_1, H_2, &c., respectively ; then we evidently have the following relations holding.

$$\begin{cases} H = C^2 R \times \cdot 24 = C^2 (B + R_1 + R_2 + \&c.) \times \cdot 24. \\ H_B = C^2 B \times \cdot 24. \\ H_1 = C^2 R_1 \times \cdot 24. \quad \&c. \end{cases}$$

Whence also . . . .

$$\begin{cases} H_B : H_1 : H_2 : \ldots = B : R_1 : R_2 : \ldots \\ H_B = \frac{B}{R} . H. \\ H_1 = \frac{R_1}{R} H. \quad \&c. \end{cases}$$

§ 7. **Heat Evolved with various Arrangements of n Cells.**—Let there be n cells, each of resistance B and *E.M.F.* E. Let R be he total resistance of the whole circuit including the battery. Let H be the total heat per second evolved ; and let H_i and H_e be he portions evolved internally to the battery, and in the external circuit, respectively.

(*a*) Let the cells be arranged m end-on and l in parallel ; and et the external resistance be r. Then we have $H = C^2 R \times \cdot 24$ calories.

We have also $C R = m E$ by Ohm's law ; and $R = r + \frac{m B}{l}$ *see* Chapter XIII. § 13, &c.). We can thus substitute for C and R in terms of known quantities ; and we have . . . .

$$\begin{cases} H = C^2 R \times \cdot 24 = \dfrac{m^2 E^2}{r + \dfrac{m B}{l}} \times \cdot 24 \textit{ calories-per-second.} \\[2ex] H_i = \dfrac{\frac{m B}{l}}{R} H = \dfrac{\frac{m B}{l}}{r + \frac{m B}{l}} H = \&c., \textit{ calories per-second.} \\[2ex] H_e = \dfrac{r}{R} H = \dfrac{r}{r + \frac{m B}{l}} H = \&c., \textit{ calories-per-second.} \end{cases}$$

(β) If the cells be so arranged that the internal resistance $\frac{m\,B}{l} = r$, then we have. . . . .

$$\begin{cases} H = C^2 R \times \cdot 24 = \frac{m^2 E^2}{2\,r} \times \cdot 24 \textit{ calories.} \\ H_i = H_e = \frac{r}{2\,r} H = \frac{1}{2} H. \end{cases}$$

§ 8. **Case of no Back-E.M.F. in the Circuit.**—Where there is in the circuit only the *E.M.F.* E of the battery, and no other E.M.F., then by Joule's law . . . .

$$\begin{cases} H = C^2 R = C\,E \times \cdot 24 \textit{ calories-per-second.} \\ \text{or } H = C\,E \textit{ watts.} \end{cases}$$

And by § 3 we know that the total electric activity is

$$W = C\,E \textit{ watts.}$$

In this case, therefore, all the electric activity runs down into the form of heat-per-second.

Experiment and theory, moreover, concur in establishing it to be a fact that in a cell where there is no local action, *i.e.* in which no chemical action occurs until the circuit is closed, all the chemical-potential-energy lost per second appears as electrical activity. Hence in this case the total chemical action gives *first* the equivalent electrical energy, and *then*, finally, the same amount of heat that would have been given had the chemical action taken place without the intermediency of a current.

§ 9. **Case of a Back-E.M.F. *e* in the Circuit.**—Now let us have in the circuit an electrolytic cell, or some other arrangement giving a reverse E.M.F. *e*; the total resistance being still R.

As argued in § 3, the battery is expending energy at the rate of $W = C \times E$ *watts*.

By Joule's law we have heat evolved at the rate of $H = C^2 R$ *watts*.

And since by Ohm's law $C = \frac{E - e}{R}$, this heat activity H may be written as $C\,(E - e)$ or $C\,E - C\,e$.

Hence of the activity C E expended by the battery, we have accounted for *part* in heat; but we have not yet accounted for the remainder $C\,e$.

'Conservation of energy' alone would, therefore, lead us to onclude that activity measured by $C e$ *watts* is being expended n the electrolytic cell. And, in complete accordance with this esult, we have the argument that in order to drive a current C gainst an E.M.F. e we must expend an activity of $C e$ *watts*.

In an electrolytic cell it is fairly evident that this *activity* can nly be expended in the storing up of chemical-potential-energy, ince no other work is being done.

Then we have expended by the battery an activity of $C^2 R$ or $C E$ *watts*; this amount of chemical-potential-energy being lost ach second in the chemical changes there taking place. Of this $C(E - e)$ *watts* reappear as heat evolved per second; while $C e$ *watts* are stored up each second in the electrolytic cell in the orm of chemical-potential-energy, chemical decomposition being here effected.

Case of charging a secondary battery.—In the charging of a econdary battery whose back-E.M.F. during charging is e, we tore up the activity $C e$, and waste in heat the activity $C(E - e)$. Hence it is more economical to charge the cell with a battery whose *E.M.F.* E is only a little greater than e; for then we store p nearly the whole activity. But if E nearly equals e, the current C will be small, and the process of charging will be a long one. As a rule we compromise matters, and do waste a good deal of ctivity in heat in order to get the cell charged within a reasonble time.

§ 10. Numerical Examples.

(i.) A battery has E.M.F. = 50 *volts*; the total resistance is 20 *ohms*. Find the current in *ampères*, and the activity (or work per econd) in *ergs per second*, in *watts*, and in *horse-power*.

Here,

$$\begin{cases} C = \dfrac{E}{R} = \dfrac{50}{20} = 2\dfrac{1}{2} \textit{ ampères.} \\ \text{Activity} = C E = \dfrac{5}{2} \times 50 = 125 \textit{ watts.} \\ \text{Activity} = C E \times 10^7 = 125 \times 10^7 \textit{ ergs per second.} \\ \text{Activity} = C E \div 746 = \dfrac{125}{746} \textit{ horse-power.} \end{cases}$$

(ii.) In the same case the external resistance R_e is 15 *ohms*, and the internal resistance R_i is 5 ohms. Find the external and internal activities both in *watts* and *calories per second*.

Here,

$$\begin{cases} \textit{External activity} = \frac{R_e}{R} \times 125 = \frac{3}{4} \times 125 \textit{ watts.} \\ \textit{External activity} = \frac{3}{4} \times 125 \times \cdot 24 \textit{ calories-per-second.} \\ \textit{Internal activity} = \frac{R_i}{R} \times 125 = \frac{1}{4} \times 125 \textit{ watts.} \\ \textit{Internal activity} = \frac{1}{4} \times 125 \times \cdot 24 \textit{ calories-per-second.} \end{cases}$$

(iii.) A battery has E.M.F. 20 *volts* and current 10 *ampères*. Find the total heat per second in *calories*.

Here $H = C \times E \times \cdot 24 = 10 \times 20 \times \cdot 24$ *calories-per-second.*

(iv.) A battery has E.M.F. = 10 *volts*; current is observed to be 6 *ampères*; and in the circuit are a set of electrolytic cells of total back-E.M.F. equal to 4 *volts*.

What is the chemical activity (or energy-per-second) stored up in the electrolytic cells? And how many calories per second are given out?

Here,

$$\begin{cases} \textit{Chemical activity stored up} = C\,e = 4 \times 6 = 24 \textit{ watts.} \\ \textit{Heat activity} = C\,(E - e) = 6 \times 6 = 36 \textit{ watts.} \end{cases}$$

§ 11. **Failure of a Smee's Cell to Decompose Water.**—If the back-E.M.F. e of an electrolytic cell would be greater than the *E.M.F.* E of the battery, then such a battery will fail to drive a current through, and decompose, such a cell.

For if the action be supposed to take place, we get . . . .

(i.) $C = \frac{E - e}{R}$ = a negative quantity; which would mean that the electrolytic cell would be driving a reverse current through the battery-cell; and that the latter would not be decomposing the former.

(ii.) Again, we should have the activity C E expended by the battery *less than* the activity C e expended on the electrolytic cell, which would be a bteach of conservation of energy.

It is for this reason that a Smee's cell cannot decompose water.

If $E = e$, then we have a stand-still; and this will be the case whatever the relative *size* of battery-cell and of electrolytic cell. The current is $C = \frac{E - e}{R} = 0$ (by hypothesis).

§ 12. **Partial Polarisation in the foregoing Case.**—In such a case, however, we get a partial polarisation. Referring to Grothüss's hypothesis (Chapter XII. § 4), we may add that we do not get the full back-E.M.F. e until the chain of interchanges of *ions* is completely established. At first, no doubt, some $\pm$ *ions* will during their temporary disengagement from their $\mp$ partners attach themselves to the kathode and anode respectively; without, however, a complete interchange of partners occurring throughout the entire chain between the electrodes. As this action occurs to a greater and greater degree, so will the back-E.M.F. rise, until (in the case of the last section) there is finally a balance and a stand-still.

§ 13. **Connection between E.M.F.s and 'Heats of Combination.'**

(I.) The passage of a unit quantity of electricity through the battery cell is accompanied by a definite amount of chemical action. Thus, in the case of a voltaic cell in which *platinum* and *zinc* are immersed in *dilute sulphuric acid*, the passage of a unit of electricity is accompanied by the solution of an electro-chemical equivalent of zinc, and the setting free of an electro-chemical equivalent of hydrogen. In all cases, if the whole action be confined to the cell and no current be sent through an external circuit, each such chemical action fixed in nature and in amount is accompanied by the evolution of a fixed quantity of heat. This heat is the full measure of the chemical potential energy lost by the cell, and its determination for each particular case is a matter of experiment.

Let us denote it, in the general case, by H *calories.* Then we have that . . . .

In the passage of one unit of electricity through the circuit the cell loses chemical potential energy whose measure in heat is H calories.

(II.) Now $E \times 1$, or E, measures the electrical work done by the battery upon unit quantity of electricity passing through it; or we must write $(E \times \cdot 24)$ if we express this work in calories. (See p. 241, second paragraph; and p. 242, last two lines.)

Hence . . . .

In the passage of one unit of electricity through the circuit the cell does electrical work measured by $E \times \cdot 24$ calories.

(III.) Now, experiments tend to show that in cells devoid of local action the former energy equals the latter; or that . . . .

$$E \times \cdot 24 = H$$
$$E = \frac{H}{\cdot 24}.$$

This theory can be and has been tested by comparing the values of E calculated in this way from an experimental knowledge of H, with values measured by electrical methods. So far, then, as experiment has been able to go, there appears to be a very intimate connection between intensity and quantity of chemical action on the one hand, and E.M.F. and quantity of electrical movement on the other, respectively.

(IV.) *Different E.M.F.s of different cells.*—The above reasoning and results throw much light on the differences in E.M.F. that exist between various cells in all of which zinc is dissolved in dilute sulphuric acid.

The difference lies in the different methods of getting rid of the hydrogen. In every case this involves an absorption of energy.

In Smee's cell the hydrogen is liberated in the free state from the platinised silver plate. The separation of this hydrogen from the sulphuric acid absorbs a considerable part of the energy supplied by the solution of the zinc. The energy of the current and, consequently, the E.M.F. are comparatively small.

In Daniell's cell the hydrogen, instead of being liberated in the free state, is allowed to decompose sulphate of copper in solution. This decomposition is accompanied by evolution of energy which restores in part that absorbed by the liberation of hydrogen from the acid. Here, therefore, the current energy and E.M.F. are higher.

In Grove's cell the hydrogen decomposes nitric acid, yielding thereby nearly as much energy as its original liberation had absorbed. Here, therefore, the energy supplied is nearly that yielded by the solution of the zinc, and the E.M.F. is very high.

Note.—It may naturally occur to the reader to ask why in the table of [Cha]pter XIV. § 15 we are given different values for the E.M.F.s of a Volta's and of a Smee's cell, although in both the hydrogen is simply set free.

We may say that the E.M.F.s as calculated chemically correspond to the [E.M.]F.s of the batteries when a current is running and when there is neverthe[less] no unknown polarisation.

It is, therefore, almost hopeless to expect agreement between the values of [E.M.]F. as calculated chemically, and as measured with open circuit, respec[tivel]y, when we are dealing with 'non-constant' cells; for we do not quite [kno]w our data for calculation. With 'constant' cells the agreement is [bett]er.

There is, however, little doubt but that there would be exact agreement [if w]e knew the exact chemical action in the cell at the time when its E.M.F. [is] electrically measured.

(V.) The expression for the E.M.F. given in (III.) shows us from [ano]ther point of view why a Smee's cell cannot decompose water. [Sin]ce in this case the back-E.M F. e is greater than the *E.M.F.* E [of] the Smee's cell, it follows that the heat per second used up in [dec]omposing the water would be greater than the heat per second [tha]t could be supplied by the Smee's cell.

CHAPTER XVI.

THERMO-ELECTRICITY.

§ 1. **Introductory.**—In the cells with which we have up to the present point been concerned, we have had certain arrangements by means of which chemical-potential-energy could be converted into electrical energy; the materials of the cell undergoing permanent chemical change. Any such arrangement (generally of two metals and one or two liquids) was called a *voltaic cell.*

If we passed a current through such a cell we either lost *chemical-potential-energy* and gained *heat energy*, or we gained *chemical-potential-energy* and used up *electrical energy* that would otherwise have appeared in the form of *heat energy*; according to the direction in which we drove the current. (We refer to the 'Ce' of Chapter XV. § 9.)

In the present Chapter we shall see that it is very easy to make certain arrangements (consisting simply of metals, without any liquid or other non-metallic body) by means of which *heat energy* may be converted direct into *electrical energy*; the metals employed undergoing no chemical change. Such an arrangement, which gives us an electric current by the application of heat only, is called a *thermo-cell.*

We shall see further that if we pass a current through a thermo-cell we have certain effects (called *Peltier* and *Thomson* effects) consisting in disengagement or absorption of heat. These effects are entirely distinct from the *Joule* effect of Chapter XV. § 4, and answer to the chemical work Ce that is done when we pass a current through an *electrolytic-cell* (*see* Chapter XV. § 9).

In what follows we shall discuss in an elementary manner the subject of thermo-cells, taking matters in the following order.

(I.) We shall first give the main facts, established by experi-

ment, as to the construction of a thermo-cell, and the dependence of the E.M.F. on the nature of the metals employed and on the temperatures at which the different parts of the cell are maintained (§§ 2 to 7).

(II.) We shall give some account of the facts with respect to the *Peltier* and *Thomson* effects referred to above (§§ 8 and 9).

(III.) We shall say something with respect to the theory of the subject (§§ 10 and 11). But, as is usual in elementary books, we shall not deal at all fully with the theoretical part of the subject.

§ 2. **The Simple Thermo-Cell.**—If a circuit be made of two metals A and B, having soldered junctions [one of which we denote by A|B, the other by B|A], we have seen that the two metals are as a rule at different potentials, but that there is no current (*see* Chapter XI. § 3); it being assumed as a condition that the whole circuit is at one temperature.

If, however, the two junctions are at different temperatures, we find in general that a current flows through the circuit, or that

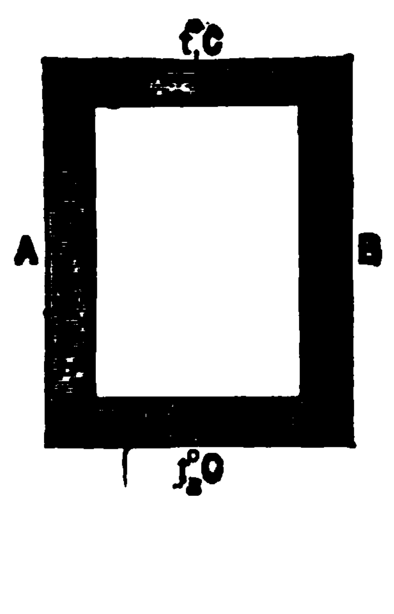


FIG. i.

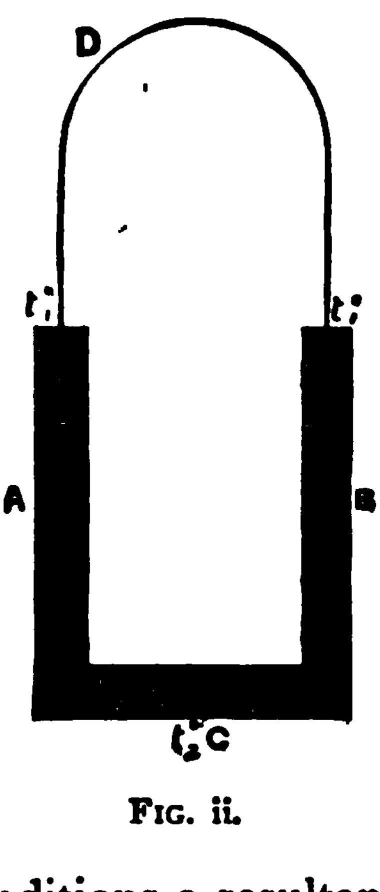


FIG. ii.

there is in the circuit under these conditions a resultant E.M.F. (The reader is requested to notice the simple manner in which we denote the junctions across which we pass from A to B, and from B to A, respectively.)

The figures here given indicate two equivalent forms of the simple thermo-cell. We may either have the two metals A and B only, with A|B at t_1° C, and B|A at t_2° C.; or we may connect A

and B by a third metal D, having both the junctions A|D and D|B at t_1° C. It can be proved experimentally, and it follows from Chapter XI. § 3, that the two forms are exactly equivalent.

§ 3. **The Thermo-Pile.**—The E.M.F.s of thermo-cells are very small, while their internal resistance, as they consist entirely of metallic bars or thick wires, is very small. Hence it is usual to couple up many simple cells in series. If n such cells be coupled up we have an E.M.F. that is n times the E.M.F. of one cell, while the internal resistance is still small.

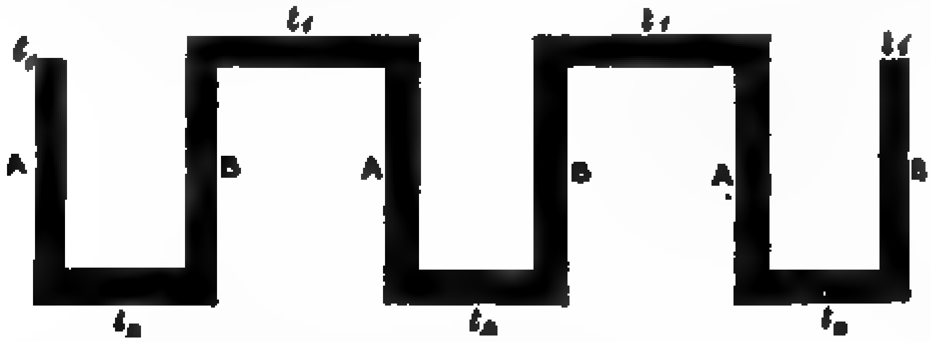


Fig. i.

The figure here given, fig. i., indicates how several cells may be joined in series. It is evident that if we have an E.M.F. in case (ii.) of the last section, then we shall in the present case have the several E.M.F.s acting in the same direction.

In the next figures, figs. ii. and iii., it is shown how such cells may be packed into a very small space. *Nobili* made his cells of thick Bi and Sb wire; and, by folding the cells into layers with insulating material between, he packed a large number of cells into the form of a cube. The opposite faces of this cube consisted of the alternate sets of junctions; so that if the two faces were exposed to different temperatures t_1 and t_2, then the alternate junctions were heated to t_1 and t_2 respectively, as in fig. i. above.

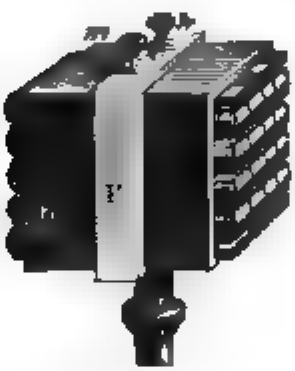


Fig. ii.

Fig. iii.

§ 4. **Thermo-Electric Series.**—Referring to the simple cell of § 2, it may be stated that the E.M.F. depends on three conditions. They are . . . .

(*a*) The nature of the metals.
(*b*) The difference of temperature of the two junctions.
(*c*) The absolute temperature of the junctions.

Thus we have (*a*) a different E.M.F. if we make our cell (*cæteris paribus*) of Bi and Fe from what we have if we make it of Bi and Sb. Again, we have in general (*b*) a different E.M.F. if the difference of temperatures of the junctions be 20° C., from what we have if (*cæteris paribus*) it be 50° C. And, finally, we have in general (*c*) a different E.M.F. if the junctions be at, *e.g.*, 20° C. and 22° C. from what we have if (*cæteris paribus*) they be at 50° C. and 52° C.

Thermo-electric series at 20° *C.*—Hence we cannot give a list of metals showing their thermo-electric relation to one another for all temperatures, but only for some stated temperature.

In the following are given in *micro-volts* the E.M.F.s of various cells, one metal in each case being *lead*, and the other the metal opposite to which the number stands. The junctions in each case are at 20½° C. and 19½° C. respectively; or there is a temperature-difference of 1° C., while the mean temperature is 20° C.

For the metals that occur higher in the list than lead, the current passes from that metal through the hotter junction to lead; for those lower in the list than *lead*, the current passes from *lead* to that metal through the hotter junction. This is indicated by the *sign* of the E.M.F. Experiment shows also that the E.M.F. of a cell composed of any two metals is simply the algebraic difference of their respective E.M.F.s with reference to *lead*.

(The list is taken from Lupton's Numerical Tables.)

E.M.F.s, in micro-volts, of cells whose junctions are at 20½° C. and 19½° C. respectively; *lead* being one of the metals in each case.

Bismuth, pressed; comm.	+ 97·0	Antimony, pressed	− 2·8
Bismuth, pressed; pure	89·0	Silver, pure; hard	− 3·0
Cobalt	22·0	Zinc, pressed; pure	− 3·7
German silver	11·75	Copper, electrolytic	− 3·8
Mercury	·418	Antimony, pressed; commercial	− 6·0
Lead	0·0	Iron wire, soft	− 17·5
Tin	− ·1	Antimony crystal. axial	− 22·6
Copper, comm.	− ·1	Antimony crystal. equatorial	− 26·4
Platinum	− ·9	Selenium	− 807·0
Gold	− 1·2		

Explanation of table.—The following two or three cases may explain further what the table means.

(i.) If we make a cell of *cobalt* and *lead*, the junctions being at $20\frac{1}{2}°$ C. and $19\frac{1}{2}°$ C., the E.M.F. will be 22×10^{-6} volts; and the current will pass from *cobalt* to *lead* through the hotter junction.

(ii.) With *lead* and *silver* the E.M.F. is 3×10^{-6} volts; and the current passes from *lead* to *silver* through the hotter junction.

(iii.) With pure *bismuth* and *cobalt* the E.M.F. is $(89 - 22) \times 10^{-6} = 67 \times 10^{-6}$ volts; and the current passes from former to latter through the hotter junction.

(iv.) With *cobalt* and pure *silver* the E.M.F. is $[22 - (-3)] \times 10^{-6} = 25 \times 10^{-6}$ volts; and the current passes from former to latter through the hotter junction.

Such a series is called a '*thermo-electric series* at the temperature (*i.e.* 20° C.) in question.'

§ 5. **Thermo-Electric Powers.**—The effectiveness of any particular cell (*i.e.* a cell composed of any two specified metals), at any specified mean temperature $t°$ C., may be reasonably measured by the E.M.F. of the cell when the junctions have a temperature difference of 1°, the mean temperature being $t°$ C.

To express this 'effectiveness' we use the term *thermo-electric power.* The meaning of this term may be seen from the following statement.

The thermo electric power of any pair of metals at $t°$ C. is measured by the E.M.F. of a cell composed of these metals when there is a temperature difference of 1° C. between the junctions, the mean temperature being $t°$ C. The E.M.F.s are usually expressed in *micro-volts.*

The more exact meaning of *thermo-electric power* may perhaps be further explained with some advantage. It is found experimentally that if the junctions are at any two temperatures $t_2°$ and $t_1°$ of which the mean is $t°$, and if $E_{t_1}^{t_2}$ be the E.M.F. of the cell under these conditions, then . . . .

$$\textit{The thermo-electric power at } t° \ C. = \frac{E_{t_1}^{t_2}}{t_2 - t_1}$$

$$\textit{where} \quad t = \frac{t_1 + t_2}{2}.$$

Example.—Thus, if 30° C. were taken as $t°$ C., we have the thermo-electric ›wer at 30° C. given by any such fraction as $\frac{E_{29°}^{31°}}{2}$, $\frac{E_{20°}^{40°}}{20}$, $\frac{E_{10°}^{50°}}{40}$, &c.

Or we may say that if we know the *thermo-electric power at C.* in micro-volts, then we get the E.M.F. in micro-volts when ıe junctions are at temperatures $(t + \theta)°$ C. and $(t - \theta)°$ C. by ıultiplying the thermo-electric power at $t°$ C. by the temperature ifference $2\,\theta$.

Note.—If de be the infinitesimal E.M.F. due to the infinitesimal difference θ in temperature of the junctions, we may in infinitesimal notation say that .

$$\textit{The thermo-electric power at } t° \text{ C.} = \left[\frac{de}{d\theta}\right]_{\theta = t}.$$

If the thermo-electric power of a single metal be spoken of, it s understood that *lead* is the other metal.

A metal A is said to have a greater thermo electric power than ı metal B at $t°$ C. when it stands higher in the list for $t°$ C.; in his case the current passes from A to B through the hotter unction.

§ 6. **The Neutral Point.**—It is found experimentally that for :very pair of metals there is a particular temperature possessing roperties to be described in the present section. This tempera-ure is, for reasons to be given later on, called '*the neutral point*' or that pair of metals, and is usually designated by the letter T. This temperature, T° C., is different for each pair of metals. Sup-osing that we are dealing with two metals A and B, whose neutral oint is T, the facts observed are as follows.

(I.) If one junction, say A | B, be at T while the other junc-ion be at some higher temperature t_1, there will be a certain E.M.F. which we may call $E_T^{t_1}$, and the current will pass from one ıetal to the other (say from A to B) through the hotter junction, *e.* through that at t_1. We should for these temperatures say that ı is above B in the thermo-electric series. The higher t_1 be-omes, the greater is the E.M.F.

(II.) If the temperature of the other junction be lowered from to some lower temperature t_2, step by step, the E.M.F. (which re may now call $E_{t_2}^{t_1}$) decreases step by step ; and when t_1

are equidistant from T, or when $t_1 - T = T - t_2$, the E.M.F. is zero.

(III.) If the temperature of the junction that was at T be lowered still further, so that $T - t_2$ is greater than $t_1 - T$, an E.M.F. acting in a reverse direction to the former is observed. This increases step by step as the temperature t_2 is still further lowered step by step.

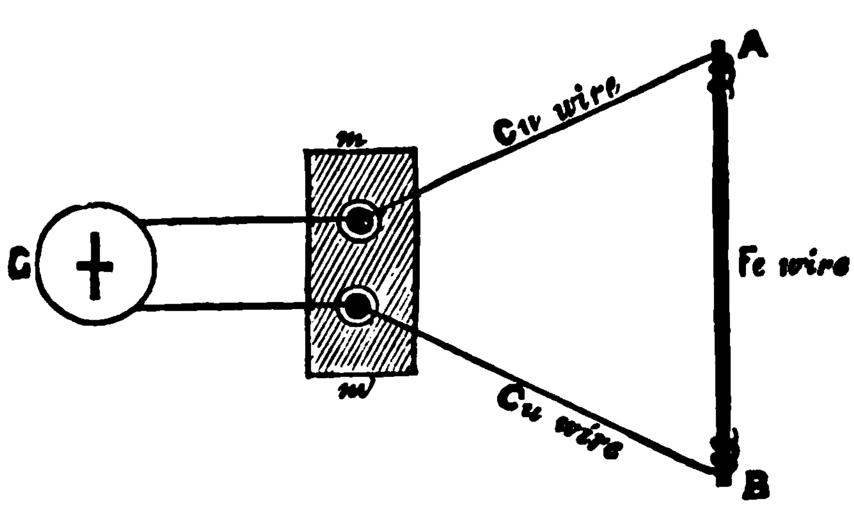
(IV.) If now the upper temperature t_1 be brought down to T, this reverse E.M.F. is still greater.

We find, in fact, that the current flows from A to B, or from B to A, across the hotter junction, according as $t_1 - T$ is greater or less than $T - t_2$ respectively; or that A and B change their thermo-electric order when the *mean temperature* $\frac{t_1 + t_2}{2}$ of the junctions crosses the neutral temperature T.

(V.) As we should perhaps have anticipated from the fact that when $t_1 - T = T - t_2$ we get no E.M.F., we find that, *e.g.*, if one junction be 20° above the neutral point T while the other is 10° below it, we have the same E.M.F. as if the former were 20° above T, and the latter were 10° above T. It is usually said that A and B have *zero* thermo-electric power at T; and that above T and below T their thermo-electric powers are of opposite sign respectively.

Note.—In more advanced treatises it will be found that certain pairs of metals have more than one neutral point. But such cases will not be considered here.

Experiment.—The following experiment will illustrate the above statement. Round the ends of a piece of *iron* wire are twisted two pieces of *copper* wire. The ends of these latter are connected, by means of mercury cups *m m′*, with a galvanometer G. This latter must be delicate, and, on account of the higher resistance due to the bad contact of the Cu and Fe, it may be of higher resistance than the form commonly used with a simple thermo-cell. The end A is left at the temperature of the air, say about 20° C., while the end B is heated in a Bunsen's flame. For iron and copper the neutral point T is about 260° C.



rises in temperature an increasing current is observed; its direction own to be from Cu to Fe across the hotter junction. This current until B is at 260°. It then decreases, and becomes zero when B is at *i.e.* is zero when $t_1 - T = T - t_2$. On raising B above 500° C. a current erse direction is observed; and this increases as the temperature rises er.

clear, from what has been said in this section and in § 4, metals A and B have *zero* thermo-electric power at their point T.

Thermo-Diagrams.—From the experimental and theoreti- stigations of Professor Tait, Sir W. Thomson, and others, nd that the thermo-relations of most metals for all temper- at least for a wide range of temperature) can be simply d in one diagram. We shall give and explain this graphic ; the reader understanding that the construction of the follows from *rimental* results n §§ 5 and 6. O v is the axis linates, along measurements at *micro-volts*; ° is the axis of along which ments repre- *rees Centigrade.* gram is drawn e suitably, so lengths of or- and abscissæ *ro-volts* and de- spectively; but inates and ab- need not be d in the same length respec-

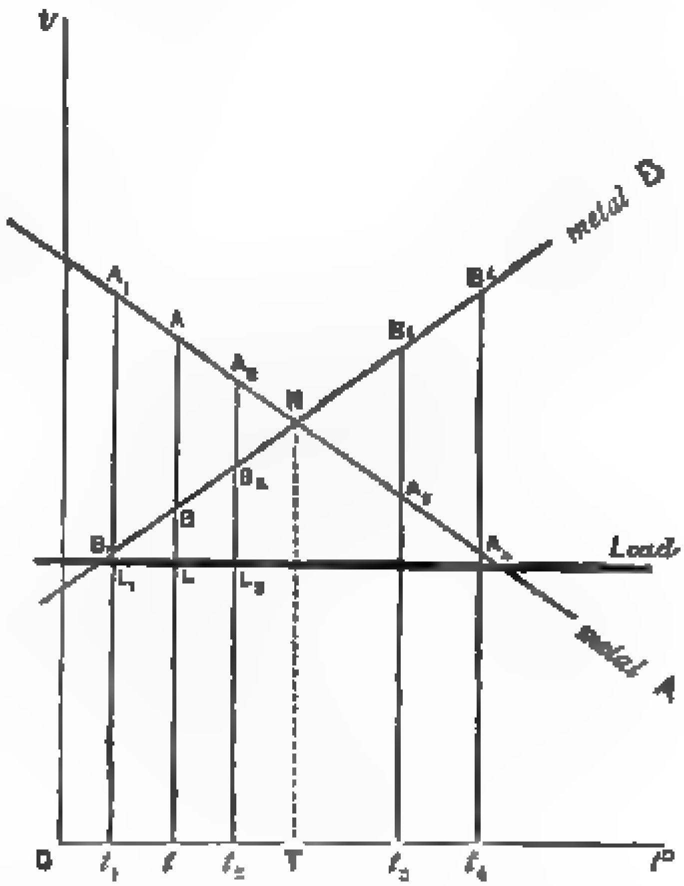


FIG. 1.

The three straight lines given represent the metals *lead*, ', and *metal B*, respectively; in what sense they 'repre- ese metals will soon be seen.

us erect *ordinates from* the points t_2, t, and t_1, on the line

O t, answering to the temperatures $t_2°$, $t°$, and $t_1°$ C., respectively; and let A L and B L measure the thermo-electric powers of *metal A*, and *metal B*, respectively, with respect to *lead* at $t°$ C., so that also A B represents the thermo-electric power of A with respect to B at $t°$ C. Let t_1 and t_2 be equidistant from t, so that $t_2 - t = t - t_1$.

Then by simple geometry the area of the trapezium $A_1 A_2 L_2 L_1$ is given by the product $A L \times L_1 L_2$; that is, by the product of the thermo-electric power at $t°$ C., measured by A L, into the difference of temperature $t_2 - t_1$ of the junctions, measured by $L_1 L_2$; where $t = \frac{t_1 + t_2}{2}$.

But by the result of experiment as given in § 5, this product gives us the *E.M.F.* $E_{t_1}^{t_2}$ of a cell composed of the *metal A* and *lead*, in which the junctions are at t_2 and t_1.

Hence we have that if the *line* A L measure the thermo-electric power of *metal A* with respect to *lead* at $t°$ C., then the *area* $A_1 A_2 L_2 L_1$ gives the *E.M.F.* $E_{t_1}^{t_2}$ of the cell for temperatures t_1 and t_2 of the junctions; where t is the mean of t_1 and t_2.

By experiment it is shown that this is true for all temperatures, *i.e.* wherever t be taken.

From mathematical reasoning it follows that if the line representing *lead* be straight, then the line representing the *metal A* must also be straight. This line can therefore be determined by finding two points; or by determining the thermo-electric power of *metal A* at two temperatures.

The same reasoning applies to *metal B*; so that the *area* $B_1 B_2 L_2 L_1$ measures the E.M.F. of a cell of *metal B* and *lead* at temperatures t_1 and t_2.

The area $A_1 A_2 B_2 B_1$ is the difference of these areas; and, as experiment shows, measures the E.M.F. of a cell composed of *metal A* and *metal B* at temperatures t_1 and t_2. If, therefore, we draw a straight line for *lead*, and determine straight lines for other metals (by determining their thermo-electric powers with respect to *lead* for two temperatures), the diagram so formed gives us measurements of the E.M.F. of any cell at any two temperatures of junctions, by simple measurement of areas. Such a diagram, then, expresses the results of experiment given in § 5.

Now we will show more clearly how the properties of the *neutral point* are exhibited in the diagram.

Suppose that the lines of the metals A and B cross at a point N lying in the ordinate from the temperature T on the axis of abscissæ.

We noticed that A B measures the thermo-electric power of A with respect to B at $t°$ C. Then at T° C. this thermo-electric power becomes *zero*, since the intercept A B is here a *point*. Hence T° C. must be the neutral point of the two metals.

On the other side of this neutral point the thermo-electric powers and E.M.F.s are reversed in sign, but are otherwise measured as shown above.

If, therefore, we reckon any area such as $N\,B_3\,A_3$ as *negative* in sign, it is clear that the *algebraic sum* of the areas $A_1\,N\,B_1$ and $N\,B_3\,A_3$, or the total area (having regard to *sign*) between the lines of the metals and the two ordinates from t_1 and t_3 respectively, will agree with the experimental results of § 6 in measuring the E.M.F. of the cell when the junctions are at $t_1°$ and $t_3°$ respectively. When $t_1\,T = T\,t_4$ the areas are equal and their algebraic sum is zero. If t_4 were still further on, so that $T\,t_4$ were greater than $t_1\,T$, the sum of the areas would change sign; indicating the change in sign of the E.M.F. of the cell.

Thus the diagram conveniently embodies the experimental results referred to in §§ 5 and 6.

On the next page we give a diagram in which the lines of many metals are given. After the above discussion it needs no comment.

Tait's formula.—The area of any trapezium, such as $A_1\,A_2\,B_2\,B_1$, is given by the product of A B into $L_1\,L_2$, or by the breadth of the trapezium into the mean between its parallel sides. But, from the triangle $N\,A_1\,B_1$, it is evident that A B is proportional to $T - t$; *i.e.* to $T - \frac{t_1 + t_2}{2}$. And the length $L_1\,L_2$ measures $t_2 - t_1$. Hence, the area $A_1\,A_2\,B_2\,B_1$ is proportional to the product $(t_2 - t_1)\left(T - \frac{t_1 + t_2}{2}\right)$; or we may say that, for the two metals in question, ...

$$E_{t_1}^{t_2} = k\,(t_2 - t_1)\left(T - \frac{t_1 + t_2}{2}\right),$$

where k is a constant depending on the units of measurement

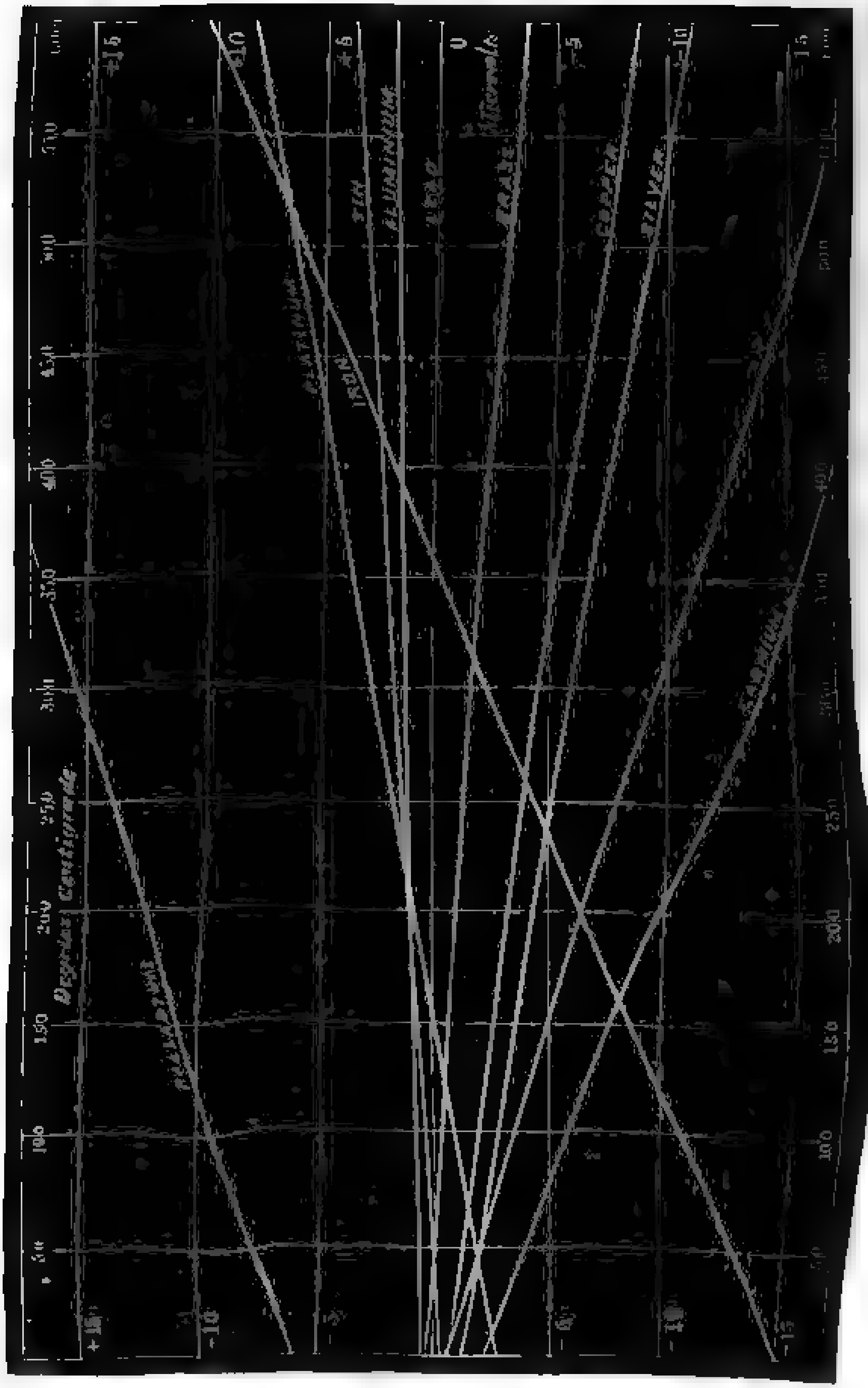

Degrees Centigrade
PALLADIUM
PLATINUM
IRON
TIN
ALUMINIUM
LEAD
BRASS
COPPER
SILVER
CADMIUM

Fig. 11.

employed and upon the situation of the lines of the metals, that is upon the metals. When k is once found for any pair of metals, the formula can then be used for all changes made in t_1 and t_2 for those two metals.

The student can, though not quite so easily, show that the above formula represents the result arrived at graphically when the two temperatures are on the opposite sides of the neutral point; attention being paid to the fact that in this case the areas in the diagram are of contrary signs.

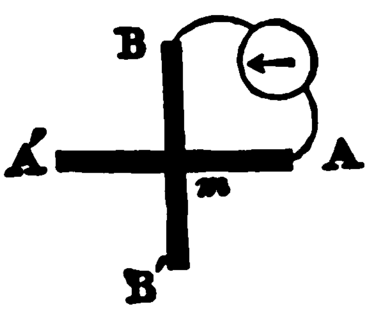
§ 8. Peltier Effect; Observed Facts.—We have seen that, when a current is passed through a conductor, heat is disengaged at a rate proportional to the *resistance of the conductor* and to the *square of the current-strength*. This is usually called the *Joule* effect.

In 1834 Peltier discovered that when an electric current passes from a metal A to a metal B across their junction, heat is absorbed or given out at this junction. The absorption takes place if A have a higher thermo-electric power than B, and the disengagement if A have a lower thermo-electric power than B. Thus, if the direction of the current be reversed, we have heating at a junction where before there was cooling, and *vice versâ*. This rate at which the heat is absorbed or evolved is found to depend upon the *nature of the metals* (not upon their resistance), upon the *temperature of the junction*, and upon the *current-strength* (not upon the square of this). It is thus quite distinct from the Joule effect. This latter may mask the Peltier effect if the current be too strong or the resistance at the junction be too great. However, if we drive the same current first in one direction and then in the other, and measure the heat per second evolved, we obtain (*Joule effect*) + (*Peltier effect*), and (*Joule effect*) — (*Peltier effect*), in the two cases respectively.

If the reader will refer to the meaning of higher 'thermo-electric power,' given earlier in §§ 4 and 5, he will see that the above-stated discovery of Peltier's involves the following: that when a current is passed across a junction this will be so heated or cooled as to originate always a thermo-electric E.M.F. *opposed to* that of the current in question. Thus, if A have a higher thermo-electric power than B, and if the current pass from A to B, we have stated that the junction is cooled; and this will create an E.M.F. tending to drive a current from B to A across this cooler junction. The

Peltier effects are thus analogous (in a degree) to the creation of counter E.M.F.s arising from the passage of a current through electrolytic cells, or through electro-motors (*see* Chapter XXIV.).

Experiment.—A cross may be made of fairly thick bars of bismuth (B B′) and antimony (A A′), having a soldered junction at *m*; bars 3 inches long and ⅛ inch diameter will answer very well. Wires from the ends A B can be connected with a galvanometer G; and wires from A′ B′ with a battery not drawn in the figure. By means of a suitable arrangement of mercury cups these connections, of A B with the galvanometer and of A′ B′ with the battery respectively, can be made alternately with ease; it being necessary only to 'rock' the cross slightly.



If the current be first passed from Bi to Sb through B′ A′ (the former metal being of higher thermo-electric power than the latter at usual temperatures), and if A and B be then connected with G, the deflexion will indicate a cooling of the junction. [We can cool the junction with ether and observe a deflexion in the same direction.]

If the current be passed through A′ B′ from Sb to Bi, and if then A B be connected with G, we observe a deflexion in the opposite direction, indicating that the junction has been heated.

Caution.—If the current be too strong the general 'Joule effect' heating may mask the Peltier effect. It is well to try two or one Smee's cells; and, if the current is too strong even with one cell, to interpose resistance by means of a rheostat or resistance box until the Peltier effect is sufficiently marked. The Joule effect falls off, as the current weakens, as the square of the current; the Peltier effect only as the current.

§ 9. **The 'Thomson Effect.'**—Sir W. Thomson pointed out the existence of another very important heat phenomenon; and Professor Tait continued and added to his work. The combined result may be given somewhat as follows.

'*In general, when a current passes through any portion of a homogeneous conductor where the temperature is not uniform, it tends to heat or to cool such a portion; the amount of the action at each point being proportional to the absolute temperature of that point.*'

Thus, in some metals, as (*e.g.*) in *copper*, the current passing from a cold to a hot region absorbs heat throughout, but most in the hotter region whose absolute temperature is the higher, thus tending to reduce temperature-difference; while when it passes from hot to cold it disengages heat throughout, but most in the hotter region, thus tending to exaggerate temperature-difference.

In other metals, as (*e.g.*) in *iron*, the effect is just the reverse; a these the passage of a current is accompanied by an absorption f heat when passing from a hot to a cold part, and by an evolution of eat when passing from a cold to a hot part. These phenomena re known as the 'Thomson effects,' and they have been found to ccompany the passage of an electric current in all simple metals with the exception of *lead*.

A little consideration will show that, so far as the Thomson effects are concerned, an electric current in a copper conductor behaves in an analogous manner to a material fluid having specific heat and flowing sufficiently slowly to acquire at each point the temperature of the conductor at that point. For this reason the effects are sometimes ascribed to a *specific heat* possessed by electricity, the amount of heat absorbed by the passage of a unit quantity of electricity in passing from one part of a conductor to another one degree higher in temperature being called the *specific heat of electricity in the given conductor.* The specific heat of electricity is not a constant quantity, but depends upon the conductor in which it flows. To account for the Thomson effect in iron, electricity must be supposed to possess a negative specific heat in that metal, while in lead it is zero. In all cases the specific heat of electricity is found to vary as the absolute temperature.

§ 10. **Theory of the Simple Thermo-Cell.**—We shall now give ome account of the theory of the simple thermo-cell. But, as tated earlier, we cannot enter at all fully into the subject.

(I.) *Two metals in contact, the junctions at the same temper-ture.*—It seems beyond dispute that at all temperatures, excepting he 'neutral temperature,' two metals in contact are at a Δ V; the mount of this Δ V depending on the temperature. Could we ake a cell of two metals A and B having one junction A | B, and hen join the other ends by some conductor that was at the same otential as *each* metal, we should have a circuit with an E.M.F. in and should get a current. But, as seen in Volta's law of Chapter XI. § 3, whether we join the other extremities direct or through nother metal, we must always have an equal and opposite Δ V at e other junction, and so have a resultant zero E.M.F. in the circuit. If $A\overset{t}{|}B$ represent the E.M.F. from A to B at the one juncon, and $B\overset{t}{|}A$ the E.M.F. from B to A at the other junction, it clear that $A\overset{t}{|}B = -B\overset{t}{|}A$. Hence, as we go completely round e circuit, we have, in the notation of § 5 . . . .

$$E_t^t = A\overset{t}{|}B - A\overset{t}{|}B = O.$$

(II.) *Two metals; the junctions at different temperatures; the E.M.F.s at the junctions only being considered; compare with (IV.).*—If now the junctions be at different temperatures t_1 and t_2, we shall in general have $A\overset{t_1}{|}B$ different from $A\overset{t_2}{|}B$. Hence, as regards the E.M.F.s that occur at the junctions only, we have a resultant E.M.F. in the circuit given by . . . .

$$E_{t_1}^{t_2} = A\overset{t_2}{|}B - A\overset{t_1}{|}B.$$

The reader must observe that we do not state that the E.M.F.s at the junctions are the only E.M.F.s in the circuit. We shall see in (IV.) that in general such is not the case. These *junction-E.M.F.s* occur *abruptly*, and may be measured by the abrupt change of potential that occurs at the junctions, the circuit being broken at some point where it is homogeneous (*i.e. not* at a junction).

(III.) *One metal; its extremities at different temperatures.*—When any metal (the one known exception being *lead*) has its extremities at different temperatures t_2 and t_1, there is in general an E.M.F. either from hot to cold or from cold to hot, according to the nature of the metal. Thus, in the two cases respectively, it will be found that the hotter end has the lower or the higher potential; the end *towards* which the E.M.F. acts being raised to the higher potential. This rise or fall of potential is *gradual.* If a complete circuit of one unequally heated metal be made, there will be no resultant E.M.F. if the temperatures change gradually from point to point; for in that case the E.M.F. due to the rise of temperature through one part of the circuit will be balanced by an exactly equal but opposite E.M.F. due to the fall of temperature through the rest of the circuit. This will be true even if the rise and fall be very different in 'steepness'; provided that there is no abrupt change of temperature at any section. If, however—either owing to two very unequal heated ends touching, or owing to a very abrupt alteration in the diameter of the conductor at any point—there be some abrupt discontinuity in the variation of temperature from point to point, then there will be in general a resultant E.M.F. in the circuit. It is generally said that this is the case when there

nite change of temperature within a distance so small that
7 be considered to be within the limits of molecular action.
ch a case there is a Δ V occurring *abruptly*, as in (II.).

V.) *Two metals; the junctions at different temperatures; the .F.s all round the circuit being considered; compare with*

f there be two metals A and B with junctions at different temtures, the total E.M.F. in the circuit will be the algebraic sum he E.M.F.s at the junctions) + (the E.M.F.s in the metals nd B). The former were considered in (II.). As regards latter, these would have a *zero* sum were the metals the same, tated in (III.). But in general a metal A with its ends at ıd t_1 does not give the same E.M.F. as another metal B with nds at t_2 and t_1; and so in general the sum of these 'gradual L.F.s' (as we may call them to distinguish them from the ıpt junction-E.M.F.s) is not zero. In the case of *lead* e is no E.M.F. due to its unequal heating. Hence, in a cell vhich one metal is lead we have a somewhat simpler sum of [.F.s in the circuit.

V.) Thus when the current is running in a thermo-cell of tions at t_2 and t_1 respectively, the distribution of potential he circuit is a resultant distribution comprising three coments.

i.) The abrupt changes of potential at the junctions. These sure what we may call the *junction-E.M.F.s* or *Peltier-f.F.s.*

ii.) The rises and falls of potential occurring gradually along conductors, due to the unequal heating of these conductors. se measure what we may call the *Thomson-E.M.F.s.*

iii.) The regular fall of potential that follows Ohm's law, ussed in Chapter XIII.

The total E.M.F. in the circuit could be measured by breakthe circuit at some place where it is homogeneous (*i.e.* not at nction) and measuring the Δ V between the broken ends.

VI.) Experiment tends to show that at the *neutral point* T° C. wo metals A and B there is no junction E.M.F. between n, though this is hardly certain; and most certainly shows that ι cell where the junctions have T° C. as their mean tempera-, the total E.M.F. in the circuit is zero.

The above statements represent, in a necessarily very im fect form, the present views as to the sources of the E.M.F. thermo-cell.

For practical purposes it is simpler to depend upon the diag and formula of § 7; since these (when fully given) embody all known results of experiment.

§ 11. **Theory of the Peltier and Thomson Effects.**

(I.) Let there be a cell of two metals A and B whose junct are at t_2° C. and t_1° C. respectively. We will suppose, for the s of more clearly defining the particular case that we at first c sider, that t_2 is higher than t_1, and that both temperatures above T; further we will suppose that, for temperatures above the metal B is of higher thermo-electric power than A. Th the case in which the total E.M.F. in the circuit is represented the area B_3 B_4 A_4 A_3 in the diagram, fig. i., of § 7.

(II.) When a current is running we have manifested electi energy. If this energy be not converted (in part) into chemi potential-energy or into mechanical or other energy, it will all converted finally into heat energy.

Now this electrical energy must have been derived from so original form of energy of which an equivalent amount must h disappeared.

In the voltaic-cell it was chemical-potential-energy that wa used up.

But in the thermo-cell the only available source of energ the heat energy supplied to the cell by a flame or other sourc heat.

(III.) When, therefore, the current does no other work it r be that (i.) heat energy is absorbed, and disappears, somewhe the circuit; (ii.) this is transformed into an equivalent amo of electrical energy; (iii.) and this again is finally transfor into an equivalent of heat energy, distributed over the ci according to the resistances of the several portions of the circ

The question arises, from what part of the circuit is the orig heat energy derived?

(IV.) In attempting to answer this question theoretically must be guided mainly by two considerations. *Firstly*, we feel sure that the well-established law of '*Degradation of ene* will hold here as in all the phenomena of inanimate nature. '

the source of heat must be *on the whole* the hotter part of the l ; and the general result of the action must be that the temratures of the cell tend to become èqualised. *Secondly*, we must member that when a current runs *against* an E.M.F. work is ne ; and, if the work takes no other form, it will appear as heat olved. Further, it seems almost certain that this heat will be olved in that portion of the conductor in which the work is done, in which the E.M.F. lies. Conversely, when a current runs *ith* an E.M.F., this E.M.F. does work on the current ; and, when heat is supplied, this work will be done at the expense of heat sorbed from the conductor ; further, it seems almost certain at this heat will be absorbed from that portion of the conductor which the E.M.F. lies.

(V.) The consideration of (IV.) will lead us to predict that, on e whole, the hotter portion of the cell will be cooled and the oler portion will be heated ; while some of the heat derived om the hotter portion of the cell will, after passing through the termediate form of electrical energy, reappear as heat distributed ound the circuit in accordance with Joule's law.

(VI.) Where the current crosses a section (*e.g.* a junction of wo different metals) at which occurs an abrupt E.M.F., or [*see* 10 (V.)] a 'Peltier E.M.F.,' there we should expect to have eat absorbed or given out according as the current flows with the E.M.F. or against it respectively.

Such an absorption or disengagement of heat, occurring at a nere section and not over any finite length of the conductor, is he true *Peltier effect* referred to in § 8.

There is probably no Peltier effect when the junction of the wo metals is at their neutral temperature T ; for it seems probable hat at that temperature the two metals are as one.

(VII.) Where the current flows through an unequally heated netal, it flows (in all metals excepting *lead*) with or against the *Thomson E.M.F.* spoken of in (III.) and (V.) of § 10.

We should then expect heat to be absorbed or given out respectively, over a finite length of the conductor in question. This s, in fact, observed in the *Thomson effect* of § 9.

Since the unequally heated ends of each metal are at the two junctions, the result of the Thomson effect will be to alter the temperatures of the two junctions. Unless, therefore, special ex-

periments are performed from which we can calculate each eff
separately, we shall in general observe the sum of these two effec
(*i.e.* of the Peltier and Thomson effects) at the junctions, and sh
not be able to ascribe the heating or cooling of the junctions t
the Peltier effect only.

(VIII.) We may then say that the transformations of energy re
ferred to in (IV.) and (V.) take place through the intermediency o
the Peltier and Thomson effects together.

When the hotter junction is at the neutral temperature T
then if the Peltier effect be zero, heat energy must be suppli
through the intermediency of the Thomson effect only.

(IX.) Now let us suppose that we cease to supply heat fro
external sources. The junctions will arrive at the same tempera
ture, by the cooling of the hotter and the heating of the coole
junction.

If now the current be continued from some external source i
the same direction as before, there is no reason for supposing tha
the above Peltier effects would cease, though there would now be
no Thomson effects, since the metals are at one temperature. We
should predict then that the junction which was the hotter would
now become the cooler, and conversely.

This would raise up an E.M.F. opposed to the former E.M.F.,
and therefore opposed to the current that is flowing.

Some such reasoning as the foregoing would therefore lead us to predict the ordinary case of the ***Peltier effect***; the case, namely, where a current is sent across a junction of two metals A and B, and where it is found that the junction is so heated or cooled as always to raise up an E.M.F. opposing the current, provided that the junction is not at the neutral temperature. In other words, there will be heating or cooling according as the current flows from the metal of lower, to the metal of higher, thermo-electric power, or *vice versâ*.

The whole of the above reasoning is necessarily rather of the nature of guessing at probable results than of strict argument. The fact is that the theory of thermo-cells is beyond the scope of an elementary book.

CHAPTER XVII.

GALVANOMETERS ; WITH A PRELIMINARY ACCOUNT OF THE MAGNETIC ACTIONS OF CURRENTS.

§ 1. **Magnetic Field about a Simple Rectilinear Current.**—We will now turn our attention to the very important class of phenomena coming under the head of '*the magnetic actions of currents.*'

On these magnetic phenomena depend the construction and use of that important class of instruments called *galvanometers*, whose use, as current detectors and current measurers, is so essential in the modern science of electricity.

In the present Chapter we propose describing various forms of *galvanometers.* But, in order the better to understand their theory, we shall give some preliminary account of the magnetic fields due to electric currents ; leaving, however, the main part of this important subject to be pursued further in Chapters XVIII.–XX.

When a conductor is charged statically with electricity, we have about it what we call an *electrostatic field.* This field acts on a + *unit of electricity* in lines of force that run to or from the conductor, as explained in Chapter X. The case of a conductor carrying a current is very different. It is true that still a + unit of electricity would in general find a field of force about the conductor, for this conductor will be in general at potentials different from that of the earth, and different from point to point of the conductor.

But this electrostatic field is quite unimportant and negligible compared with the new field of force that springs into existence directly the 'electricity' moves, or directly there is a *current.* This new field is a *magnetic field* ; and we shall therefore consider its action, not on a + unit of electricity, but on a + *unit magnetic pole.*

It is easily shown by direct experiment that the lines of force in which this field urges our unit pole form circles round the wire that carries the current, so that a pole is urged, not to or from the wire, but continually round and round it (the wire being assumed straight). Each line of force is a closed circle (not a spiral), lying in a plane perpendicular to the rectilineal wire, and having its centre in this wire. The field is weaker further from the wire, and stronger nearer to the wire.

As with the electrostatic field in Chapter X. §§ 13 and 14, so with a magnetic field we can mark out a certain number of lines of force and leave in the field such a selection that the number piercing 1 sq. cm. held perpendicularly to the lines at any place in the field measures the strength of the field at that place, *i.e.* gives the number of *dynes* with which a unit magnetic pole would be urged at that place.

Experiments.—(i.) A hole is bored (with a bow drill and turpentine) in a sheet of glass, and a wire carrying a current is passed through this hole perpendicularly to the glass plate.

On passing a strong current, and scattering iron filings on the plate, these latter will (on tapping the plate) be observed to arrange themselves in con-

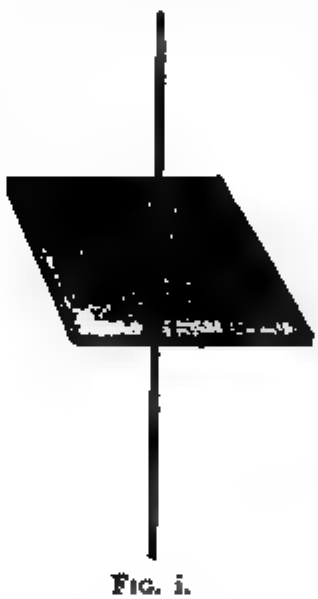


FIG. i.

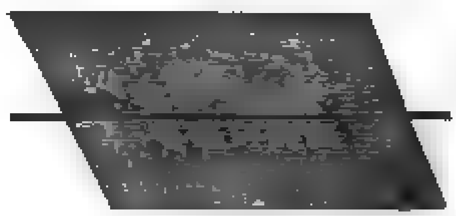


FIG. ii.

centric circles about the hole as centre. If the wire be not perpendicular to the plate, the lines will be ellipse-shaped. This indicates that the lines of force are circles lying in planes perpendicular to the wire.

(ii.) If the wire lie on the plate, we find the filings arranged in straight lines perpendicular to the wire. These straight lines are the sections of the concentric circles, made by the plate.

Now let a magnetic needle, so balanced as to turn any way, be placed near such a current; and let us for the present consider only the field due to the current, leaving the earth's field out of the question. It is clear that the needle can be in equilibrium only when it lies in the plane passing through its point of suspension and perpendicular to the wire carrying the current; and when, further, its poles are equidistant from the wire. If it is very small it may be said to come to rest when it lies in (or is tangent to) one of the above circular lines of force. (Compare Chapter II. § 11.) In such a position its poles are urged by equal forces tending in opposite directions round the wire; these equal forces being inclined at equal angles with the needle, since this forms a chord of the circular line of force.

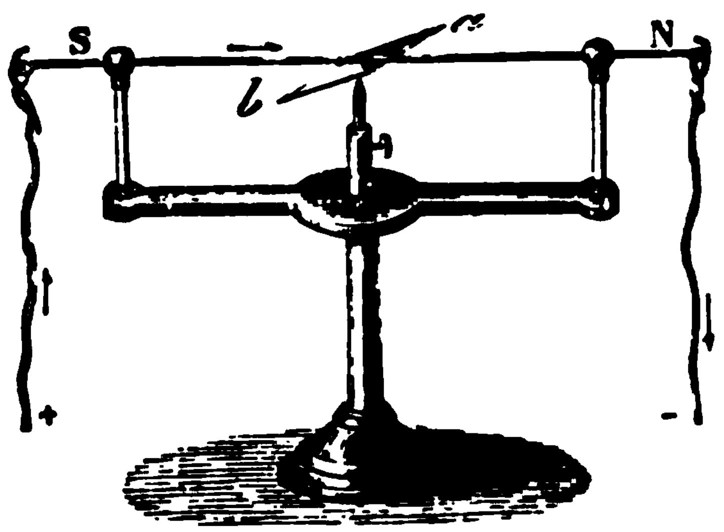
This pair of forces will direct the needle as stated, and will also give a resultant urging the needle broadside-on towards the wire. The reader should make the above clear to himself by means of a figure.

(iii.) A magnetic needle is fixed on a cork, and so floats on the surface of water. To one side of the needle is placed a wire perpendicularly to the surface of the water, and a strong current is passed through the wire. It is better so to place the wire, and to pass the current in such a direction that the action of the current is not opposed to that of the earth, but acts with it. It will then be observed that the needle will be dragged into a position in which its poles lie on the circumference of a circle which has for its centre the point where the wire cuts the water, *i.e.* lie on the same circular line of force; and that then the needle will be urged broadside-on towards the wire.

(iv.) In the same way are steel or iron filings urged broadside-on towards a wire carrying a current, and caused to adhere to it, the filings having become 'magnetic needles' by the inductive action of the field. They are thus arranged in very close circles *round* the wire. (In the case of a magnetised steel wire the filings adhere *end-on*, pointing along the lines of force that in this case radiate *from* the wire.)

§ 2. **The + and − Directions of the Lines of Force.**—It is easily shown, by experiment with a magnetic needle, that the + and − directions of the lines of force are given by either of the following rules.

(I.) *Ampère's rule.*—'*If one swims with the current* (*i.e.* so that it flows from feet to head) *and looks at a n-seeking* (or +) *pole, this will be urged to one's left hand. A s-seeking* (or −) *pole will be urged to one's right hand.*'



Oersted's experiment.—The historic experiment of Oersted illustrates well this rule of Ampère. The figure here given sufficiently explains this simple experiment.

(II.) Another form of the rule is often very useful. No one who has driven in an ordinary (or *right-handed*) screw can forget how the hand turns as the screw advances away from the person driving it. As one drives it from one, the hand turns as do the hands of a clock that faces one. Now the above experimental rule of Ampère means that if the current advances with the screw, a + pole is urged round it in the direction in which the screw turns. And since the direction in which a + pole is urged gives us the + direction of the lines of force, we may give the following rule.

'*The + direction of the lines of force about a wire carrying a current is associated with the direction of the current in just the same way as the direction of rotation of an ordinary (or right-handed) screw is associated with the direction of the onward movement of the screw.*'

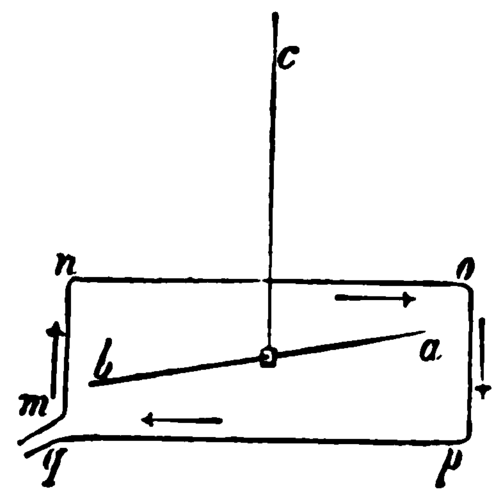
(III.) *Field due to a circle of wire carrying a current.*—If the wire be bent into a circle it will easily be understood that at the centre of the circle all the lines of force there combine to give a line of force running perpendicularly to the plane of the circle. To any side of this line the lines of force bend away; they ultimately curve completely round the wire and run into themselves again. Thus there is, if we wish to be very exact, only one *straight* line of force, viz. that which runs through the centre of the circle perpendicularly to its plane. [Mathematicians would say that the two ends of this line join at infinity, so that it also forms a 'closed curve.'] But practically, if we take a portion of the field that is near the centre of the circular current and is small as compared with the diameter of the circle, we may consider this portion to be uniform, and to have its lines all running perpendicular to the plane of the circle.

A little consideration will show us that the same 'right-handed screw relation' holds here in this somewhat converse case. If we

follow this central line of force in its + direction (*i.e.* if we travel as a + pole is urged), then the current round the circle is associated with our direction of movement as the direction of rotation of a screw is associated with its onward movement. *E.g.*, if we face a circle in which the current goes *clock-wise* (*i.e.* as do the hands of a clock when we face it) our + pole is urged *towards* this circle.

§ 3. **Simple Form of Galvanometer.**—If we balance a magnetic needle so that it moves in a horizontal plane, it will come to rest when it is in the plane of the magnetic meridian. If we deflect it from this position of rest, there will be a couple due to the horizontal component of the earth's field tending to restore it to its original position of rest.

Now let a loop of wire be passed round the needle, above and below it, so that its plane coincides with the plane of the magnetic meridian. The figure here given exhibits this simple arrangement.



If a current be passed round this wire loop, it is easy to see that the upper and the lower wires give fields about the needle in the same direction, tending to set it at right angles to the plane of the loop, *i.e.* at right angles to the plane of the magnetic meridian.

Now the earth's couple is greatest when the needle stands E. and W., and is zero when the needle stands N. and S.; while the converse is the case with the current's couple.

Hence the needle will settle at some angle θ of deflexion from the magnetic meridian; the magnitude of θ (*cœteris paribus*) depending upon the current-strength.

Such is the principle of the *galvanometer*. In §§ 5, 6, 12, 13, 14, 15, we shall describe instruments designed for *measurement* of currents. In §§ 7, 8, 9, 10, 11, the instruments described are mainly for *detection* of very small currents. (For further general remarks on galvanometers, *see* § 16.)

§ 4. **Relation of Strength of Field to Current-Strength.**—It can be shown that *strength of field is* (*cœteris paribus*) *directly*

proportional to strength of current; the strength of field being measured, *e.g.*, by the vibration method of Chapter III. §§ 3 and 6.

We make a vertical frame. round which insulated wire may be wound, and place it in the plane of the magnetic meridian. In the centre of this frame the needle vibrates under the field due to earth and current together.

We then try the effect of having *one*, *two*, *three*, &c., turns of wire round the frame. By including in the circuit a rheostat, by means of which more or less resistance may be introduced, and an auxiliary galvanometer whose deflexion we thus keep constant, we can insure the constancy of the current that we employ.

We then compare the fields, as in Chapter III. § 3, and show that the above law holds. It is easy to show that the field depends strictly upon *current-strength* and *not* upon *current-density*; that is, upon the quantity per second passing over any section of the conductor, not upon whether this current forms (so to speak) a broad slow stream or a quick narrow one.

Experiment.—One way of proving the above is to show that when a current flows through a thick wire, and returns through a thin wire wound round the former, the total resultant magnetic field is *zero*. This is another example of proving equality by means of the 'zero method.'

The reader will observe that from this it follows, as assumed above, that with the same current *n* turns of wire act as one turn carrying *n* times this current. Hence, the law stated at the beginning of this section is proved by the method given, in which we employ *one*, *two*, *three*, &c., turns of wire.

§ 5. **The Tangent Galvanometer.**—The simplest form of true galvano*meter* (or current-*measurer*) is that called the *tangent-galvanometer*. Referring to § 2 (III.), we shall see that about the centre of a circular wire carrying a current the field may be considered uniform. Or, if a needle whose length is small as compared with the diameter of the circle (*see* note (iii.) at end of this section) be suspended at the centre, then, however much it be deflected it will remain practically in the same strength of field as long as the current remains constant.

If then we can measure the strength of this field by the amount of deflexion, we measure at the same time the current that gives rise to the field; whereas, if the needle were too long or situated unsymmetrically with respect to the circle, the needle would, for

the same current, pass into different strengths of field according to the extent of its deflexion; in which latter case we could measure the fields but not the current.

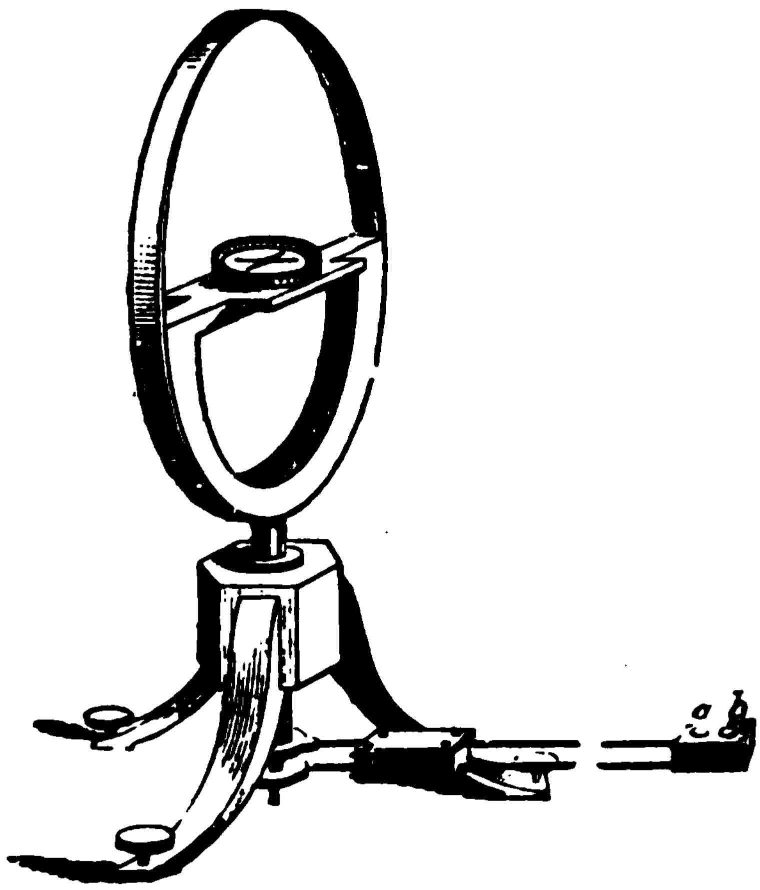


FIG. i.

Fig. i. gives a general view of the tangent galvanometer. There is usually a short needle provided with a long and light index that moves round a graduated circle. Very often arrangements are made by means of which we can either use one thick circle of copper for very powerful currents, or two or more turns of wire for currents not so strong (*see* § 7). This circle or coil is placed in the plane of the magnetic meridian. In fig. ii. we are supposed to be looking down from above, and so to be viewing the top of the circle of copper and the needle, projected together on to a horizontal plane.

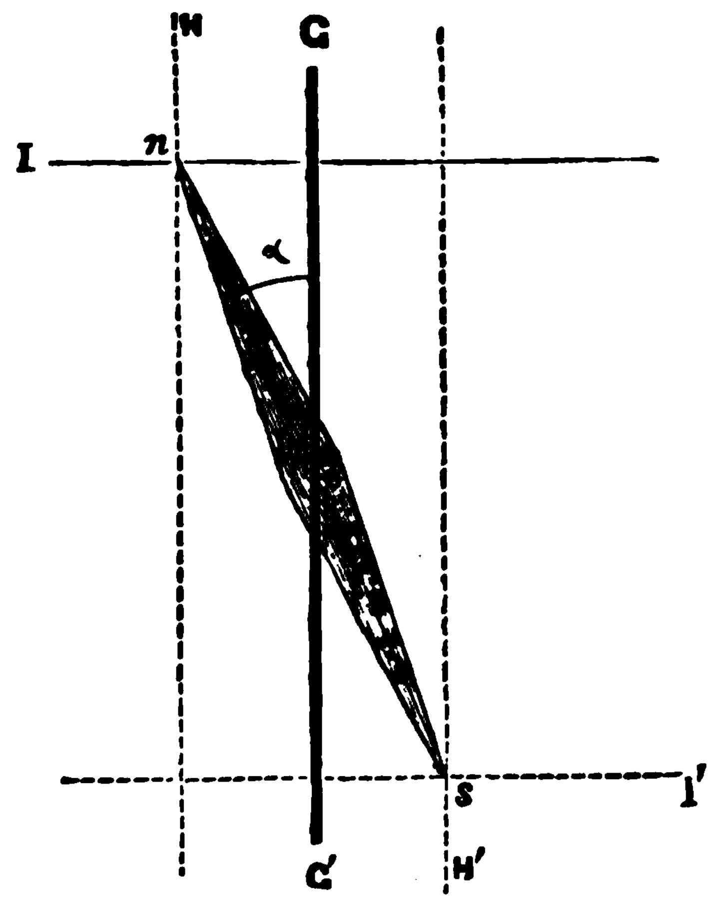


FIG. ii.

G G′ is part of the projection of the copper circle; it lies, as mentioned in § 3, in the magnetic meridian. *n s* represents the needle; this lies in reality below G G′, at the centre of the circle of which G G′ is the top. The dotted lines running parallel to G G′ represent the direction of the earth's horizontal field, H; those perpendicular to G G′ represent the field I due to the current.

Using the notation of Chapter III. we have that . . . . .

(i.) Earth's couple acting on needle is H $\mu\, l \sin \alpha$.

(ii.) Current's couple acting on needle is I $\mu\, l \cos \alpha$.

When there is equilibrium we must have these couples equal; or

$$H\,\mu\, l \sin \alpha = I\,\mu\, l \cos \alpha.$$

Whence $$I = H \,.\, \tan \alpha.$$

Now, in the present case we have said that whatever be the deflexion of the needle we may consider I to be constant while the current is so, and to be directly proportional to the current C. Hence we may write $C = k\,I$, where k is some constant depending on the dimensions of the coil, the number of turns of wire, and the unit of current that we employ.

We have then $C = k\,H \,.\, \tan \alpha$.

Hence with the same instrument, and with H the same, the current is proportional to the *tangent* of the angle of deflexion; whence the name of this instrument.

It is clear that it can be used for currents up to any strength, since $\tan \alpha$ can have any value. But it fails to give accurate measurement if α exceed (say) about 70°, since then large changes in current give small changes in α. The best angle of deflexion is about 45°, as may be shown mathematically.

Notes.—(i.) *Other forms of the instrument.*—In Gaugain's form we have the needle placed in the axis of the coil, at a distance from the centre of the coil about one-fourth of its mean diameter. In Helmholtz's modification of Gaugain's instrument there are *two* coils lying symmetrically on either side of the needle, and the wires are wound on small portions of cones, of which, if completed, the centre of the needle would be the common apex. Such an arrangement gives a more uniform field, and permits one to use a somewhat larger needle.

(ii.) *To set the coil in the magnetic meridian.*—When the coil is in the magnetic meridian, the same current, passed in opposite directions, should give opposite deflexions of equal magnitude. The coil should be adjusted until this is the case.

(iii.) As regards the relative dimensions of needle and diameter of coil, giving results accurate enough for practical purposes, we quote here the following authorities.

Wiedemann ('Die Lehre von der Electricität,' iii. 247) gives length of needle at most one-eighth diameter of coil.

Kempe ('Handbook of Electrical Testing,' p. 18) gives needle about ¾-inch for a 6-inch or 7-inch ring, as accurate enough for most purposes.

S. P. Thompson gives length of needle about one-tenth of diameter of coil.

Brough gives needle one-sixth of diameter of coil.

Andrew Gray gives needle 1 *centimètre* for coil of diameter 15 *centimètres*, in a standard instrument.

Hence, needle one-tenth of diameter of coil seems a good relative dimension.

§ 6. **The Sine Galvanometer.**—Fig. i. shows us a somewhat different form of galvanometer. Here the coil turns round on a vertical axis, the movement being measured over a horizontal graduated circle. As before, the coil is initially in the plane of the magnetic meridian.

We may here turn the coil until the current does not affect the needle at all. In this position it must be that the lines of force due to current coincide with those of the earth, or the coil is due magnetic east and west. We then turn the coil back through 90° over its graduated scale, and it will lie in the plane of the magnetic meridian.

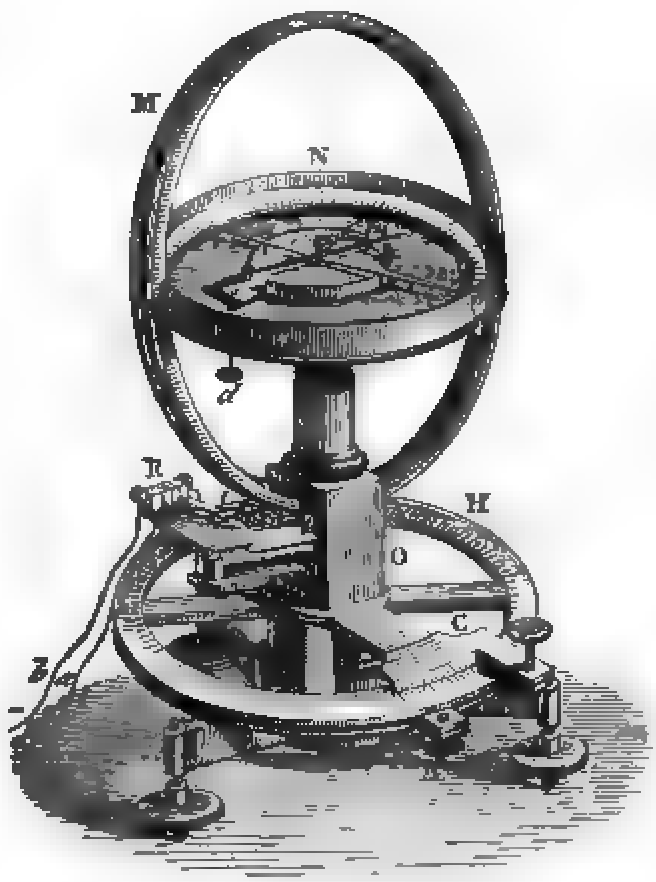


FIG. i.

When the needle is deflected the coil is turned in the same direction; and finally, if the current be not too strong, can be brought again directly over the needle. When this is the case we note the angle α through which the coils have been turned. The current will be proportional to *sin* α.

Fig. ii. shows us the theory of this instrument. In it is represented the final position of equilibrium, in which the coil G G′ lies again over the needle at an angle α from the magnetic meridian N S. Since now the lines of force due to the current are perpendicular to the needle, we have . . . .

$$\begin{cases} \text{(i.) Current's couple} = \mathrm{I}\,\mu\, l. \\ \text{(ii.) Earth's couple} = \mathrm{H}\,\mu\, l\,.\, \sin\alpha. \end{cases}$$

With similar reasoning to that given in the last section we have finally . . . .

$$C = k\text{H} \, . \, sin\, a$$

whence the name of the instrument.

It is evident that we cannot, without using shunts, measure any current exceeding k H, since the greatest value of $sin\, a$ is 1, which it has when $a = 90^c$, or when the needle stands E. and W. With a larger current we should chase the needle completely round.

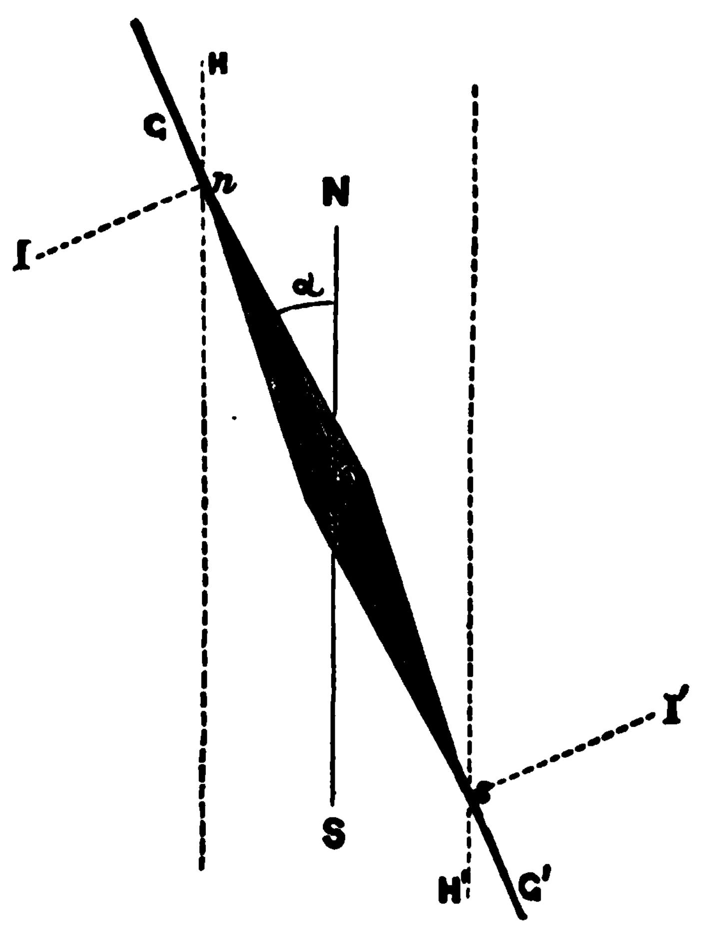


FIG. ii.

Since the coil is always over the needle we may have this of any length we please, for it will always be in the same part of the current's field.

§ 7. **The Multiplying Galvanometer.**—The extent of deflexion of the needle depends upon the strength of the magnetic field due to the current.

With the same current, this can be multiplied many-fold by passing the wire many times round the needle. In fact, we wind the wire in a coil and suspend the needle inside it.

The advantage of this can be demonstrated as in § 4.

A figure of a multiplier is shown in the next section.

§ 8. **Astatic Galvanometer; Two Needles.**—In the last section we showed how to increase the current's action on the needle by *increasing the field-strength due to the current.*

There is another very effective manner in which the deflexion for a given current may be almost indefinitely increased. This other method is based upon the device of *making the earth's restoring couple as weak as we please*, while we leave the current's deflecting couple unaltered.

We cannot do this by weakening the magnetic moment of a

ıeedle ; since, as we see in the formulæ of §§ 5 and 6, we then weaken equally the current's action on the needle.
we can so combine two needles that they form with re- › the earth a single needle of as small a magnetic moment lease ; while, by coiling the wire round one needle only, we ıse the current to act on a single needle of considerable ic moment.
ystem of needles on which the earth has little (or, strictly, ion, ıs called an *astatic* system ; and a galvanometer in ıuch a system is ed is called an *galvanometer.*
ere are two com- ns that give us ient astatic sys-

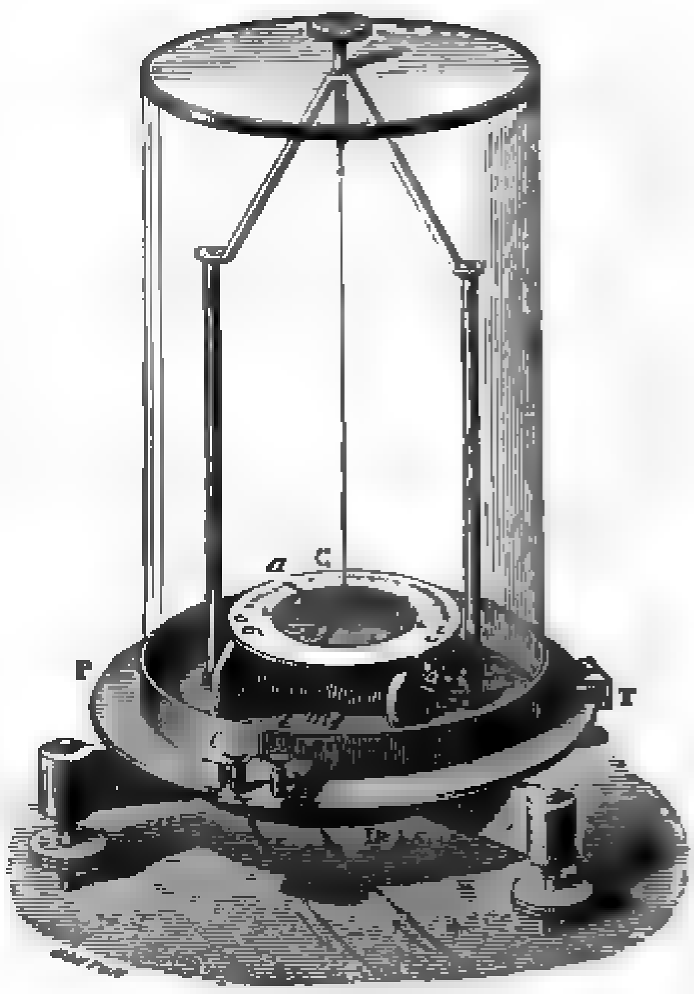


FIG. I.

Let us connect two needles $n\,s$ s' of nearly equal h, so that, when are suspended, re in the same l plane ; the poles directed opposite If $n\,s$ be some- he stronger, the edles will act *with to the earth* as edle of the same l, but of pole- .h measured by fference $(\mu-\mu')$; ıt the magnetic nt with respect to the earth is $l\,(\mu-\mu')$, and may be very ndeed by having the pole-strengths μ and μ' nearly equal.
e coils of the galvanometer, however, as seen in the figure, arranged that one needle of moment $\mu\,l$ is in the strong ithin the coil ; while the other needle, $\mu'\,l$, inasmuch as ıt rsed in direction and is also in the reverse field above the

coil, is acted upon by a weaker couple in the same direction as is the lower needle.

Thus we can, by making the pole-strengths μ and μ' very nearly equal, cause the earth's restoring couple to become very small, without diminishing the deflecting couple; and so can cause the galvanometer to become very sensitive.

(ii.) Next let two needles be fixed and suspended as before, but let the vertical planes through them make some small angle θ with each other. In fig. ii. we give a projection of the two needles as viewed from above. Let them further be of equal strength, so that $\mu = \mu'$.

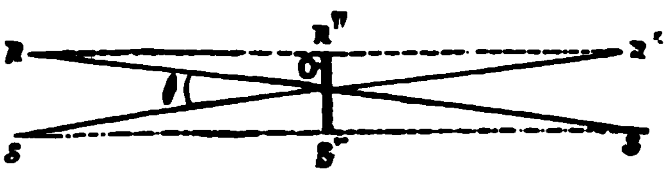


FIG. ii.

Then it is pretty clear that with respect to the earth they will act as a short needle $n'' s''$ whose axis is at O, standing at right angles to the bisector of the angle θ; its pole-strength being 2μ, its length being $n'' s''$ or $l \sin \frac{\theta}{2}$. It follows that its magnetic moment will be $2\mu . l \sin \frac{\theta}{2}$. This can be made as small as possible by making θ as small as we please. It is to be observed that the needles will stand so that $n'' s''$ is in the magnetic meridian; or the general direction of the two needles themselves will be E and W. The action of the current is as before.

§ 9. **The 'Controlling Magnet' Method.**—A needle balanced so as to move in a horizontal field is influenced by the horizontal component of the earth's field only, as we have said before. Now we may superimpose on this horizontal field another due to a magnet; and, if the lines of force of this field run parallel to those of the earth's, but in an opposite direction, we may weaken, to any desired extent, the resultant field in which the needle moves, by suitably adjusting the strength of the magnet's field. The most convenient arrangement is to have a magnet directly above the needle, capable of being slided up or down a metal stem so as to make the field about the needle weaker or stronger; and also capable of revolving in a horizontal plane about the stem, so that it may either stand in the magnetic meridian or may make any angle with the meridian.

For our present purpose our object is to leave the needle directed in the magnetic meridian by a very weak resultant field.

We thus should have the magnet's field a little weaker than the earth's, and opposite in direction.

The needle is acted upon by the current as before; and, as the restoring couple is very small owing to the weakness of the restoring field, the instrument may be very sensitive.

We may add that in the most sensitive instruments the methods ... are combined.

Thomson's Mirror Galvanometer.—The trans- ... ıls by means of interruptions and reversals of currents ... e described in a future Chapter. At present we will merely observe that galvanometers will indicate such reversals and interruptions, and so will serve as instruments to receive signals of this nature. We shall see in a later Chapter that when such signals are sent through long submarine cables, the currents are very small on account of the great resistance; and further, that interruptions and reversals lose their abrupt character and become mere fluctuations in the weak current transmitted.

To indicate such weak currents and such slight fluctuations, Sir W. Thomson invented a form of galvanometer without which signals by cable would hardly have been possible.

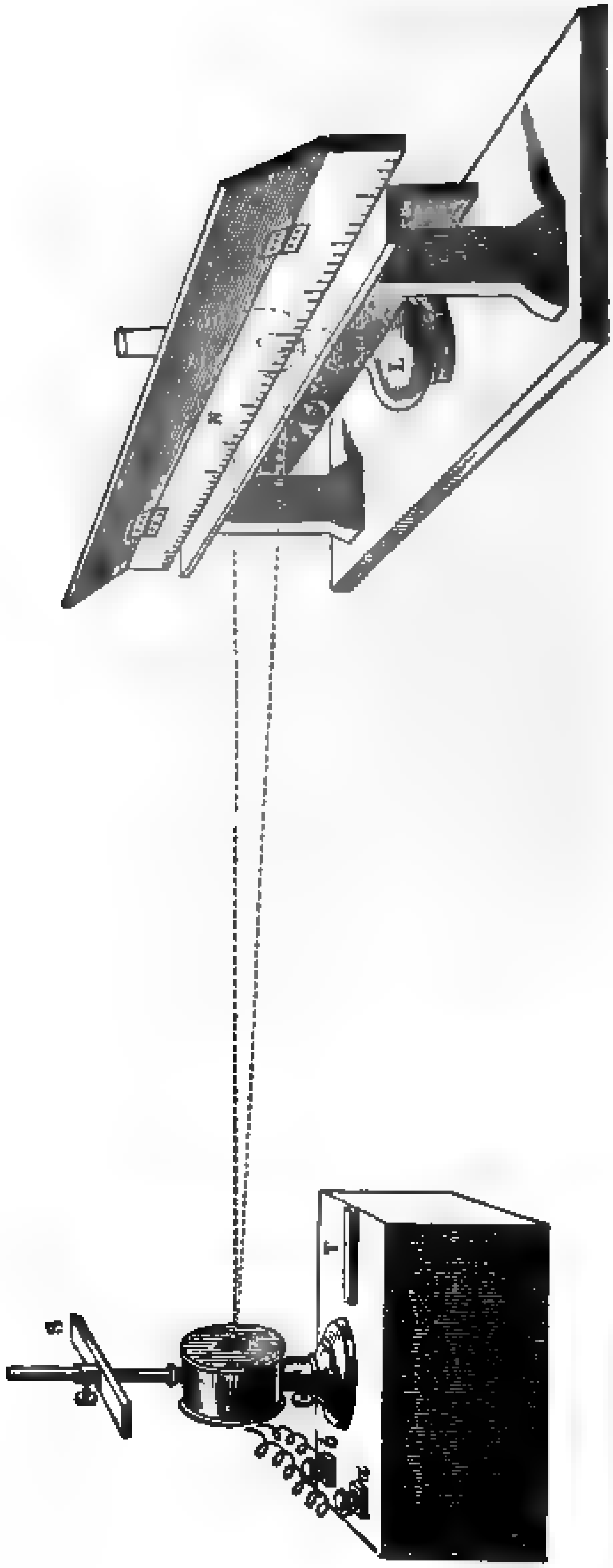
In his *mirror galvanometer* we may notice the following main points of interest.

(i.) As the instrument is usually intended to be used in circuits where the resistance is already very great, there are often many thousand turns of (necessarily) fine wire; this 'multiplies' the strength of the field due to the current without perceptibly diminishing this latter. For the total resistance will not relatively be much increased.

(ii) The 'needle' is a very small bit of magnetised watch-spring. Having little *mass* and inertia, its movements follow any fluctuations in the current almost instantaneously.

(iii.) A controlling magnet enables one to vary the sensitiveness of the instrument at will, as explained in § 9.

(iv.) The same controlling magnet enables one, if need be, to bring the needle to rest in any position; and, aided by the method of suspension mentioned below, to work the instrument even on



oard ship. In such cases we *over-compensate* the earth's field nd direct the needle by the magnet alone. The fibres supporting he needle are fixed at top and bottom, and so prevent it from winging against the sides of the coil in which it works.

(v.) To the needle is attached a very light *mirror*. A beam rom a lamp falls upon this, and the reflected ray gives as index spot of light that moves over a graduated scale.

We thus (as in the case of the quadrant electrometer of Chapter X. § 33) can have an index as long as we please, possessing no mass or inertia.

The figure on the opposite page indicates the coil, lamp, beam id its reflection, scale, and controlling magnet placed above the oil. The needle and mirror are inside the coil and are not seen. n additional controlling magnet T is sometimes of service in inging back the spot of light to the *zero* mark.

Notes.—(i.) The mirror may be concave or plane. In the latter case an xiliary lens is used, in order to bring the reflected beam to a focus on the ale.

(ii.) Either in the reading of, or in the construction of, the scale, we must ake allowance for the fact (proved by elementary optics) that if the needle id mirror be displaced through an angle $\theta°$, the reflected beam is displaced rough an angle $2\theta°$.

§ 11. **The Differential Galvanometer.**—In the differential alvanometer the needle is suspended symmetrically between two oils. These coils must fulfil the following requirements.

(i.) Their resistance must be exactly equal, so that equal E.M.F.s at the terminals give equal currents.

(ii.) Equal currents in the same direction must give equal but pposite fields about the needle; so that, when the same current i the same direction is passing through the two, the needle is unffected.

An exact test that these conditions are fulfilled is to send the *ame* current through two coils coupled end-on, so that the current asses in the same direction through each coil. The needle iould not be affected. Usually a small portion of one coil is left oveable, allowing exact adjustment to be made when the inrument is used.

It is clear that by coupling the coils suitably this instrument

may be used as an ordinary galvanometer, the two coils giving fields acting on the needle in the same direction.

The differential galvanometer has several uses. (i.) One use is to compare resistances ; the method is somewhat as follows. A current is divided into two branches, passing through the two coils respectively. Since the resistances of these are equal, the currents will also be equal and the needle will be unaffected. The unknown resistance is now introduced into one branch, thus causing an unequal distribution of the two currents. Then *known* resistances are introduced into the other branch until the needle returns again to *zero*. When this is the case the resistances must be equal. In this method we are independent of changes in the E.M.F. of the battery.

(ii.) Another, less simple, use of this instrument is given in Fleeming Jenkin's 'Electricity,' p. 242, to which book we refer the student for details as to this method.

We will only say here that by *shunting* one branch of the galvanometer we may either measure a small resistance to a small fraction of an *ohm*, or may measure a resistance which is a large multiple of the greatest resistance contained in the resistance box.

§ 12. **The Ballistic Galvanometer.**—As we have seen, a current can be measured by the couple that it exerts (if we may use the expression) on a needle in an instrument of known construction ; this couple depending also in a known way on the angle that the needle makes with the plane of the coil.

Now *quantity* of electricity is given by (*current*) × (*time*). Hence we can, in the case of steady currents, measure the total quantity of electricity that has passed by means of the tangent galvanometer, by observing both current and time of duration.

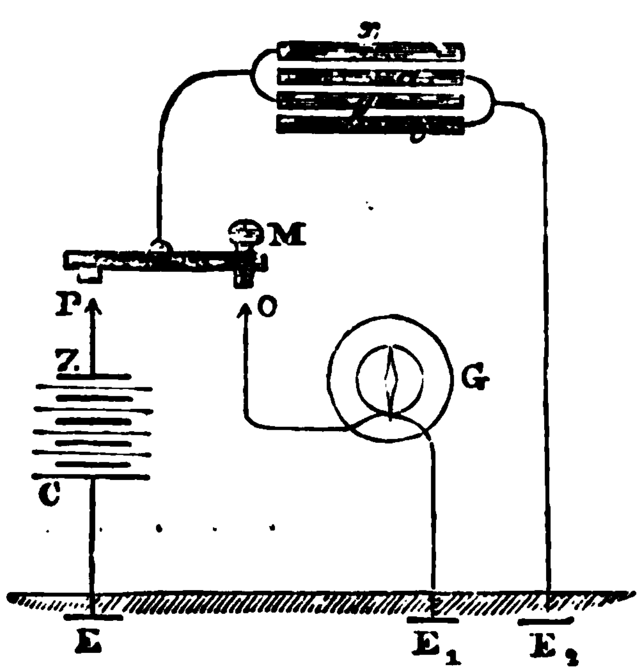
But supposing that we wish, *e.g.*, to measure the quantity of electricity in a condenser by discharging it through a galvanometer. In the ordinary case we have a current whose strength rises from *zero* to a maximum, and then falls again to *zero*. While it passes the needle is deflected, and part acts when the arm of the couple is the full length l of the needle, part when the arm is $l \cos \theta$ (*see* § 5), where θ rises from zero to a maximum and then decreases again. It is impossible, under these conditions, to calculate in any simple way the total quantity that has passed. In order to do this an instrument is devised analogous to the *ballistic pendulum* used in mechanics. In this latter instrument we can estimate what are usually called 'impulsive forces,' or 'impulses,'

by allowing them to act on such a massive and slowly-moving pendulum that there is no appreciable change of position until the impulses have ceased to act. The impulses thus act on the pendulum in one known position (viz. when it is hanging vertically); and the sum total of these impulses can be deduced from the extent to which the pendulum swings, its mass and dimensions being known.

In the *ballistic galvanometer* we have a needle of such a mass that it does not move appreciably from zero until the discharge has entirely passed. Thus each portion of the discharge acts on the needle with couples whose arms are all the length l of the needle. The extent of swing of the needle then depends solely upon the sum of (*each current-strength*) × (*the infinitesimal time during which it remains constant*). In spite of the fact that the current changes continuously, thus making this '*sum*' a matter for the 'infinitesimal calculus,' even a beginner can see that the sum of all the products above given measures the *total quantity of electricity that has passed.* If Q be this quantity, and θ be the maximum angle of deflexion of the needle, it can be shown that . . . .

$$Q = k \, . \, sin \frac{\theta}{2}$$

where k is a constant depending upon the construction of the instrument and upon the strength H of the earth's horizontal field. This constant can be found or calculated.



Note.—The student can find the proof of the above formula in more advanced books.

Use of ballistic galvanometer.—We may use the foregoing instrument to measure, or compare, the capacities of condensers. In the accompanying figure $x\,y\,a\,b$ is a condenser. The plates $a\,b$ are to earth, while the opposite plates $x\,y$ can be connected, by means of a key M, *either* with the pole P of a battery, *or* with the one terminal of a ballistic galvanometer G. The other pole C of the battery, and the other terminal of the galvanometer, are to earth. When the plates

xy are connected with the pole P of the battery, the condenser is charged so that xy are at a potential V, while ab are at *zero* potential. (It is clear that V measures the unpolarised E.M.F. of the battery, for the other pole C is to earth, and there is no permanent current.)

Then if K be the capacity of the condenser, the quantity Q of the charge will be

$$Q_1 = K_1 . V$$

where Q_1 is in *coulombs*, if V is in *volts* and K_1 is in *farads* (see Chapter XVIII. § 4).

Now let the condenser be discharged through the ballistic galvanometer.

The total impulse given to the needle is, we have seen, directly proportional to Q; and this impulse is also, we have stated, proportional to $sin \frac{\theta_1}{2}$, for mechanical reasons that we have not given; θ_1 being, as above, the 'angle of throw' of the needle.

Hence we have $K_1 = \frac{k}{V} . sin \frac{\theta_1}{2}$.

So, for another condenser, we have $K_2 = \frac{k}{V} . sin \frac{\theta_2}{2}$; whence

$$\frac{K_1}{K_2} = \frac{sin \frac{\theta_1}{2}}{sin \frac{\theta_2}{2}}.$$

We can thus compare any condensers with standards, or with each other.

Note.—Let t be the time in seconds of a complete to-and-fro oscillation of the needle when no current passes. Let θ measure the angle of throw in some unit of angle such as *degrees*. Let r_1 measure in *ohms* that resistance through which the E.M.F. measured by V (*see* above) would drive a steady current giving a permanent deflexion of one unit of angle—the same unit as is used in measuring θ. And let K be the capacity of the condenser in *farads*. Then it can be shown that . . . .

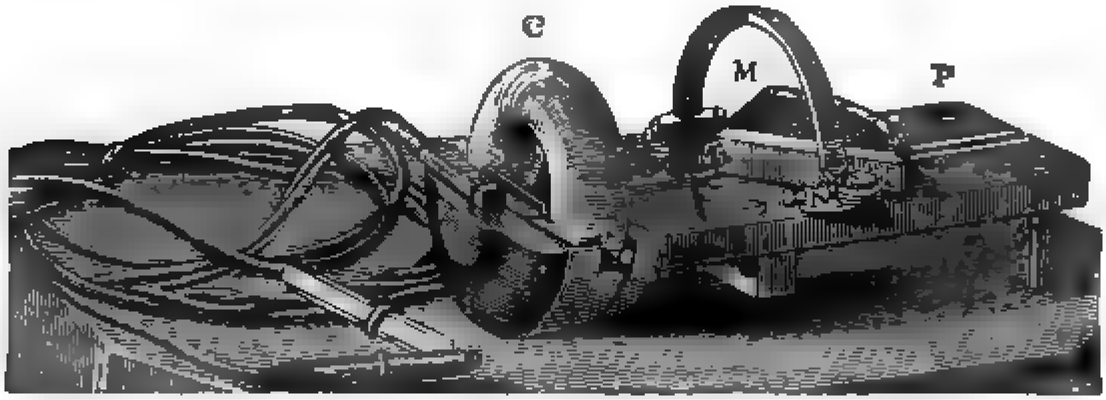
$$K = \frac{t . sin \frac{\theta}{2}}{\pi . r_1}.$$

This formula is given in Fleeming Jenkin's 'Electricity.' The more massive the needle, and the less friction there is in its movement, the truer is the formula.

§ 13. **Sir W. Thomson's 'Graded' Potential Galvanometer**—To meet the present requirements of exact electrical measurement, Sir W. Thomson has invented two galvanometers of convenient form and of wide range of sensibility, the one designed especially for measurements of E.M.F.s or of potential differences, the other for measurements of currents. In both there are arrangements for altering to a known extent and through a wide range the sensibility of the instrument; and both are so marked by the makers that, with simple corrections for the strength of the earth's horizontal field H at each place, results can be read off in *volts* and *ampères* respectively. For these reasons the instruments are called '*graded* galvanometers. A very full account of them is given in Gray's 'Absolute Measurements in Electricity and Magnetism.' They have not, however, for reasons alluded to in § 17, satisfied practical requirements. Sir W. Thomson has since invented other instruments, alluded to at the end of Chapter XXV.

The volt-meter galvanometer.—The general principle of a voltmeter galvanometer has been already explained in Chapter XIV. § 11.

In the figure C is the coil. This has a great number of turns of wire, and a large resistance, generally exceeding 5,000 *ohms.*



M is the *magnetometer*, or system of needles. This consists of a system of short needles, parallel to one another, disposed symmetrically about a common pivot. We can consider this system to be equivalent to one short needle, and shall often speak of it as 'the needle.' To this is rigidly fixed a long and light aluminium index, standing at right angles to the direction of the needle. It is this index that is seen in the figure. It moves over a scale properly graduated.

Over the magnetometer and resting on the magnetometer box, but removable at pleasure, is seen a semicircular magnet. By the use of

this magnet the '*restoring-field*' (which would otherwise be only the earth's horizontal field H) may be much increased in strength. Further, when the magnet is used, any error in the measurement of H at any particular place will be a smaller percentage of the whole field than it would be if H alone were the whole field, and so will cause a smaller error in the measurements made with the instrument, providing that the field due to the magnet is known. (This latter has, in practice, to be re-determined from time to time.) But it must be remembered that, according to the principle of § 9, any strengthening of the restoring-field diminishes the sensibility of the instrument.

The instrument is so constructed that the axis of the coil C, along which runs the central line of force (*see* (III.) of § 2), passes through the point of suspension of the needle, and thus the current tends to direct the needle along the line of the axis of the coil.

On the other hand, the magnet is so placed that its magnetic axis passes through the point of suspension of the needle, in a direction perpendicular to the last.

When used, the instrument is levelled, and is so placed that the planes of the coil and of the magnet are in the magnetic meridian. When this is the case, the index should stand at *zero* when no current passes, and equal currents in opposite directions should give equal deflexions from zero.

In this position we have as a *restoring-field* the earth's horizontal field H, to which may be added at pleasure the stronger horizontal field due to the magnet, coinciding in direction with H. The *deflecting field* due to the coil is horizontal, and is perpendicular to the last field. Thus the theory of the instrument, since its needle is relatively short and moves in a uniform field, is that of the tangent galvanometer.

The sensitiveness of the instrument can be varied, both abruptly, by means of the magnet, which may be used or removed, and also, step by step, by sliding the magnetometer box nearer to, or further from, the coil. The feet of the box slide in a groove in such a way that the whole moves parallel to itself, while the point of suspension of the needle moves along the axis of the coil. Instructions as to the conversion of the readings into *volts* under all conditions of sensitiveness are sent with each instrument.

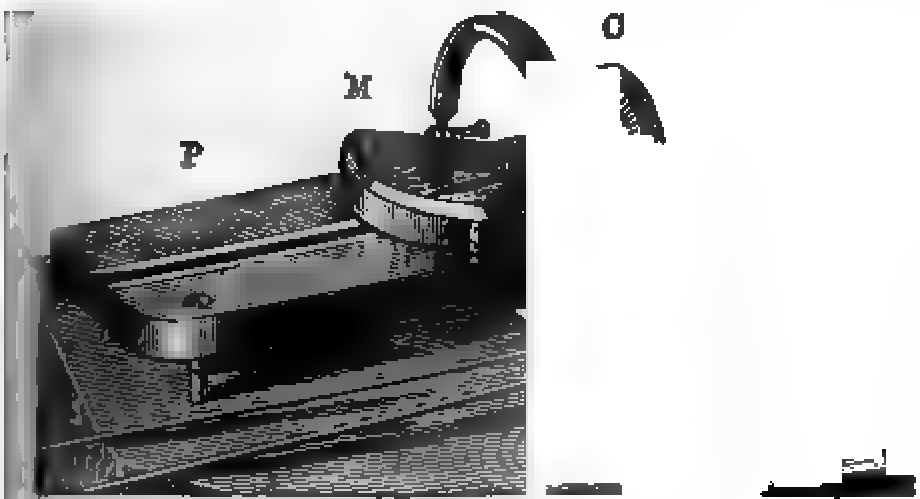
The terminals are of an ingenious construction, designed partly to obviate accidents in measuring very large ΔV.s, and the 'leads' are so arranged that the current passing through them has no action on the needle.

§ 14. **Sir W. Thomson's Graded Current Galvanometer.**—In the construction of the last instrument it was desired to measure

the ΔV between two points without appreciably diminishing this ΔV by lessening the resistance between them. Hence we saw that the coil was made of a very high resistance.

In the present case it is desired to measure currents without appreciably diminishing them by the introduction of resistance. The coil, therefore, must be of as low a resistance as is consistent with the requisite sensitiveness.

The *current galvanometer*, or *am-meter*, is in general construction almost identical with the last. But the coil is composed of a few turns



of thick copper wire, or copper strip, of a resistance equal to about $\frac{1}{1000}$ *ohm*. There is also a special arrangement of the terminals, to obviate inconveniences or accidents in dealing with large currents, and the 'leads' are so arranged that the current passing through them has no action on the needle.

§ 15. Weber's Electro-Dynamometer.

It is sometimes desired to measure currents by some instrument that does not depend upon the magnitude of the earth's field of force at any particular place—in fact, to construct an instrument that can be used without reference to the variable quantity 'H,' that has appeared in the formulæ of the preceding instruments.

Now we shall see in later Chapters that a coil carrying a current acts as a magnet of the same shape whose poles answer to the two faces of the coil. It is found that the magnetic moment of such a coil depends solely upon its shape and the strength of the current flowing through it; so that, as long as these remain constant, we have an absolutely invariable magnetic needle.

Let the large coil of § 5 (or one with more turns of wire, if suitable for the purpose) stand with its plane vertical and coinciding with the

plane of the magnetic meridian. And let there be suspended at its centre by a 'torsion wire' a relatively small coil, whose *faces* are turned due magnetic N and S, and whose plane therefore is perpendicular to that of the larger coil.

If a current be passed in the right direction through the small moveable coil, it will be in equilibrium with respect to the earth's horizontal field, since its faces (or poles) are turned N and S. If another current be passed through the larger coil, the field due to this will act on the smaller coil, tending to set it parallel to the larger. We *might* have given the small coil free suspension, and have used the small coil as we used the needle in § 5. But since we wish to get rid of H from the formula of calculation, we twist the torsion wire until we bring the small coil back to its zero position, in which the earth's field did not act on it. The angle of torsion then measures the couple with which (to speak intelligibly enough, even if not very accurately) the large coil acts on the smaller.

We have then a balance between—(i.) the torsion of the wire on the one hand, and (ii.) the electro-magnetic couple between the coils on the other. The former can be determined by ordinary mechanical methods, and the latter depends (in a manner known to those who have studied the theory of electro-magnetic actions) on the dimensions of the coils, and on the product of the strengths of the two currents. Thus, for each instrument a formula can be once for all constructed, giving the value of one current when the other is known.

If we pass the *same* current through both coils, then we can measure this current by measuring only the above given angle of torsion. This last form of the instrument is adapted for the measurement of currents that pass in rapidly alternating directions, since in this case the reverse current in the large coil finds a needle of reverse polarity upon which to act. The angle of torsion will in this case be proportional to the square of the current-strength.

§ 16. **Some General Observations on Galvanometers.**—We will end this Chapter with a few general remarks on galvanometers.

(I.) *Long-coil and short-coil galvanometers.*—Fleeming Jenkin uses these two very clear terms to indicate instruments in which the current passes many times, or few times, round the needle respectively.

For obvious reasons we cannot increase the field acting on the needle by the device of passing the current many times round without at the same time unavoidably increasing the *resistance* of

the instrument, for we cannot make coils having many turns of any but fine and therefore highly-resisting wire.

In using an instrument we must, then, consider whether the resistance R of the rest of the circuit be large or small as compared with the resistance G of the galvanometer. If G be negligible as compared with R, the field is multiplied n-fold by using n turns of wire, and the deflexion for a given current is thus made much greater. But if R is negligible as compared with the resistance of one turn of wire, then we get but $\frac{1}{n}$th of the current when we use n turns, and thus gain nothing but inconvenience in using a *long-coil* instrument.

§ 17. **Galvanometers for Practical, or Commercial, Use.**

Now that electrical measurements have to be made every day in places where the presence of large magnets and powerful electrical currents renders the magnetic field unknown and variable, it is necessary to have instruments whose action is independent of the earth's field of force, and whose accuracy is not perceptibly impaired by such disturbances as are likely to occur.

At the end of Chapter XXV. will be found some explanation of the principles of several of the more recent types of instruments.

§ 18. **Calibration of Current-meters and of Voltmeters**

The system of units employed in modern electrical science is based upon the electro-magnetic actions of currents, as will be explained in Chapter XVIII. And the practical unit of current employed, the *ampère*, can be determined by observing the attraction or repulsion between accurately constructed coils of wire carrying a steady current. [It was (*e.g.*) in some such way that Lord Rayleigh measured in ampères the currents he employed.] The ampère being thus measured electro-magnetically, it can be determined how much silver or copper is set free in 1 second by a current of 1 ampère.

From the experiments of Lord Rayleigh and others it appears that *about* 1·118 *milligramme of silver is set free by* 1 *ampère in* 1 *second.*

In practical text-books the student will find it explained how this result renders it a comparatively easy matter to calibrate any given *ammeter* in *ampères.*

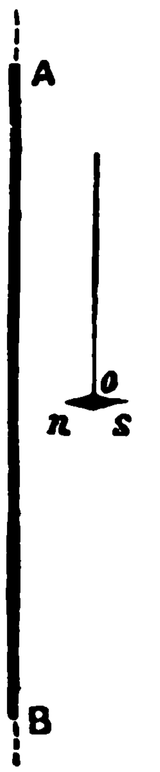
As regards *voltmeters*, we may [*see* Chapter XIV. § 11] graduate them also by means of currents measured electro-chemically. Or we may make use of the known *E.M.F.s* of certain standard cells; these E.M.F.s having been previously determined once for all, in absolute value, by long and careful laboratory experiment.

For further on 'Calibration' we refer the student to practi(text books.

CHAPTER XVIII.

ACTIONS BETWEEN CURRENTS AND MAGNETIC POLES.—MAGNETIC EQUIVALENT OF A CURRENT.—ACTION BETWEEN CURRENTS AND CURRENTS.

§ 1. **Action of an Infinite Rectilinear Current on a Magnetic Pole.**—If we pass a current through a vertical rectilineal wire A B, and observe its action on a horizontally-balanced magnetic needle *n s* placed near it, we arrive experimentally at several important results.



(i.) We find (as also was indicated by the experiments of Chapter XVII.) that the lines of force form circles whose centres lie on the wire and whose planes are perpendicular to it.

(ii.) We find that when once the wire is so long that it makes the angle A O B nearly 180°, then, as far as its action on the needle is concerned, it is practically infinite.

This suggests the idea that the current gives a field at O only so far as it gives a 'broadside-on' projection when viewed from O; and that a current running directly towards O would give no field there.

(iii.) We find that the field-strength at O is directly proportional to the current-strength, as has been otherwise shown in Chapter XVII. § 4.

(iv.) We find that, for the same current, the field-strength is inversely proportional to the perpendicular distance from O to the wire.

§ 2. **Action of an Element of a Current on a Pole.**—If we wish to be in a position to calculate the action of a current of any shape on a pole having any position with respect to the current,

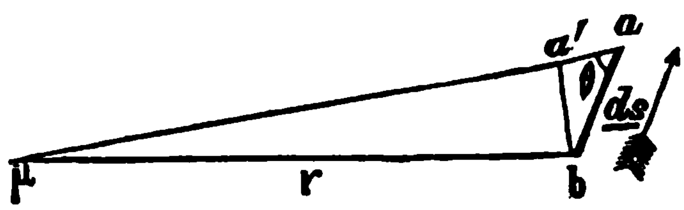
we must know the action of each little bit (or *element*) of the current on the pole, and must sum up all these actions in order to get the total action of the entire current.

Such summing-up belongs to the *integral calculus* in general; but in one case it can be done very simply.

For the benefit of those readers whose mathematical knowledge is elementary, we shall explain further the meaning of an *element of a current*, and shall then discuss in a simple manner the law of action that is found to hold, and the application of this law to determine the field at the centre of a circular current (*see* § 6).

Let $a\,b$ represent a bit of a current so small that the following results may be considered to be true.

(i.) $a\,b$ may be considered to be straight, although it be part of a curved circuit. Its length will be designated by ds, where in general ds is a very small fraction of a centimètre.



(ii.) The length $\mu\,a$ (where μ represents the position of the pole acted on, the pole-strength being μ units) may be considered as equal to $\mu\,b$, in whatever position $a\,b$ stand with respect to μ. Thus, in the figure, r *centimètres* is the length of either $\mu\,a$ or $\mu\,b$. This is because we suppose ds to be exceedingly small with respect to r.

(iii.) If we draw $b\,a'$ so as to make an isosceles triangle $b\,\mu\,a'$, we may consider $b\,a'$ to be at right angles to both $\mu\,a'$ and $\mu\,b$.

(iv.) And, following from (iii.), we may speak of $b\,a$ as making 'the angle θ' with either $\mu\,a$, or with $\mu\,b$ produced; we shall generally say 'with r.'

When the above conditions hold, $a\,b$ is called an *element*. These conditions do *not* hold in the figure, since that is exaggerated in order to make details sufficiently plain.

From the experiment of § 1 it was not difficult for mathematicians to guess at the law of action of each element of current on the pole or on the short needle. The law guessed at was such as, applied to the case of § 1, would enable us to predict the results there experimentally arrived at. It was then applied to other cases. The total action of a circuit was predicted by means of the *integral calculus* from the assumed law of action of an element. In each case thus tested, experiment fulfilled the prediction. When any assumed law stands thus the test of experiment,

we must be satisfied to consider it to be true. We now give the law, thus established, of the action of an element of current on a pole.

Let μ be the pole-strength, r the distance in *centimètres* of the element $d\,s$ from the pole, $d\,s$ the length of the element, $d\,s\,.\,sin\,\theta$ the projection of $d\,s$ perpendicular to r (or the length $b\,a'$ in the figure), C the current-strength. Then it is found that *the pole is urged with a force proportional to* $\frac{\mu\,.\,C\,.\,ds\,.\,sin\,\theta}{r^2}$ *in a direction perpendicular to the plane containing the element* $d\,s$ *and the pole* μ, *the direction of this force being further determined by Ampère's rule.*

If we analyse this law with the aid of the figure given, we see that

1stly. The force acting on μ is directly proportional to μ and to C. This fact is what we should expect. In the last Chapter it was partly proved, and partly assumed as following from the definition of μ and C.

2ndly. For the same length $d\,s$ of the element, the force is proportional to $sin\,\theta$; that is, the element acts solely in so far as it presents a 'broadside-on projection' (which is $a'\,b$ in the figure) to the pole.

3rdly. The force varies as $\frac{1}{r^2}$; thus following the same law as we found to hold in gravitation, in electrostatics, and in magnetism.

4thly. If the current have the direction of the arrow, and the pole be + or north-seeking, this latter is urged straight up from the plane of the diagram.

5thly. The action is not in the line joining the pole and the element, but is perpendicular to the plane containing the two. The action then is very different from any with which we had to do in electrostatics or in magnetism.

Note on the experiment of § 1.—In the case of the infinite rectilinear current, we found the total field at $n\,s$ to vary as $\frac{1}{r}$, whereas the elemental law has $\frac{1}{r^2}$. This may perhaps perplex the learner.

The reason is as follows. As we recede further from the current A B, we certainly increase the distance from each element. But at the same time the upper and lower parts of the wire begin to present an increasingly great 'broadside-on projection' to the pole or needle.

§ 3. **The Absolute System of Electro-Magnetic Units.**—The reader should at this point look again at Chapter XIV. § 1, and Chapter XV. § 2. We intend now to explain what are the fundamental units to which we there referred.

(I.) *Unit of current.*—If a wire be bent into the arc of a circle, and a pole be placed at its centre, we see by the results of § 2 that the forces due to all the separate elements will act in the same direction on the pole; and, further, since each element is the same distance from the pole, the *total force* on this will be simply proportional to the *current-strength*, the *pole-strength*, the *length of the arc*, and *the inverse square of the radius.*

Now we have already defined the units of *force*, *pole-strength*, and *length.* It is, therefore, simplest to define the electro-magnetic unit of current to be such as makes the force unity when all the other above quantities are unity. Or . . . . .

The electro-magnetic unit of current is such that flowing through an arc of unit length (*i.e.* 1 centimètre), *whose radius is unity* (*i.e.* 1 centimètre), *it acts with unit force* (*i.e.* 1 dyne) *on a pole of unit strength* (*see* Chapter II. § 7) placed *at the centre.*

As has been already stated, the practical unit, called the *ampère*, is one-tenth of the above absolute unit.

(II.) *The unit of E.M.F.—If the E.M.F. between two points in a circuit be such that unit current flowing for unit time does unit work between these two points, then this E.M.F. is called unit E.M.F. in the electro-magnetic system.*

It will be evident to the student that this definition of unit E.M.F. is, in theory at least, simple and clear; for the work can be conceived of as measured by observation of the number of calorimetric heat units given out between the points (as in Chapter XV. § 4); while the current can be measured in the above given absolute units by means of a galvanometer of known constants.

Such a definition of unit E.M.F. is far better adapted to the requirements of electro-dynamical measurements than is the electrostatical definition given earlier.

As has been already stated, the above given unit is inconveniently small. The practical unit, called the *volt*, is 10^8 times this absolute unit.

(III.) *The unit of resistance.*—Ohm's law defines the *un* *resistance as that through which unit E.M.F gives unit cur* The practical unit of resistance, called the *ohm*, is 10^9 time absolute unit. Thus, as stated, we have . . . .

$$\text{The ampère} = \frac{10^8}{10^9} = \frac{1}{10}\text{th absolute unit of current.}$$

§ 4. Summary of Electro-Magnetic Units (*see* § 3).

Absolute unit of current . . . . as defined
Practical unit of current, the *ampère* . . . . $= \frac{1}{10}$ absolut

Absolute unit of E.M.F. or of Δ V . . . . as defined i
Practical unit of E.M.F., the *volt* . . . . $= 10^8$ absolut

Absolute unit of resistance . . . . as defined in
Practical unit of resistance, the *ohm* . . . . $= 10^9$ absolut

Absolute unit of activity is the rate of work when unit curren under unit E.M.F., and is 1 *erg per second.*
Practical unit of activity, the *watt*, is the rate of work wh *ampère* works under one *volt* E.M.F.; it $= 10^7$ *ergs per sec*

We get *watts* by multiplying the *number of volts* by the *num* *ampères*; or, if we are considering only the heat given ou conductor, by $(\textit{number of ampères})^2 \times (\textit{number of ohms})$.

1 watt = about ·24 *calorie per second,* or about $\frac{1}{746}$ *English horse-power.*

Absolute unit of quantity is that which crosses any section of ductor in one second when absolute unit of current flows.
Practical unit of quantity, or the *coulomb*, is the same wh *ampère* flows.

Absolute unit of capacity is that of a condenser in which the charge is absolute unit of quantity when the ΔV of the pl absolute unit of E.M.F. or ΔV.
Practical unit of capacity, or the *farad*, is that of a conder which the bound charge is *one coulomb* when the ΔV of the is *one volt.* We may add that the more usual practical uni *micro-farad* (*see* below).

The *meg*ohm, *mega*-volt, &c., are respectively 1,000,000 tim ohm or volt, &c.
The *micr*ohm, *micro*-volt, &c., are respectively $\frac{1}{1000000}$ of th or volt, &c.

Note.—Determination of the units.—The three quantities, C, R, and E, 'e related to each other by Ohm's law. It may be well to indicate how the osolute units, or any convenient multiple of them, might be determined idependently of one another.

Current.—When the dimensions and construction of a galvanometer are nown, and when the magnetic field H in which it is situated has been deterined by the method indicated in Chapter III. §§ 15 and 16, it is possible to ieasure current in absolute electro-magnetic units by observations of its action pon the galvanometer needle. For such a measurement no knowledge of E r of R is either directly or indirectly implied.

Resistance.—The absolute measurement of the resistance of a conductor is ne of the most important experiments in physical science, and one of extreme ifficulty. Of the several methods that have been employed to obtain a *direct* ieasure of the resistance of a wire, the simplest consists in passing a current irough the wire which is contained in a calorimeter similar to that shown in ie figure on page 241. Theoretical considerations give for the heat developed er second in the calorimeter the value $JH = C^{2}R$. Here C is the current, the alue of which can be calculated from the indications of a tangent galvanoieter placed in the circuit; H is the heat developed per second, which can be ound from the rise of temperature of the liquid by the principles of calorimetry; nd J is the mechanical equivalent of heat (p. 239), which is known from oule's experiments to be very near to 41,750,000. The only quantity remainig is *R*, which can therefore be calculated from the other three. Other methods epend upon the laws of electro-magnetic induction given in p. 357; for the etails of these the student must refer to more advanced works upon the subject.

Electromotive force.—Having methods of measuring current and resistance bsolutely, E.M.F. may be found by the arrangement shown on p. 234. *e* is he cell whose E.M.F. is required; a tangent galvanometer is placed between l and B, and the current Ae_0GB is not required. The current given by the iattery P is first adjusted to a convenient strength by the rheostat R, then the ontact piece Q is moved along the wire A B until no current flows in G. Vhen this is the case the reasoning of § 13, p. 234, shows that $e = aC$; *a* being he resistance of the wire A Q, and C the current in the circuit P R B Q A P, vhich is given by the tangent galvanometer: thus *e* is determined.

§ 5. The Dimensions of the Derived Units.

When any system of units has been constructed, each derived unit :an be expressed in the fundamental units. The manner in which :ach derived unit involves the fundamental units can be exhibited in simple form, giving what is called the *dimensions* of that physical uantity. Thus, in Chapter II., the *dimensions* of *force* were given by he relation . . . .

$$F = M.L.T^{-2}.$$

In any one system of units physical quantities are of different iatures if their dimensions (*i.e.* the way in which they involve the

fundamental units) are different. This statement may be taken as axiomatic.

But the same physical quantity may have different *dimensions* in two different systems of units respectively. This is not so easy a matter to understand, but the fact is clear enough, as will be seen from the table given on the next page. Thus the dimensions of *current* are different according as we regard it from the point of view of electrostatic quantity (*see* Chapter V. § 1, passing across a section in unit time, or from the point of view of magnetic actions.

If two physical quantities have different dimensions in the same system, it follows that not only are they different in essential nature, but that in general any alteration of the fundamental units will alter in different ratios the numerical values of the two quantities.

On p. 301 we give a table exhibiting the dimensions of the various physical quantities in the two systems, *electro-magnetic* and *electrostatic*, respectively. It would be out of place in this Course to show how the dimensions of each are found, but we give one example.

Example.—The dimensions of magnetic pole-strength.—By definition and experiment we have . . . .

$$\textit{force between two equal poles} = \frac{(\text{pole-strength})^2}{(\text{distance between them})^2}.$$

Using symbols to represent the units of pole-strength, force, and length, we may write . . . .

$$\mu^2 = \text{F} \times \text{L}^2 = \frac{\text{M L}}{\text{T}^2} \times \text{L}^2 = \text{M L}^3\text{T}^{-2}.$$

And therefore $\mu = \text{M}^{\frac{1}{2}} . \text{L}^{\frac{3}{2}} . \text{T}^{-1}$ gives the dimensions required.

With respect to the fact that in the electro-magnetic system the dimensions of R are L T^{-1}, or are those of a velocity, the reader is referred to Chapter XXI. § 7 (II.), note.

The ratio-velocity v.—We must now call attention to the very remarkable fact that when a quantity is expressed both in electrostatic and in electro-magnetic measure, the ratio between the two sets of dimensions always involves simply the dimensions of a *velocity*; this we have designated by v in the last column of the table on the next page. Direct experiment shows (within the limits of experimental error) that this velocity is a constant.

If C and C′ represent the numerical magnitude of the same current measured electro-magnetically and electrostatically respectively, and if the same convention be assumed with respect to the other symbols, then by the table on the next page we have . . . .

$$v = \frac{\text{Q}'}{\text{Q}} = \frac{\text{C}'}{\text{C}} = \frac{\text{E}}{\text{E}'} = \sqrt{\frac{\text{R}}{\text{R}'}} = \sqrt{\frac{\text{K}'}{\text{K}}}.$$

Table of Dimensions of Units.

Symbol	The magnitude measured	In the (magnetic or) electro-magnetic system	In the electrostatic system	Ratio of latter to former
μ	Strength of magnetic pole	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}$	—	—
V	Magnetic potential . . .	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$	—	—
I	Strength of magnetic field	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$	—	—
m	Magnetic moment . . .	$L^{\frac{5}{2}}M^{\frac{1}{2}}T^{-1}$	—	—
ρ	Intensity of magnetisation or (magnetic moment) ÷ (volume)	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$	—	—
Q	Quantity of electricity . .	$L^{\frac{1}{2}}M^{\frac{1}{2}}$	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}$	$L^{1}T^{-1}$ $(=v)$
C	Current	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}$	$L^{1}T^{-1}$ $(=v)$
V, ΔV and E	Potential, difference of potential, and electromotive force	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}$	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$	$L^{-1}T$ $(=\frac{1}{v})$
R	Resistance	$L\,T^{-1}$	$L^{-1}T$	$L^{-2}T^{2}$ $(=\frac{1}{v^2})$
K	Capacity	$L^{-1}T^{2}$	L^{1}	$L^{2}T^{-2}$ $(=v^{2})$
σ	Specific inductive capacity	—	[a simple ratio or number; zero dimensions]	—

The value of the constant v has been determined from double measurements of the same quantity, the same E.M.F., or the same capacity, respectively; and each investigation agreed in giving to v a value approximately agreeing with the velocity of light, and of the propagation of electro-magnetic induction (*see* Chapter XX. § 18). It also agrees with the velocity with which two bodies carrying electrostatic charges must move parallel to each other in order that the electrostatic repulsion between them may just balance their electromagnetic attraction (*see* Chapter XXII. § 15 (ϵ)). It would appear that when rapid motion is given to electrostatic charges of electricity, these give electro-magnetic fields as do currents; but the velocity given to them must be very great in order to render this action at all sensible.

We may then regard the velocity v as one of the great constants in nature, that which expresses the physical constitution of the ether with respect to the propagation of waves in which the vibrations are transverse.

§ 6. **Field due to a Circular Current.**—We will now complete what was said about a circular current in Chapter XVII. § 2. Near the wire the lines of force will practically be small circles about the wire, as in Chapter XVII. § 1, since the rest of the circuit will be relatively unimportant. But in mean positions the lines will be the resultants due to the whole circuit, and will not be circles, but curves of some other form situated symmetrically with respect to the central line and to the plane of the circular current.

Field at the centre of the circular current.—Now consider the field at the centre of the circle. Referring to §§ 2 and 3, we see that the following considerations will determine the field.

(i.) All the elements of this circular current are the same distance r from the centre, and therefore the field-strength at the centre is directly proportional to the length $2\pi r$ of the circumference.

(ii.) The field will also be proportional to the inverse square of the common distance r, or to $\frac{1}{r^2}$, where r is the radius of the circle.

(iii.) If the current be given in electro-magnetic measure, it will be easy to find the magnetic field-strength at the centre, or the number of *dynes* with which unit pole would there be urged in a direction perpendicular to the plane of the circle.

From the above it follows that if c be the current in absolute measure, and I the magnetic field-strength at the centre, then . .

$$\mathrm{I} = \frac{2\pi r . c}{r_2} = \frac{2\pi . c}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(i.)}$$

The force with which a pole of strength μ is urged will be . .

$$\mathrm{F} = \frac{2\mu\pi c}{r} \textit{ dynes} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii.)}$$

The couple acting on a *small* needle of magnetic moment m, placed at the centre and lying in the plane of the circuit, will be .

$$m\,I = \frac{2m . \pi . c}{r} \textit{ units of couple} \quad . \quad . \quad . \quad . \quad . \quad \text{(iii.)}$$

If the current C be given in *ampères*, we must divide each of the above results by 10.

§ 7. **Magnetic Shells; and the Fields due to Them.**—If we regard only the external magnetic field produced by an electric circuit, we find that we can replace the circuit by a thin steel sheet

ıose faces are of opposite polarities; such a sheet magnet, *magnetic shell*, giving exactly the same magnetic field as did the rren', provided that certain conditions as to its thickness and degree of magnetisation be observed.

But it will be necessary first to give some account of the .ture of a *magnetic shell*, or, as it is otherwise expressed, of *mellar* distributions of magnetism. We return to 'currents' § 9.

Consider an enormous number of small straight magnetic :edles placed side by side, until they form a sheet whose area is :ry large as compared with the length of each little needle; that , a sheet very large as compared with its thickness. If the needles : all turned one way, so that the one side of the sheet be rmed of their north poles and the other side of their south poles, ıen we have what is called a *lamellar* distribution of magnetism; r we have a *magnetic shell*.

In the case of magnetic needles, whose length was very great as ɔmpared with their thickness, it was convenient to speak of their ɔles and of their *length*; the product of the length and pole-:rength giving us that *magnetic moment* of the needle (called $\mu\, l$ r m) upon which so much was found to depend. The reader ill see from Chapter III. § 16, and from Chapter XVII., that pon the magnetic moment of the needle depended the external eld that it gave, and also the couple with which a magnetic field :ted upon it in turn.

Now in the case of a magnetic shell we must express ourselves ɔmewhat differently. We must here consider the intensity ρ of the ırface magnetisation, and the thickness t of the shell.

To take the case with which we shall be concerned, viz. that f a uniform shell, we may suppose all the little needles to ıve been equal both in length t, and in pole-strength. Then, if ıe sum of all the little poles that form 1 *square centimètre* of the ırface of the shell would, if concentrated at one point, form a ngle pole of strength measured by ρ units, we may very naturally ıy that the *density of magnetism* on the surface of the shell is easured by ρ; the word 'density' being here used in the same ınse as in electrostatics; viz. quantity [in this case magnetic ɿantity] per unit area.

If the thickness of the shell be called t, the product $t\,.\,\rho$ is

called the *strength* of the shell. It is the same as the magnetic moment of a needle of length t and of pole-strength ρ, representing the concentrated magnetism of 1 square centimètre of the surface. We will use the single symbol j for the *strength* of a shell.

Now let us consider any point P external to the shell. Let the boundary of the shell subtend a solid angle[1] measured by Ω at the point P; and let the strength of the shell be j. Then it can be shown that the field at the point P depends upon j, upon Ω, and upon the way in which Ω alters in value as we move from point to point. As long as the thickness t is relatively very small, it is found (by mathematical methods belonging properly to the *integral calculus*) that we are concerned with the strength j; not with t and ρ separately.

The reader must notice that (according to the mathematical investigation not given here) the field at P does not depend upon the shape of the shell, so long as its edge remains the same. If the shell be bent into a surface nearly closed, but having a hole of a certain shape and size left unclosed, it will give the same external field as would a shell that just fitted this hole; since the two would have the same edge. And a completely closed shell gives no external field.

§ 8. **Magnetic Potentials due to Magnetic Shells.**—The reader is now referred to what was said of *potential*, and of *force*

[1] *Note on solid angles.*—Let us consider a point P and a sphere of unit radius described about P as centre. Any portion of the surface is said to subtend a *solid angle* at the centre, this angle being measured by the fraction

$$\frac{\text{portion of surface in } \textit{square centimètres}}{(\text{distance from centre in } \textit{centimètres})^2}.$$

Since the radius is one *centimètre*, we may say that the solid angle is measured by the area of the portion of the surface considered, this area being expressed in *square centimètres*. Now let lines be drawn from P to the edge of the magnetic shell. These will intercept a portion of the surface of our unit sphere. The solid angle Ω subtended by the boundary of the magnetic shell at the point P is measured by the portion of the surface, expressed in *square centimètres*, which is thus intercepted on the surface of the unit sphere described about P as centre.

Thus, since the total surface of unit sphere is 4π *square centimètres*, the total solid angle about any point is 4π.

If a point is relatively *very close* to the surface, the surface subtends (very nearly) a solid angle of 2π at the point; and subtends exactly 2π when the point is actually on the surface.

as measured by *rate of change of potential*, in Chapter X. §§ 9-14 and elsewhere. If '+ unit pole' and '− unit pole' be substituted for '+ unit electricity' and '− unit electricity,' all that was there said applies to magnetic fields. (Thus, for example, a pole μ gives out $4\pi\mu$ marked lines of magnetic force.) It is then readily understood that if we know the magnetic potential at a point P due to a magnetic shell or due to an electric circuit, and if we know at what rate this potential changes in different directions, then we know all about the magnetic field at P.

In the case of the magnetic shell, it can be shown that the magnetic potential on unit pole at P due to this shell is measured by the product $j\Omega$. The force at P in any direction will, for the general reasons given in Chapter X., be measured by the rate at which this potential changes its value in this direction; and this, since j does not alter for the same shell, depends upon the rate at which Ω changes in that direction. Thus a small shell close by and a large one far off may give the same value of Ω at P, or the same *potential at P*; but in general the rate at which Ω changes, or the *force at P*, will be different in the two cases.

Inside a completely closed surface formed of a magnetic shell, the potential is everywhere $4\pi . j$, as explained; since now $\Omega = 4\pi$. It does not, therefore, change from point to point; or there is a constant potential and no force in the hollow of such a closed surface. Outside it the potential and force are both *zero*, since, as explained, Ω now equals zero.

As was seen in Chapter X., the direction in which the potential changes most rapidly is the direction of the resultant force, or is the direction of 'the' lines of force of the field.

Of course the potential on a pole of μ units will be measured by $\mu j \Omega$.

It follows also that the work done between two positions of the pole μ, at which the shell subtends solid angles Ω_1 and Ω_2 respectively, will be measured by

$$\mu j (\Omega_1 - \Omega_2).$$

The mathematical investigation of the potentials and fields due to a magnetic shell demands a knowledge of the infinitesimal calculus. But an investigation in which infinitesimal notation, at any rate, is not used, can be found in Cumming's 'Theory of Electricity' (Macmillan).

§ 9. **Magnetic Equivalent of an Electric Circuit.**—We now return to that which was the main reason for the digression of §§ 7 and 8.

If we have on the one hand a circuit carrying a current measured by C in absolute electro-magnetic units, and on the other hand a magnetic shell of the same boundary and of a 'strength' j numerically equal to C; and if we examine the potentials and fields at any external point, due to the two respectively, then it is found that the two give identical results.

The proof of this equivalence demands a knowledge of the infinitesimal calculus; and in the present Course the reader must be content to accept as a result of mathematical analysis confirmed by experiment the statement that . . . .

'*As regards the external magnetic field produced, a circuit of current-strength C* (as measured in absolute electro-magnetic units) *is exactly equivalent to a magnetic shell having the same boundary and having an uniform strength j numerically equal to C.*'

So the formulæ of the last section become, for an electric circuit, $\mu C \Omega$ and $\mu C (\Omega_1 - \Omega_2)$.

Experiment.—De la Rive's floating battery.—A small battery-cell is constructed, composed of two strips the one of zinc and the other copper immersed in dilute acid in a short wide test tube. The circuit is completed by an insulated wire wound into a small circular coil that has its plane vertical when the test tube is vertical. This small cell and circular current is floated on the surface of a large vessel of water, by means of a cork or other float through which the test tube passes.

We thus have a vertical circular current capable both of rotation on its vertical axis and of horizontal translation. If we test it by means of a large bar magnet we shall find that it acts as would a thin circular magnetic shell. As could be deduced from Chapter

XVII. § 2, that face in which (as one stands opposite to it) the current appears to circulate *clockwise* will act as a south-seeking magnetised surface, while the face in which the current appears to run *counter-clockwise* acts as a north-seeking surface. The coil will thus turn until the face of opposite polarity to that of the magnetic pole presented is turned towards this latter, and will then be 'attracted' and move towards it.

If the current be strong and the water be still enough, the coil will under the earth's action only 'set' with its faces turned north and south respectively.

The equivalence of a magnetic shell and an electric circuit was discovered and stated by *Ampère.*

§ 10. **This Equivalence is for the External Field only.**—There is one obvious difference between the circular current and its 'equivalent' magnetic shell. In the circuit, the field is continuous; and a pole is urged *through* the circuit as well as up to it. In the magnetic shell the external field stops at the surface of the shell.

§ 11. **Principle of Sinuous Currents.**—Referring to § 2, we see it stated as the result of experiment that the element *a b* acts on the pole as does the element *a' b.*

Extending this to the case of wires of finite length we should predict that a circuit of any form in which the wire has small sinuosities, loops, and other irregularities, will act on an external pole just as does a circuit of the same general form in which the wire is without such irregularities; provided that the irregularities are of very small size as compared with their distance from the pole acted on.

This equivalence of simple and sinuous wires is called the '*Principle of sinuous currents.*'

Experiment.—Employing the 'zero method,' as before, it is easy to show that a circuit, in which the current goes one way round a simple wire, and returns by another wire wound over the former in small sinuosities, has *zero* action on a pole. This shows that simple and sinuous currents, which have the same general form, are equivalent as regards action on an external pole.

Thus, a complete circuit in which the wire has sinuosities that are very small as compared with the size of the circuit, is equivalent to a magnetic shell of the same average boundary as the circuit and same 'strength.'

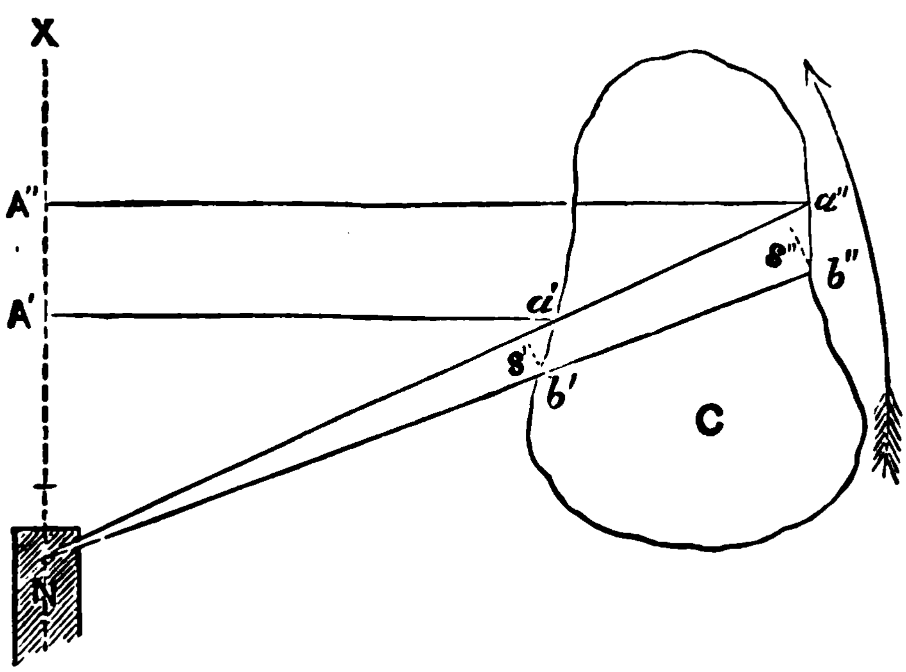
Thus, we may regard the arrangement of the above experiment to be equivalent to two equal shells superimposed on one another,

X 2

the faces being turned contrary ways; such a magnetic arrangement giving neutrality.

§ 12. **Reaction of a Pole on an Element of Current.**—We have spoken of the force exerted on a pole by an element of current, assuming the pole to be moveable and the current to be fixed. But by Newton's laws, which may be regarded as so confirmed by universal experience that they are *axioms*, there must be between the pole and the element a reciprocal action of which either half becomes manifest as we fix the pole or fix the current respectively. The pole and the element are urged *round each other* in a direction given by the 'screw' analogy of § 2. If we fix the pole, then the current will be urged round it, the rotation following the same direction as before. We can thus give the rule that . . . .

'*If we stand in a + pole, so that we are exactly parallel to, and facing, a person swimming with the current in the element* (see Ampère's rule, § 2), *then the element is urged off to our left in a direction perpendicular to the plane containing the element and the pole; the force being measured by* $\frac{\mu \cdot C \cdot ds \cdot \sin\theta}{r^2}$ *as in* § 2.' Of course a − pole gives the force in exactly the opposite direction.



§ 13. **Action of a Pole upon a Closed Circuit.** — What follows applies to any closed circuit; but, since the mechanical reasoning is then simpler, we will take only the case of a *plane* closed circuit.

Let N be the north pole of a magnet, and N A′ A″ X a straight line through N; we suppose the pole to be concentrated at the point N and to be of strength μ. Let the circuit C be so fixed that it can only turn about the axis N X, swinging round upon the points A′ A″ as a door swings upon its hinges.

We will consider whether or no the pole N has any tendency to make the circuit C revolve as a whole round the axis N X.

Take two lines N $a'a''$ and N $b'b''$, so close together that $a'b'$ and $a''b''$ may be considered to be *elements* (*see* § 2), and drop perpendiculars on N X from a'' and a'.

Since the circuit is closed, each such pair of lines will intercept two elements $a'b'$ and $a''b''$, in which the current flows opposite ways.

As seen in § 2, we may, as regards the action of N, replace $a'b'$ and $a''b''$ by the intercepts made on *circles* through b' and b'' respectively; these intercepts are represented by dotted lines, and will be called s' and s'' respectively, their distances from N being called r' and r''.

Now the pole N urges $a''b''$ perpendicularly down into the page with a force measured by $\frac{\mu . C . s''}{(r'')^2}$; while $a'b'$ is urged upwards from the page with a force $\frac{\mu C . s'}{(r')^2}$.

But from plane geometry we have the proportion $s'' : s' = r'' : r'$; and hence the *force on* $a''b''$: *force on* $a'b' = \frac{1}{r''} : \frac{1}{r'}$.

The effect these will have in turning the circuit as a whole about N X as axis depends upon the product of these forces into the arms a''A$''$ and a'A$'$ respectively.

But, again by simple geometry, it is clear that

$$arm\ a''A'' : arm\ a'A' = r'' : r'.$$

Combining this result with that obtained for the two forces, we have that the two moments are equal, being in the proportion of $\left(r'' \times \frac{1}{r''}\right) : \left(r' \times \frac{1}{r'}\right)$ or of 1 : 1. They are also opposite in direction. Hence they neutralise each other. In the same way the whole closed circuit may be divided into pairs of elements that, together, give *zero* moment about N X. Hence the whole has zero moment about the axis N X.

By similar, but more advanced, reasoning it can be shown that for *any* closed circuit [whether plane or not] there is *zero* moment about *any* axis through a pole N.

By the above method of investigation it is shown that a complete circuit has no tendency to revolve as a whole about any axis through the pole of a magnet; of which axes one particular case is the magnetic axis that joins the N and S poles of the magnet. This means, when interpreted mechanically, that the two poles N and S act on the closed circuits in lines passing through the poles;

since, if they did not act in such lines, we should have a 'moment' about axes through the poles. In other words, the action between the circuit considered as a whole and the poles is a *direct action*, and not such as that considered in § 2. This result, arrived at from the consideration of the action of a pole on the elements of current forming the complete circuit, agrees with the view which regards the circuit as equivalent to a magnetic shell. For the action between a pole and a magnetic shell is also a *direct action*.

The line of action between N and the circuit C will pass through some point in C; and, if the circuit be moveable about some axis through which the line of action does not pass, the circuit will revolve into a position of equilibrium, or will 'set itself,' as would a magnetic shell. In this position it will then rest, being prevented, owing to the fixed axis, from moving up to the pole.

§ 14. **Action of a Pole on an Incomplete Circuit.**—If there be such an arrangement that the circuit external to the pole be not

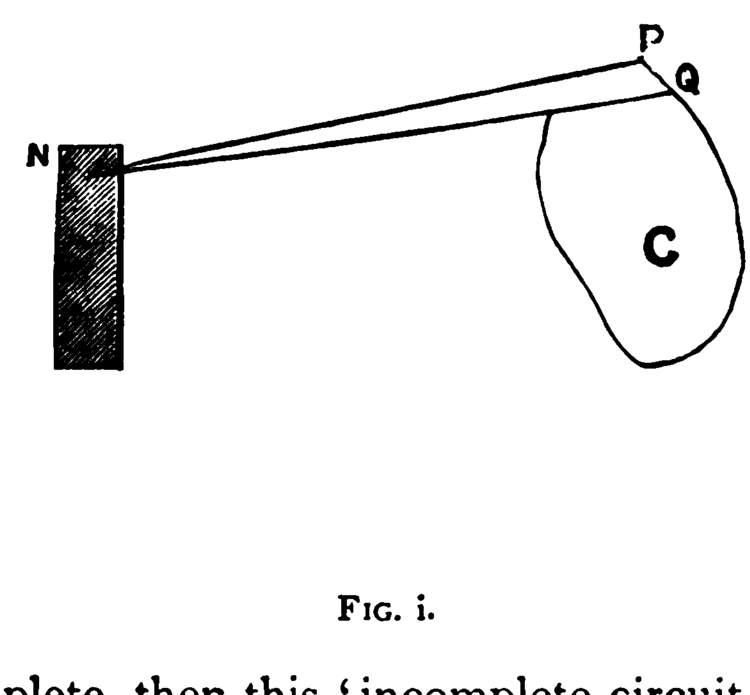


FIG. i.

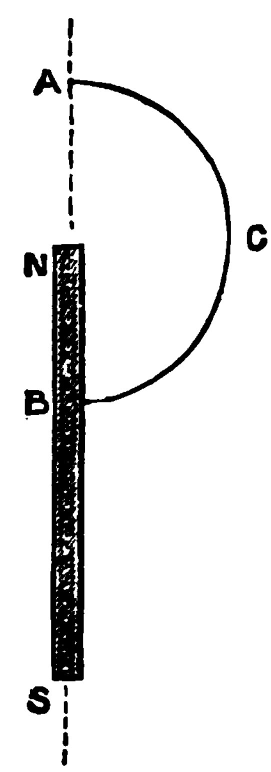


FIG. ii.

complete, then this 'incomplete circuit' *will* be urged round the pole. Every voltaic circuit must be complete; but part of it may be completed through the magnet itself, leaving the portion external to the magnet incomplete.

In fig. i. we see that, as regards rotation round the pole N, the portion P Q of the circuit C is uncompensated, and therefore the whole will be urged round an axis through N.

In practice this can be effected by arranging a circuit A C B A that the part C is mobile about the axis A B ; the part A B of e circuit being the axis of the magnet N S.

It can be shown that if the points A B lie both between N and S, both outside N S, the resultant action of the two poles N and S will such that C does not revolve about the axis. In the case drawn it ill so revolve.

§ 15. **Action of the Earth's Field on Currents Completely r Partly Mobile.**

When currents are arranged so as to be completely or partly mobile bout vertical or horizontal axes, there will be in general movements ue to the action of the earth's field.

If we resolve the earth's field into its vertical and horizontal components, the nature of the movements referred to can be predicted ithout much difficulty, by means of the principles explained in hapter XIX. §§ 4, 5, and 7.

There is, however, no especial interest attaching to these actions, nd we shall accordingly omit further discussion of them.

§ 16. **Actions Between Currents; Ampère's Laws.**—The first aw arrived at by means of direct experiment is very simply expressed. It is as follows.

Law I.—Parallel currents in the same direction attract, parallel urrents in opposite directions repel, one another.

There are many pieces of apparatus by means of which this act can be shown ; the form that is perhaps of the most general se is the 'Ampère's stand.' It is not necessary to describe in letail the arrangements in this or in similar pieces of apparatus. t is sufficient to say that, by means of pivots and mercury conections, wires bearing currents are given, to a greater or less legree, freedom of movement ; and that thus the action of other urrents on them can be observed.

Experiment.—The figure on the next page represents the portion M of a ectangular coil M N acting upon the portion B of the moveable piece B C. As rawn, the currents in M and in B are in opposite directions, and repulsion etween M and B will be observed.

When two wires bearing currents are not parallel, but are inlined to one another, the following law is found to hold (*see* also hapter XIX. § 8).

Law II.—When two currents make an angle with one anoth they attract one another if they run both towards or both from t vertex of the angle, and repel one another if they run the one towar and the other from the vertex.

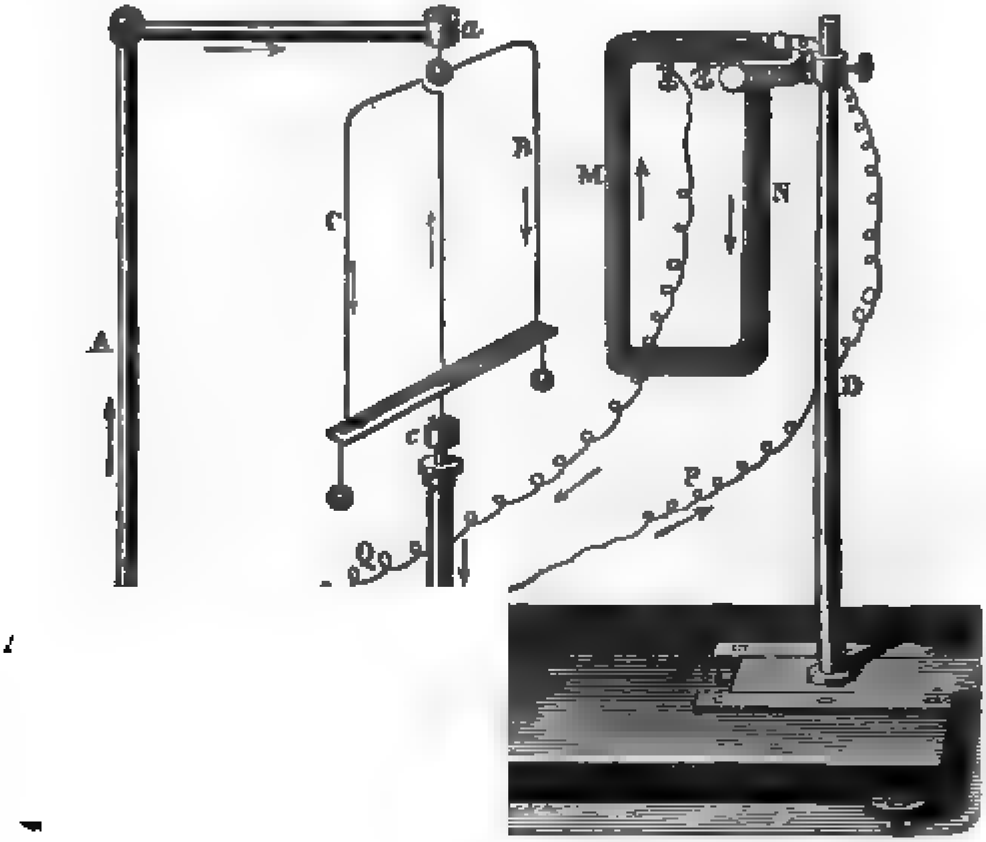


FIG. i.

Or, *two currents crossing one another tend to move into a ,tion in which they are parallel and in the same direction.*

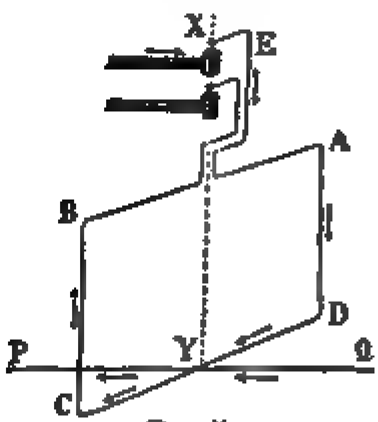


FIG. ii.

Experiment.—This may be illustratec means of the arrangement here indicated. current P Q acts mainly on the portion C the mobile current, and movements ensue v are in accordance with the above law.

It may be observed that P Y acts or portions Y C and C B, and the portion Q Y on D Y and on A D, so as to give moveme one and the same direction. The action o upper part B A will be in a contrary direc but this action will be negligible if B is rela very far from P Q.

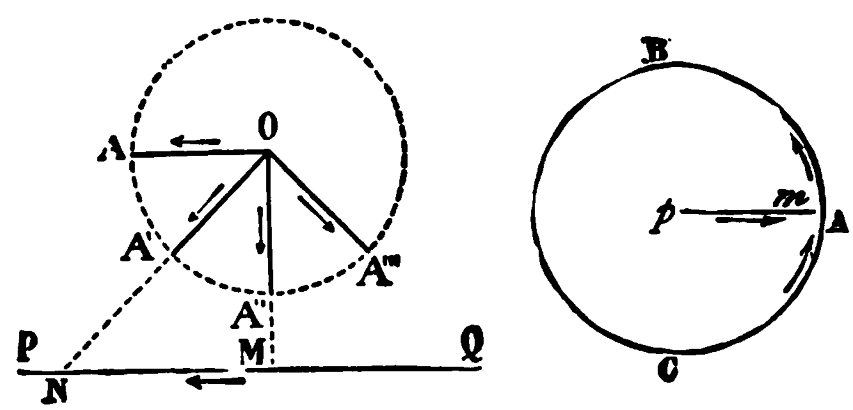
[*Law III.—In a rectilinear current there is repulsion between two consecutive elements of the current.*—Ampère devised experin

in which it appeared that collinear elements of a current repelled one another, as if, indeed, this were an extreme case of Law II., in which the angle between the two currents was 180°. It is, however, doubtful whether any experiment has shown directly this action between two elements of currents that are truly collinear. It is indeed true that calculations based upon the assumption of this action have been verified. But the simple statement of this law, as given above, does not fit in well with the modern theory of electro-magnetic and electro-dynamic actions, and, as no direct application of it will be made in this Course, we think it best to pass it by without laying any stress upon it.]

For the more modern view of Laws I. and II. we refer the reader to Chapter XIX. § 8.

§ 17. Continuous Rotations of Currents.

If O A be a current mobile about O as centre, and if P Q be a rectilinear current, the above first two laws show us that there will be continuous rotation of O A through the positions O A′, O A″, O A‴, &c., round and round.



So again, if A B C be a circular current and pm be a current mobile about p as centre, then from the same laws we can again predict continuous rotation.

Experiments have been devised that illustrate these actions. For the more modern view of the same we refer the reader to Chapter XIX. § 9.

§ 18. Ampère's Laws of the Actions Between Elements of Currents.

By means of experiments on finite currents, and by deductions from these aided by arguments based upon the principle of 'symmetry,' Ampère investigated the laws governing the action of 'elements of current' (*see* § 2) upon one another.

These laws once established, he employed them to calculate the actions of circuits upon each other in various cases, the calculations being as a rule long and laborious. In fact he worked out, in a manner which commands our admiration, a very complete system and method of attacking electro-dynamical problems—a method based upon the 'action at a distance' between the current elements.

Now, however, this method is not employed. Faraday introduced,

and Clerk Maxwell further worked out and systematised, the electromagnetic method of treating all electro-dynamical matters. In this method the currents are treated as magnetic systems giving magnetic fields and acted upon by magnetic fields. This view is far more fruitful than is Ampère's, and probably represents more accurately the physical nature of the actions. The views and methods of Ampère were rather those of a mathematician than of a physicist.

With respect to his views we must, however, allow that for many purposes, *e.g.* for predicting the nature of the movements that will ensue in certain cases, the Laws I. and II. given in § 16 are very convenient; and they should certainly be committed to memory by the student, as expressing and summing up a group of experimental results.

CHAPTER XIX.

LAWS OF THE MOVEMENTS OF CURRENTS AS DEDUCED FROM THE CONSIDERATION OF MAGNETIC FIELDS AND POTENTIALS.

§ 1. **Magnetic Fields and Potentials.**—In Chapter X. we discussed, with reference to electrostatic phenomena, various matters of importance connected with fields of force, lines of force, potentials, equipotential surfaces, and the like. The reader must observe that all the general propositions there given hold good, *mutatis mutandis*, for any field where the forces that give rise to the field vary as $\frac{1}{r^2}$. They hold good, *e.g.*, for magnetic fields. Thus we can test and investigate a magnetic field by the magnitude and direction of the force acting on + unit-magnetic-pole in the various parts of the field: and again, we can map out a magnetic field in just the same way as, in Chapter X. §§ 13, 14, we mapped out the electrostatic field.

So also magnetic-potential-differences can be measured by the work done on + unit-pole, the work being measured in *ergs*. And the 'tubes of force' property given in Chapter X. § 16 applies also to magnetic tubes of force.

§ 2. **Movements are from Higher to Lower Potentials.**—When a pole, system of poles, or electric circuit equivalent to a magnetic system, is placed in a magnetic field, it will as a rule be urged by forces. By the very definition of 'potential' *it will be urged from a place or position of higher, to one of lower, potential*; or it moves from a place or position to which it could have been brought from an infinite distance with the expenditure of *more* work, into a place or position to which it could be brought from infinity with *less* work. This, in the case of electrostatics, amounts only to saying that a + charge moves down, and a — charge moves up, the lines of force; for, in electrostatics, we deal always with

charges that are either + or —. In the case of magnetic fields we may be dealing with simple + or — poles, in theory at least; and with regard to these the same may be said. But we may be dealing with magnetic shells or with electric circuits that are equivalent to magnetic shells; and in such cases the movements may be of both rotation and translation, or of either alone. The italicised statement given above is therefore the more general form of the law.

To make matters clearer we will consider the case of a field due to a + magnetic pole N. A simple + pole placed in this field will move off to infinity from N, and a simple — pole will move from infinity up to N; in both cases the final position is that to which the + pole could be brought from infinity with least positive (or with most negative) work. A magnetic shell placed in the field will turn until its — face is turned towards N, and will then move up to N; the final position being that into which it could have been brought from infinity with the most negative work possible. If the shell were compelled to slide on rails so as to keep its + face towards N, it would then move off to infinity; this being, under the assumed conditions of constraint, the position of lowest potential possible.

Whether the final position of the pole or magnetic system be one of low positive-, of zero-, or of negative-potential, depends upon the conditions of each particular case. But this final position will always be that from which the system could be restored to its initial position only with the expenditure of the greatest amount of work that was, under the existing conditions as to freedom of movement, &c., possible.

§ 3. **Potentials on Poles and on Circuits.**—Where we have magnetic *poles* in fields due to magnetic shells, or to electric circuits equivalent to magnetic shells, it is convenient to use for the *potential on these poles* the expression given in Chapter XVIII. §§ 8 and 9. This expression, *mutatis mutandis*, is derived mathematically from the fundamental form of Chapter X. §§ 7 and 9.

If we have a magnetic shell, or an electric circuit equivalent to this, in a field, it is convenient to derive another form for the expression giving the *potential on the shell.* Referring to the method of mapping out fields, given in Chapter X. § 14, we may consider the number of marked lines of force that pierce the shell or

circuit. These are said to *pierce the circuit in a + direction* if they run into its + face and out at its − face; that is, *if they run in the opposite direction to the circuit's own lines of force.*

By considering various cases the reader will perceive . . . .

(i.) That when a shell or circuit is moved from infinity into a position in which lines of force pierce it in a + direction, then positive work is done on the shell.

(ii.) And when moved into a position in which on the whole no marked lines of force pierce it, or in which [if plane] it lies 'edgewise' to the field, then *no* work is done.

(iii.) And when moved into a position in which the lines pierce it in a − direction, then negative work is done. If then we so move the shell that more lines pierce it in a + direction, or fewer lines pierce it in a − direction, we do positive work on the shell; and conversely. The former position is the one of higher potential.

The foregoing is reasoning of a general nature: we will now examine this matter in a more exact manner. It was stated in Chapter XVIII. §§ 8 and 9 that the potential of a shell or circuit on a pole, or of a pole on a circuit—(this potential must by 'Conservation of energy' be *mutual*)—is given by the expression $\mu\, j\, \Omega$, or $\mu\, C\, \Omega$, respectively. Here C is in absolute electromagnetic units.

Now a pole μ gives out $4\pi\mu$ lines of force all round it, or over the whole solid angle 4π; this following, *mutatis mutandis*, from Chapter X. §§ 14 and 15. Hence unit solid angle will enclose μ of the lines, and a solid angle Ω will enclose $\mu\, \Omega$ lines.

Hence we may write the expressions $\mu\, j\, \Omega$ and $\mu\, C\, \Omega$ in the equivalent forms $j\, N$ and $C\, N$; where N is the number of marked lines, due to the pole μ, that are included within the boundary of the shell or circuit.

Since this expression for the potential on the shell or circuit holds for each such pole as μ, and since we may consider that any magnetic field is due to a system of poles, it follows that when a circuit of current-strength C is in a magnetic field, the potential on the circuit is $\pm$ C N.

So, again, the work done in any change of position will be measured (in *ergs*, of course) by the value of . . . .

$$\pm\, C\, (N_1 - N_2)$$

where N_1 and N_2 are the number of marked lines embraced by the

circuit in the first and second positions respectively; C being expressed in absolute electro-magnetic units. The *sign* of the work done is easily settled in any particular case by the principle explained earlier in this present section.

§ 4. **General Law of Movement of (Magnetic Shells or of) Electric Circuits.**—From §§ 2 and 3 considered together it follows that . . . .

'A circuit (bearing a current) when placed in a magnetic field tends to place itself in such a position that as many lines of force as possible pierce it in the negative direction.'

In other words, it tends to move into the position of greatest negative potential. If so constrained that it cannot turn itself so as to embrace the lines of force negatively, then it will move so as to embrace as few positive lines of force as is possible; or will move into the position of lowest positive potential.

And a circuit, completely free to move, tends to place itself, firstly, so that the lines of the field run with its own lines, or run into its south-polarity face and out of its north-polarity face; secondly, so that as many lines as possible are enclosed by the boundary wire.

§ 5. **The Case of a Uniform Field.**—In a uniform field the lines of force are parallel and equidistant.

Hence a circuit cannot here gain more lines by moving from one part of the field to another; only by turning into a suitable position.

Thus in the earth's uniform field the floating circuit, described in Chapter XVIII. § 9, will only 'set' itself so that its plane, which is vertical, may be perpendicular to the horizontal component of the field; it will have no movement of translation. In this it behaves as would its equivalent magnetic shell.

It is possible to place the circuit so that the field lines run *exactly* against the circuit's lines; in this case it is in unstable equilibrium, and cannot so turn as to obey the above 'law' until slightly displaced.

§ 6. **The Case of a Field not Uniform.**—In this case the circuit can, by moving from one part of the field to another part, embrace more or fewer lines of force.

If quite free to move it will turn itself so that the field lines run with its own lines, and will also move into the strongest part

of the field where the lines are closest together. It will then be embracing as many — lines as possible. If constrained by any means to keep so turned that its lines run against those of the field, it will then move into the weakest part of the field; for there it will embrace as few + lines as possible, which is all that it can do under the assumed conditions.

§ 7. **The Case of Incomplete Circuits.**—What we have said above gives very simply the law by means of which we can predict the general nature of the movements of magnetic shells, or of complete electric circuits equivalent to magnetic shells.

But we have, in Chapter XVIII. §§ 15, 16, and 17, given cases of movements on which it is not at all easy to see at first glance how the general law of § 4 can have any bearing at all. In the cases referred to we seem to be dealing with portions of currents, often rectilinear; not with circuits.

It is more direct, and sometimes simpler, to treat the movements of rectilinear currents (and of other incomplete circuits) from the point of view of the fundamental law of action given in Chapter XVIII. § 12. We may throw the law, there given, into the following somewhat more general form . . . .

'*A rectilinear current placed in a magnetic field is urged to move at right angles to its own length and perpendicularly to the lines of force; the direction of movement being to the left for a person who is swimming with the current and looking down the lines of force.*'

The word 'down' means along the direction in which a + pole is urged; or along the + direction of the lines.

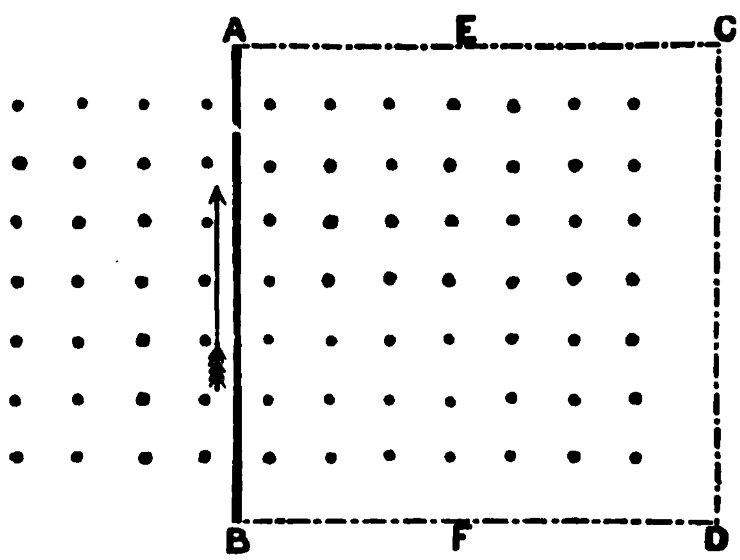
It is, however, important to observe that we can also treat these movements, and the above law, from the point of view of § 4.

If we consider an indefinite rectilinear current, we have as we know a field of force composed of lines that form circles about the wire, the planes of the circles being perpendicular to the wire (*see* Chapter XVII. § 1).

Now if we imagine a plane magnetic shell whose edge coincides with the wire, and which has no other boundary excepting this edge but extends to infinity, it can be shown that the field of force due to this magnetic shell would also consist of lines forming circles about the edge; in fact, it can be shown that the fields due

to the wire, and due to an infinite magnetic shell whos bounding edge coincides with the wire, would be exact same; provided, of course, that the shell has a suitable 'str So long as the edge of the shell coincides with the v makes no difference in what direction from the wire the s supposed to extend. Thus, if the wire were vertical the shell extend to infinity in any azimuth, to N., to S., to E., or to V field given by it remains constant. But it is an accepted pr that if the fields due to two magnetic systems are the sai may for all electro-magnetic purposes consider these system equivalent.

We may thus consider any indefinite rectilinear curren replaceable by a magnetic shell as described above; or, as v fairly deduce from the results summed up in Chapter XVI: we may consider it to be one side of an electric circuit the which is at an infinite distance. We will now show that t of movement given (in above could have be dicted from the pi enunciated in § 4.



In the accompanyi gram the dots repre section a field of foi lines running from down into the plane diagram.

A B represents a rectilinear current, the direction of the current being from

Let A C and B D extend 'to infinity' in a direction e pendicular to the line A B; and let C D, which comple circuit, be at infinity, or at least so far away as to lie outs magnetic field considered.

Then, as far as movements in this field are concern may consider the wire A B to be one side of the 'infinite' A C D B A.

In the case drawn, the circuit 'embraces — lines of forc § 3, end) for the lines of force are running down into a which the current runs clockwise, *i.e.* into a S face. Henc should be a movement of A B to the *left*, so that the circu

embrace more − lines of force, according to the principle of § 4. And this movement, thus predicted, is what would follow from the principle of Chapter XVIII. § 12, as expressed in the italicised law given above.

If the circuit A C D B lay away to the left of A B then the lines would be piercing the circuit in a + direction; and A B would still move to the left, in order that A B C D might embrace fewer + lines of force. And so for all positions of our infinite circuit.

We have thus shown that the simple law of Chapter XVIII. § 12 does fit in with the more general principle of § 4 of this Chapter, though perhaps the agreement may seem a little artificial.

§ 8. **Reconsideration of Ampère's Laws.**—We will now reconsider, from the point of view that we have explained in the present Chapter, the two main laws of currents enunciated by Ampère; viz. the laws of parallel, and of angular, currents. The reader is referred back to Chapter XVIII. § 16, Laws I. and II.

For simplicity we will suppose the currents to be rectilinear and indefinite in length, and we will consider them to be replaced by infinite plane magnetic shells as explained in the last section, or by the equivalent infinite circuit. Each will give a field of the nature explained in Chapter XVII. § 1, and elsewhere. And each circuit will embrace more or fewer + or − lines of force due to the other. If the wires are parallel and the currents run in the same direction, then the lines of each current cut the other circuit negatively; while if the currents run in opposite directions, the lines of each will cut the other circuit positively. This will be true in whatever 'azimuths' the two infinite circuits lie. Hence in the former case we should, from the principles of § 4, predict that the wires would move towards each other; and in the latter case, from each other. In the former case the movement causes each circuit to embrace more − lines of force; and in the latter case the movement causes each to embrace fewer + lines of force. The movements actually observed would thus have been predicted.

In the case of 'angular currents' the same general reasoning would be followed. If the currents are at some small angle, and are in opposite directions, then each circuit is embracing + lines

of force due to the other current. If this angle opens out, the number of + lines embraced is decreased; and it becomes zero when the currents are at right angles to each other, since now each circuit is edgeways to the lines of force due to the other. Further rotation will cause each circuit to embrace − lines due to the other; and the number embraced will be greatest when the currents are parallel and in the same direction.

§ 9. **Cases of Continuous Rotation.**—Referring to the cases of continuous rotation given in Chapter XVIII. §§ 14 and 17, we may say that it is not easy to apply to them in a direct manner the principle of § 4 above. Such direct application will always appear to be forced and artificial.

It is better to apply the italicised law of § 7 of the present Chapter, being content with having shown that this law is in accordance with the more general law of § 4.

Applying this law we see that . . . .

(i.) In Chapter XVIII. § 14, fig. ii., the mobile portion of the circuit is urged continually across the lines of force that radiate from the magnet pole.

(ii.) In Chapter XVIII. § 17 the mobile radial wire is continually urged across the lines of force due to the circular current; these lines, as seen earlier, cutting the plane of the circuit at right angles.

§ 10. **Potentials Due to Circuits** (*continued*).

We have said that, as regards the external field, circuits are equivalent to magnetic shells of the same boundary and 'strength.'

But there is an important difference between the two; inasmuch as, in the case of circuits, a pole can thread its way through the circuit and can come back to its original position. It is clear then that we can do any amount of + or − work on a pole by causing it to perform complete tours through a circuit, without any change in the initial and final solid angle Ω subtended by the circuit. The question arises, What work is done in one such complete tour of a pole μ?

Without going fully into the matter we may explain to some extent as follows. In a plane, a line may perform a complete revolution round a point and may come back to its initial position; it will be in the same place as before, but will have described a plane angle measured by one complete revolution, *i.e.* by 2π. Similarly, when the pole has performed a complete tour through the circuit, the solid angle subtended by the circuit has really changed by what we may

from analogy call 'a complete solid revolution'; and this is a solid angle of 4π.

Hence, if in one position of the pole μ the solid angle subtended is Ω_1, and in another position it is Ω_2, and if further the pole have also performed *a* complete tours in one and the same direction through the circuit, then the solid angle has changed by an amount measured by $(\pm 4a\pi + \Omega_1 - \Omega_2)$; the double sign being due to the fact that the tours may have taken place in the one or the other direction respectively.

Hence, from §§ 8 and 9 of Chapter XVIII., we have for the work done on or by the pole in the above movement the expression . . .

$$\mu C(\pm 4a\pi + \Omega_1 - \Omega_2).$$

If the pole comes back to the same position, so that $\Omega_1 = \Omega_2$, we have for the work the expression . . . .

$$\pm 4a\pi\mu C.$$

In the case of an indefinite rectilinear current, the same formula holds with respect to complete tours round the wire; the case of incomplete tours will not be noticed here. We regard the indefinite rectilinear current as the edge of an indefinitely large circuit; and the work done in *a* complete tours of the pole μ will be given by the formula $\pm 4a\pi\mu C$. [Here C is in absolute electro-magnetic units.]

§ 11. **Potentials on Circuits** (*continued*).

So, again, the expression . . . .

$$C(N_1 - N_2)$$

of § 3 does not give the total change of potential on a circuit due to a magnetic field, if this field be due to a pole which performs tours through and round the circuit.

As in statical electricity (*see* Chapter X. § 14) we saw that $+Q$ gave $4\pi Q$ marked lines of force, so a pole of strength μ gives $4\pi\mu$ lines of force. Each time that the pole performs the tour through and round the circuit, all these lines of force are caused to pierce the circuit. Thus each tour may be regarded as adding $4\pi\mu$ marked lines to those piercing the circuit. Hence, to the above expression we must add $\pm 4a\pi\mu C$, if the pole μ have performed *a* complete tours between the two positions, all in one or the other direction. As before, the infinite rectilinear current may be regarded as a particular case. Hence, *a* complete revolutions of the pole round the current give work measured by $\pm 4a\pi\mu C$.

CHAPTER XX.

SOLENOIDS, ELECTRO-MAGNETS, DIAMAGNETISM, AND ELECTRO-OPTICS.

§ 1. **Cylindrical Magnet built up of Circular Laminæ.**—Just as in Chapter I. § 5 we considered the ideally perfect linear magnet as built up of small linear elements each of which was a perfect magnetic needle, so in the case of solid cylindrical magnets perfectly magnetised we may consider the whole to be built up of a series of thin magnetic laminæ in each of which the magnetic distribution is 'lamellar' (*see* Chapter XVIII. § 7).

Each element will be a magnetic disc in which the one face is uniformly +, and the other uniformly −. The total external action of the whole uniformly magnetised cylinder will be that of the two end surfaces, the one + and the other −.

§ 2. **The Ideal Solenoid.**—Now by what we have stated in Chapter XVIII., a circular current is exactly equivalent to a plane circular magnetic shell. We should suppose, then, that if we built up a cylinder composed of such circular currents, placed with their + faces all turned in the same direction, we should have the equivalent to a cylindrical magnet.

In such a system of plane circular currents the opposed + and − faces of the successive elements would neutralise each other with respect to external action, leaving only the two end faces, + and − respectively, to act.

An arrangement of this sort is called the *ideal solenoid.*

§ 3. **The Practical Solenoid.**—It is not, however, practicable to form such an arrangement of plane circular currents, each complete in itself.

The nearest approach to it that we can make is a current of a spiral form, the turns of the spiral being flat and close together.

By the principle of Chapter XVIII. § 11, each turn A B c of this flat spiral is approximately equivalent to two components: (1) a plane circular current B b c, the projection of the spiral; (2) a linear current of length A c, lying perpendicular to the plane of the circle, and passing through its centre (*see* fig. i.).

Thus, the whole spiral will be equivalent to (i.) an ideal solenoid composed of plane circular currents, its length and diameter

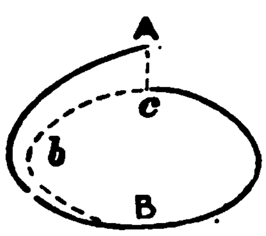


FIG. i.

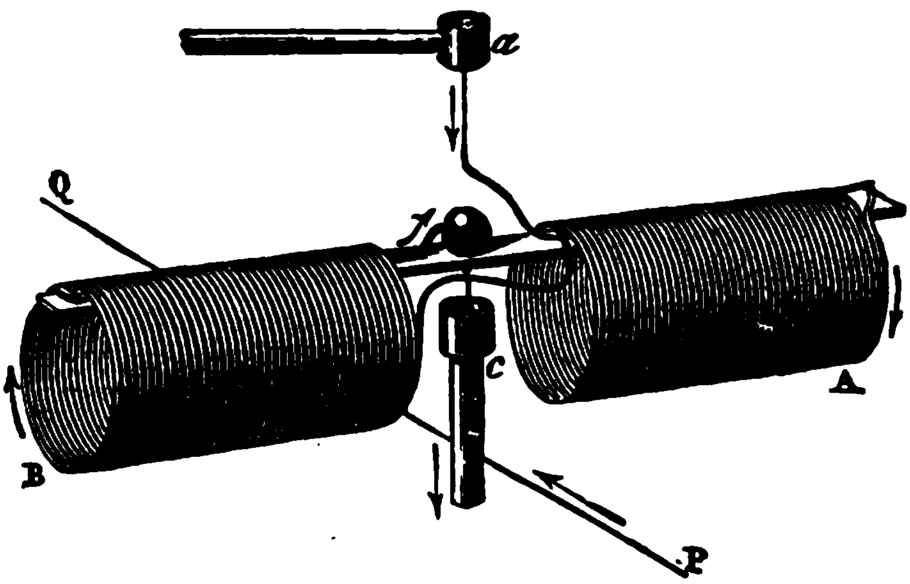


FIG. ii.

being those of the spiral; and (ii.) a linear current, coinciding with the axis of the spiral.

If, then, we cause the current to return along the interior of the spiral, we can, for the external field, neutralise the linear component of the spiral. Such an arrangement, represented in the accompanying fig. ii., is the *practical solenoid.*

A spiral in which the linear component is unneutralised will give us both a field similar to that of a cylindrical magnet, one in which the lines of force lie in planes passing through the axis of the spiral, and also a field similar to that of Chapter XVII. § 1, composed of circles about the axis of the spiral and lying in planes perpendicular to it. These two components give, of course, a single field which is nearly that of a cylindrical bar-magnet, but in which the lines of force do not lie *exactly* in planes containing the axis of the spiral.

Experiments.—(i.) Experiment with a spiral current, in which the linear component is uncompensated, on a balanced magnetic needle. It can be shown that as a whole the action of a spiral, if its coils be flat and close together, is *mainly* that of a weak bar-magnet of the same shape.

Now hold the spiral vertically, so that its axis is situated with respect to

the needle as was the linear current in Chapter XVII. § 1. It will be found that there is an action on the needle showing the existence of the circular lines of force about the axis.

(ii.) Experimenting with 'practical solenoid,' or spiral in which the linear component is neutralised by the return current led through the interior, we find that the second action disappears; the field is that of a weak bar-magnet.

(iii.) Experimenting with one solenoid on another balanced upon an Ampère's stand, we can show the resemblance to the action of two bar-magnets on each other.

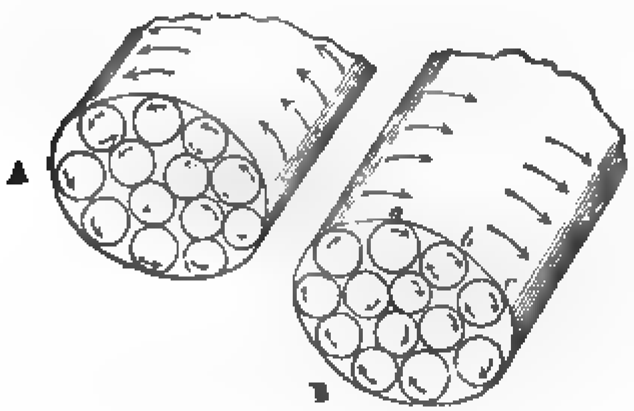
(iv.) A magnet acts upon a balanced solenoid as on a balanced magnet.

(v.) A solenoid suitably balanced will show the declination and inclination.

That end of the solenoid in which, as one faces it, the current runs clockwise will be — or S-seeking; that end in which the current is counter-clockwise will be + or N-seeking.

(vi.) If we hold a solenoid vertically and place a sheet of glass horizontally above it, iron filings will show us lines of force radiating from its poles, as in the case of a bar-magnet. If we treat a common spiral current, with the linear component uncompensated, in a similar manner, the lines of force radiating from the poles will have a slight spiral twist.

§ 4. **Ampère's Theory of Magnetism.**—Struck with the close resemblance between a solenoid and a bar-magnet of the same shape, Ampère constructed a theory of magnetism which should account for this resemblance. He considered that in each molecule of the magnetic substance there circulates continually an electric current; this current being self-sustained and constant, and pursuing an invariable path round the molecule.



As in all molecular hypotheses of magnetism (for which *see* Chapter I. § 5), *neutrality* or *evident magnetism* are merely a matter of the arrangement of the molecules.

In Ampère's theory, a bar would be magnetised to a maximum when the molecules had so turned that all their currents were parallel.

This condition of things is represented in the figure, where the two ends of the bar are shown. It is reasonable to suppose that, except on the surface, the currents of neighbouring molecules would neutralise each other with respect to external effect; since, as shown, the neighbouring pairs of contiguous currents would be equal and opposite. There would, therefore, be left only the outside ring of molecular currents; and these may be supposed to be equivalent to one continuous circular current. Thus, if we consider the bar to be cut into flat discs as in § 1 above, each disc would be equivalent to a plane circular current; and the whole bar would be equivalent to a solenoid, *as regards the external field.*

§ 5. **Solenoid, and Hollow Cylindrical Magnet, Contrasted.**—In fig. i. we represent in section a solenoid, and a few of its lines

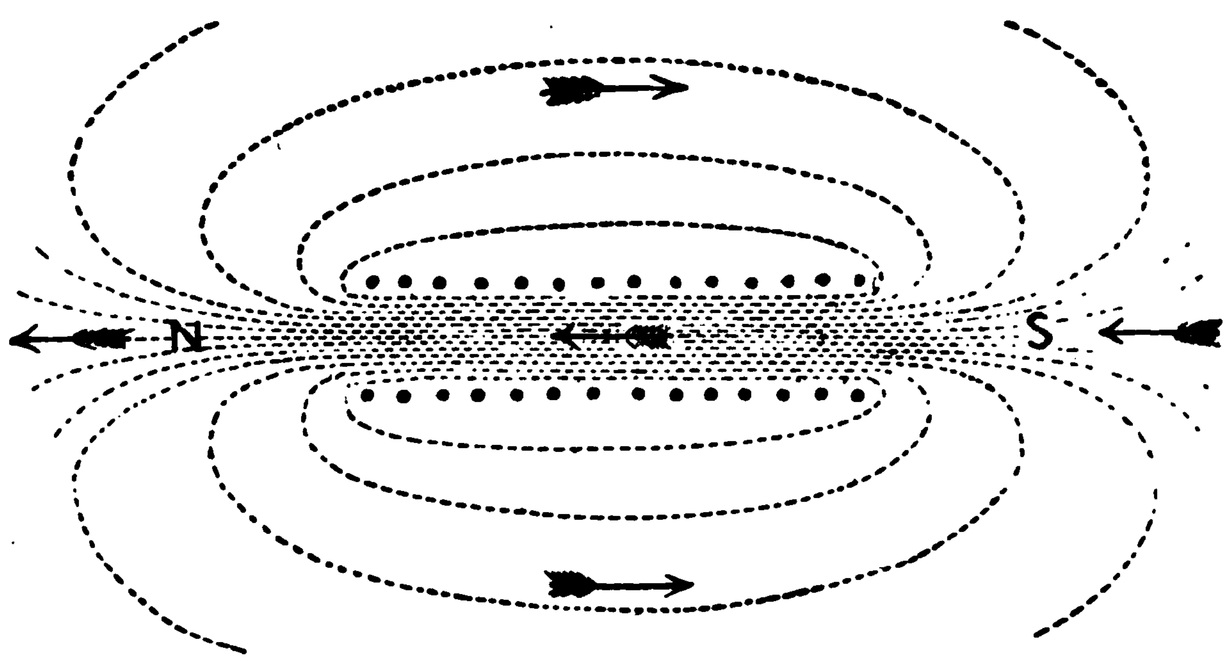


FIG. i.

of force are also given, the arrows representing their + direction. It will be seen that inside the solenoid the lines run in a contrary direction to that which they have outside.

In fig. ii. is given in section a hollow cylindrical magnet, in like manner. Here, the lines along the inside have the same direction as outside. The outside field has the same general character. But while all the lines of the solenoid run continuously through and round the hollow tube, in the case of the hollow magnet, on the other hand, all the lines run into, and end in, the solid steel that forms the side of the hollow cylinder. In fact, we must remember that the *complete* solenoid is equivalent to a magnet, as regards external field, but any longitudinal slip of it taken alone is not equivalent to a magnet; while each such

longitudinal slip of the hollow magnet is itself a complete magnet, the hollow cylinder being not a simple whole, but being a compound arrangement formed of a system of magnets arranged as are the staves in a barrel.

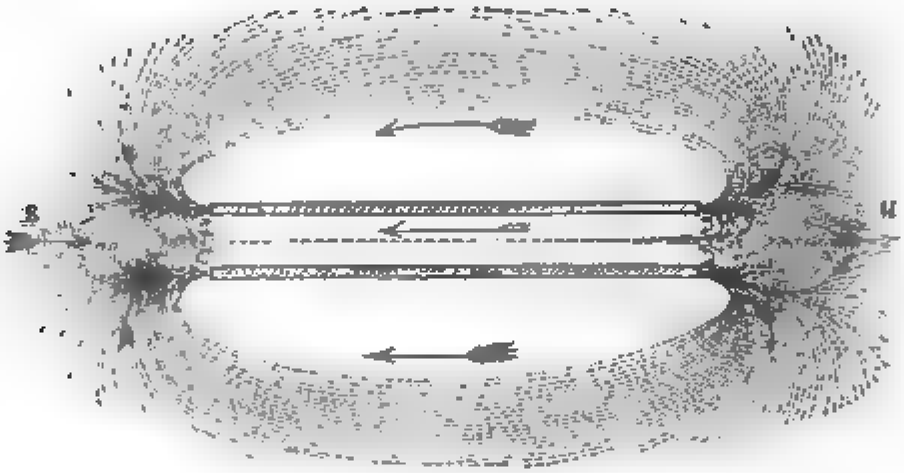


FIG. II.

§ 6. **Matter Placed in a Uniform Magnetic Field of Force.**—Let us consider a uniform field of magnetic force, and a cylinder of any material, whose two ends are plane faces standing perpendicular to its axis, so placed as to lie with its axis along the lines of force of the field.

In the case of soft iron, or of steel previously unmagnetised, we find that the magnetisation is such that we have evident magnetism at the two end-surfaces only; if we neglect, as relatively unimportant, the irregularities that occur at the edges where the molecules are free towards the outside and in contact with other molecules towards the inside. Over these end-surfaces the density ρ of magnetisation (*see* Chapter XVIII. § 7) will be approximately uniform. There is no experimental reason for supposing but that cylinders of any material are, if sensibly magnetised at all, magnetised in a similar manner to the above.

The value of this density ρ depends (i.) upon the field-strength I, and (ii.) upon the nature of the material of which the cylinder is composed. Now experiment indicates that, so long as the bar is far from saturation (*see* Chapter I.), then ρ is directly proportional to the field-strength I. Thus we may write . . . .

$$\rho = k\,\mathrm{I}$$

where k is a quantity depending upon the nature of the material. When the field-strength $\mathrm{I} = unity$, then k is numerically equal to ρ.

This quantity k is called the *coefficient of magnetisation* of that material, and is measured by the value which ρ has when I = *unity*.

Without at present discussing whether the following assumptions are physically possible (we shall see later that they are), let us assume that k may be a + quantity, zero, or a − quantity; and let us consider what would be the observed condition of the cylinder in the three cases respectively.

(I.) *Let* k = *a + quantity.*—This would make ρ positive. That is, remembering what is the + direction of the lines of force, we should observe a north-seeking polarity at the end lying furthest down the lines of force, and a south-seeking polarity at the other end. Or this case is the usual one of soft iron or other magnetic matter placed in a field of force.

(II.) *Let* k = *zero.*—Here we observe no polarity, since $\rho = 0$. That is, the material is one whose presence in the field makes no difference to it.

(III.) *Let* k = *a − quantity.*—In this case ρ will have a contrary sign to that which it had in case (I.). Or the material would be one in which induction takes place in a contrary direction to that observed in iron; thus a north-seeking pole of a magnet would, at any rate apparently, induce in a bar of such material a north-seeking pole at the end nearest to the former.

In order that we may now use convenient names for different classes of bodies, we shall to some extent forestall what will be discussed more fully later on in this Chapter.

We may, therefore, state that there is in the first place a class of bodies for which k is +; or for which induction takes place down the lines of force, as in the case of iron. Such bodies, of which *iron* (including steel) is by far the most important, are called *magnetic*, or more properly *paramagnetic*. In the case of very pure soft iron, k has a large value; thus the presence of a bar of such iron in a magnetic field may increase the field-strength near the poles even fifty-fold.

There is, secondly, another large class of bodies for which k is −; or for which induction takes place up the lines of force, or in the contrary direction to the above. For such bodies k is very small. For example, if a bar of *bismuth* be placed in a magnetic field, this field will appear to be slightly weakened near the poles of the bar, in virtue of the opposed induced polarity of the bar;

but, from the smallness of k, the field as a whole is but very slightly affected. Such bodies are called *diamagnetic.* (For further discussion *see* §§ 14 and 15.)

§ 7. **Movements of Small Bodies in a Non-Uniform Magnetic Field.**—Let us now consider a field that is not uniform, and a small body placed in the field. By *small body* we here mean one so small with respect to the whole field that it can be considered to be all of it in a stronger or weaker part of the field at the same time; and yet not so small but that one side of it is, in our non-uniform field, in a part of the field of somewhat different strength to that in which the other side finds itself. We will consider what will be its behaviour.

(I.) *Small magnetic bodies.*—It can be shown that a small magnetic body, such as an *iron* pellet, for example, is urged from weaker to stronger parts of the field.

Thus, if a small iron pellet be presented to a pole of a magnet (the usual case of a non-uniform field) there will be induced an opposed polarity on the side next to the pole, and a similar polarity on the side more remote. The former will be attracted, and the latter will be repelled, by the pole; these two polarities are equal in magnitude but opposite in sign. Now the former polarity is in a stronger field than is the latter, and hence attraction will predominate, and the pellet will move towards the pole.

(II.) *Small diamagnetic bodies.*—For similar reasons a small diamagnetic body is urged from a stronger into a weaker part of the field.

§ 8. **The 'Setting' of a Long Body in a Uniform Magnetic Field.**—Next let us consider the case of a long cylinder placed in a uniform magnetic field, at an angle with the lines of force of the field. We will represent our cylinder as composed of a series of small spheres placed near to one another. This is a convenient representation, and though not an accurate one, will not invalidate the very general results at which we shall arrive.

(I.) *A magnetic cylinder.*—Fig. i. represents a cylinder of iron. Each of the little spheres A B C D would, if it stood alone, be magnetised in the direction of the lines of force of the field. This is represented by the lettering n s in each. There is, however, inductive action between the spheres, each n or s inducing an s or n

respectively in the nearest portion of the neighbouring sphere. The total result will be that each little sphere is magnetised, not along the lines of the field, but in a direction represented by n' s' in fig. ii., lying between the direction of the field and the direction of the cylinder.

Thus each little sphere is acted upon by a couple tending to drag n' s' into the direction of the lines of the field (*see* Chapter II. § 12). Hence there will be a couple acting upon the whole

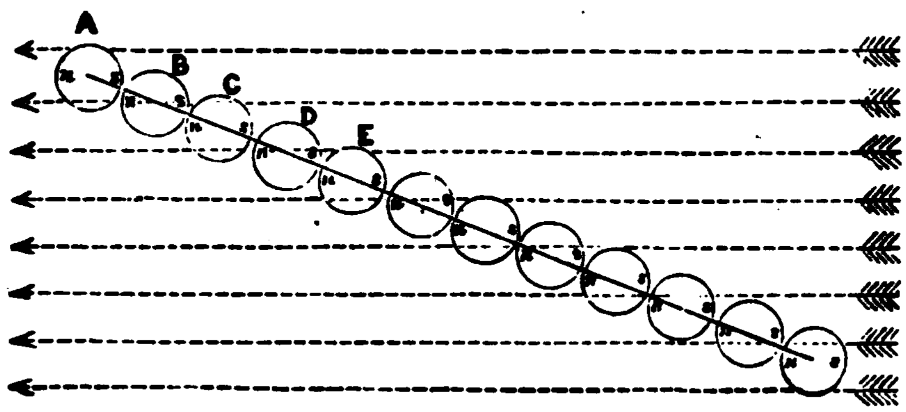


FIG. i.

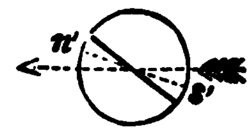


FIG. ii.

cylinder, and there will be stable equilibrium only when this lies along the lines of force. It is to be noticed that in this position the external and internal actions concur to give the maximum magnetisation.

When the cylinder is perpendicular to the field there is also equilibrium, but unstable. In this position the internal induction acts against the external, and the magnetisation is at a minimum.

We may therefore state that . . . .

A magnetic cylinder tends to set along the lines of force of a uniform field; that is, to assume the position in which its magnetisation is at a maximum.

(II.) *A diamagnetic cylinder.*—In this case the external and internal induction will both be the reverse of what it was in case (I.). Thus we must interchange the letters n and s in the above given figures; and must further remember that each n or s in one sphere induces an n or s respectively in the nearest portion of the neighbouring sphere. The total result will be that each little sphere will be magnetised

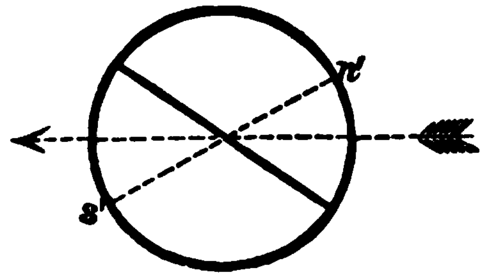


FIG. iii.

somewhat as in fig. iii. Hence there will be a couple acting on each sphere ; and this will be in such a direction that the whole cylinder will be urged to lie along the lines of force. In this position the internal and external inductions act against one another, and the magnetisation is at a minimum.

When the cylinder is perpendicular to the field there is unstable equilibrium, and the magnetisation is at a maximum.

Hence *a diamagnetic* (*see* § 6, end) *cylinder tends to set along the lines of a uniform field ; that is, to assume the position in which its magnetisation is minimum.*

We may, however, add that with diamagnetic bodies the above action is very feeble ; and no observation has as yet detected any 'setting' of such bodies in a *uniform* field.

§ 9. **A Long Body in a Non-Uniform Field.**—When a *magnetic* cylinder is suspended in such a non-uniform field as that between the two poles of a powerful magnet, the results of §§ 7 and 8 concur to show that it will set *along* the lines running between the two poles.

When a *diamagnetic* cylinder is so suspended, the action described in § 7 would urge it to stand in the weakest part of the field ; *i.e. at right angles* to the lines of force between the poles. But the action given in § 8 would tend to make it set along these lines. In all cases that occur in practice, the former action gives rise to the greater couple ; and so the diamagnetic cylinder stands at right angles to the lines running between the poles.

To exhibit these phenomena, very powerful magnets are needed. We therefore proceed to consider how powerful temporary magnets may be obtained.

§ 10. **Solenoid With, and Without, an Iron Core.**—According to the view that appears to be most in accord with experiment, magnetic matter possesses innate magnetism. This magnetism is, however, molecular ; and we have evident magnetism, or an external field, only when the molecules are suitably arranged.

We see that, on this view, there is nothing very surprising in a moderate field 'producing' a strong magnet ; for the magnetism is in the soft iron already, and the field may be able to arrange the molecules suitably.

We may contrast the magnetic action of an ordinary solenoid

with that of one in which there is a soft iron core. It is often said 'the core strengthens the solenoid.' But it is perhaps more in accordance with facts to say 'the solenoid renders evident the innate magnetism of the iron.'

Experiments.—(i.) We can compare the action of the two on a balanced needle.

(ii.) We can examine the fields of the two respectively by placing them under glass and sprinkling steel filings above. It will be seen that in the case of the solenoid the field is weak, and that some of the lines of force 'leak out' between the turns of the wire. In the case of the solenoid with iron core, the field is far stronger, and the strength is more concentrated at the poles.

The iron is, from its symmetrical position, magnetised in the direction of the lines of force due to the solenoid; *i.e.* in the direction of the solenoid's axis. The north pole of the core will be that at the north end of the solenoid, where the current, to one facing the end, appears to run counter-clockwise.

§ 11. **Electro-Magnets.**—Thus, when a soft iron core is wrapped round with many turns of wire, and a current is passed through the wire, the core becomes temporarily a magnet. Such magnets are called *electro-magnets*, and can be made far more powerful, mass for mass, than can any permanent steel magnets.

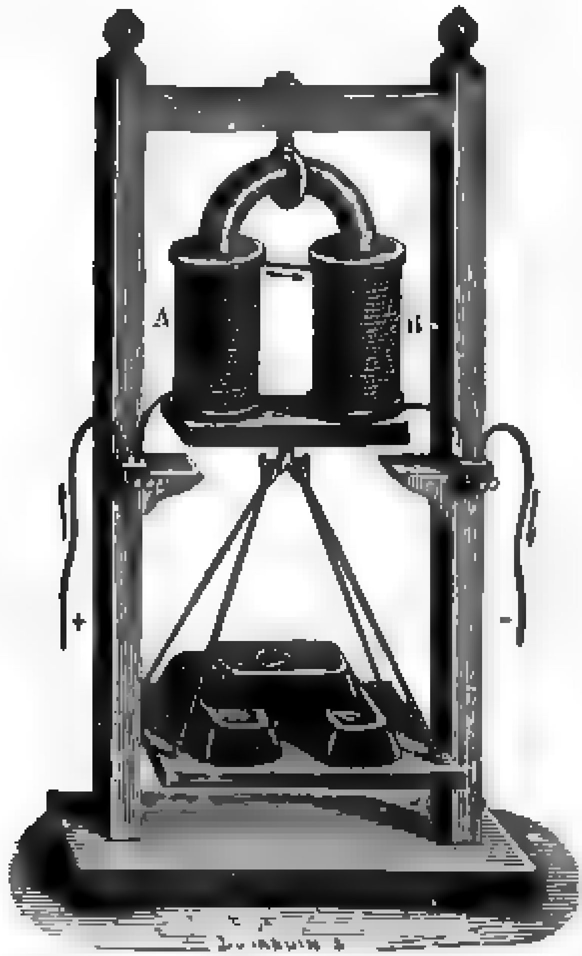
We may regard the external field to be made up of two components; the one due to 'evident magnetism' now evoked in the iron, the other due to the spiral or solenoidal current. These two fields are superimposed upon one another. As long as the iron is far from saturation, the field due to it is approximately proportional to the field-strength (*see* § 6) and therefore to the current-strength. But when saturation is reached, any further increase in current will only increase the comparatively insignificant component field due to the spiral alone. In winding the wire about an iron core, and in passing a current through the wire, it is necessary to have regard to the following considerations.

(i.) The wire must not be too thick, or it will not be possible to give a number of turns sufficient for the production of a strong field with a current of reasonable magnitude.

(ii.) The wire must not be so thin as to give great resistance and consequent loss of energy in heat.

(iii.) It is of little use producing a field-strength greater than that necessary to magnetise the iron nearly to saturation.

(iv.) The distribution of the wire about the core must be adapted to the shape and dimensions of this latter.



The accompanying figure shows one form of electro-magnet.

Experiment.—It is possible, in the case of a powerful electro-magnet, to trace the lines of force in a very striking manner. Instead of filings we may use small pieces of steel knitting-needles. If the field be powerful, the effect of gravity on these pieces of steel will be relatively insignificant; and we may cause them to attach themselves end to end to one another, and so trace out the lines of force in any direction in space, while with permanent magnets we were able to trace the lines only over a horizontal plane.

Note.—Field due to an electro-magnet.—In calculating the field due to an electro-magnet we can consider the solenoid, and the core which has become a cylindrical magnet, separately. But in general the field due to the former is relatively insignificant, and we need only regard the core.

§ 12. **Paramagnetic and Diamagnetic Phenomena.**—When powerful electro-magnets are employed it is found that all bodies are influenced by the magnetic field (*see* § 6).

If pellets of various materials are suspended, by means of a light and long thread, near one of the poles of such a magnet, it is found that certain substances are attracted by the pole, while others are repelled.

Those bodies which are attracted are called *paramagnetic*, or *magnetic*; such are iron, nickel, cobalt, manganese, platinum, carbon, many salts of magnetic metals, solutions of such salts, and oxygen gas.

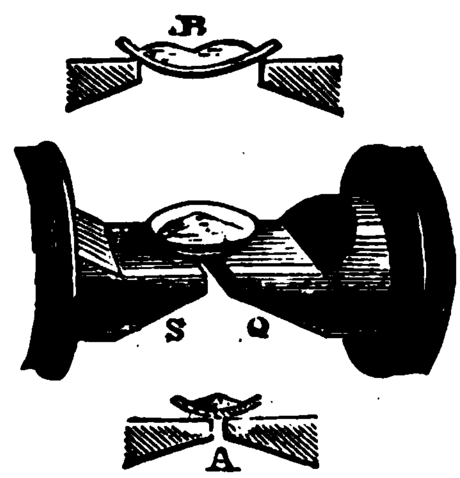
Those bodies which are repelled are called *diamagnetic*; such are bismuth, antimony, zinc, tin, mercury, lead, silver, copper,

[illegible]d, phosphorus, glass, quartz, alum, sulphur, sugar, hydrogen, nitrogen, water, alcohol, and most other liquids and gases not here named.

Iron is the most strongly *magnetic*, and *bismuth* the most strongly diamagnetic, body known.

If we make bars of various substances, those which are magnetic will set axially, or in a line with the poles; while those which are diamagnetic will set equatorially, or at right angles to the line joining the poles (*see* § 9).

The accompanying figure represents experiments with magnetic and diamagnetic liquids respectively. The liquid is placed in a watch-glass and rests on the poles. When the current is passed, the magnetic liquid B rises up in a heap over each of the two poles; while the diamagnetic liquid A is repelled into a heap between the two poles. Such effects are very small, and must be magnified by means of reflected light if they are to be made clear. The difference between the two classes of liquids is clearer if we employ *thin* glass tubes filled with the one or the other respectively, and observe whether these set axially or equatorially. It must, however, be remembered in this case that the glass itself is diamagnetic; its action can be allowed for.



§ 13. **Pseudo-Diamagnetic Phenomena.**—In certain cases a bar may set equatorially when its material is magnetic, or axially when its material is diamagnetic, owing to peculiarity of structure.

Thus, a bar may be made composed of short steel needles separated from each other, lying side by side, running transverse to the length of the bar. Such an arrangement will, as a whole, set equatorially; each little needle lying axially. So again, if in a bar of bismuth the crystallisation have a certain direction with respect to the length of the bar, this may set axially. The repulsion or attraction of pellets from or to a pole is the best way of dividing bodies into the two classes.

§ 14. **Relative Magnetism or Diamagnetism.**—By Archimedes' principle we know that bodies immersed in any fluid medium appear to have a +, zero, or − *weight* according as they

displace less than their own, their own, or more than their own weight of that medium respectively. By this principle we can predict, *e.g.*, whether a body immersed in water will sink, remain where it is, or be forced upwards.

Similar reasoning applied to the case of a magnetic field leads us to predict—what can be verified by experiment—that when a body, whose coefficient of magnetisation with respect to vacuum is k, is immersed in a medium whose coefficient is h, the body will behave as though it were in vacuo and had a coefficient k' equal to $k-h$. If this be true reasoning, then if $k>h$ the body will appear to be magnetic; if $k=h$ it will be neutral, and if $k<h$ it will appear to be diamagnetic.

These predictions have been experimentally tested and verified. Thus, a weaker solution of ferric chloride appears diamagnetic when in the midst of a stronger solution, though in vacuo it is distinctly paramagnetic.

§ 15. **Is there Absolute Diamagnetism?**—The question naturally arises: 'Is there then such a thing as true diamagnetism, or is it merely that some bodies are less magnetic than that which we call "vacuum"?'

Some bodies which appear diamagnetic in the magnetic medium oxygen, may very well be found to be magnetic when tested in vacuo. But most diamagnetic bodies (*e.g.* bismuth) are still diamagnetic in vacuo.

It can be shown that all phenomena of repulsion and of equatorial-setting with which we are acquainted could be accounted for by supposing 'vacuum' to be a medium slightly magnetic.

But the phenomenon referred to in § 16, viz. the contrary directions in which a ray of plane polarised light is rotated in different media, seems to imply an essential difference in the sign of k for the two classes of media respectively. Before, therefore, we can accept unreservedly the view that all phenomena come under the head of paramagnetism or relative paramagnetism, it will be necessary to show that such a view can be reconciled with the fact quoted.

The whole question is at present unsettled. Quite recently (1886) experiments have been tried tending to show that bodies may change from diamagnetic to paramagnetic behaviour, or *vice versâ*, according to the strength of the field in which they are

placed. It has even been suggested that perhaps these qualities are not permanent, but change with time under the action of a field. Curiously enough, Ampère's theory of 'molecular currents' has been revived as a somewhat fruitful view.

§ 16. Rotation of the Plane of Polarisation in a Magnetic Field.—In giving some account of certain phenomena that show a remarkable connection between magnetic and electric stresses on the one hand, and radiant energy on the other, we must assume that the reader has some acquaintance with the elements of physical optics. If this is not the case he is advised to read enough of the subject to understand (i.) what is meant by a ray of light or of other radiant energy; (ii.) what is meant by *a plane polarised ray*, and by *the plane of polarisation*; (iii.) what a *Nicol's prism* is, and how it is used to obtain a plane polarised ray; (iv.) how a second Nicol's prism can be used as an *analyser* to detect whether a ray is plane polarised, and whether the plane of polarisation has been *rotated* or has changed its 'azimuth.'

Now it is found that if a plane polarised ray be passed through a transparent medium that ordinarily has no power to rotate the

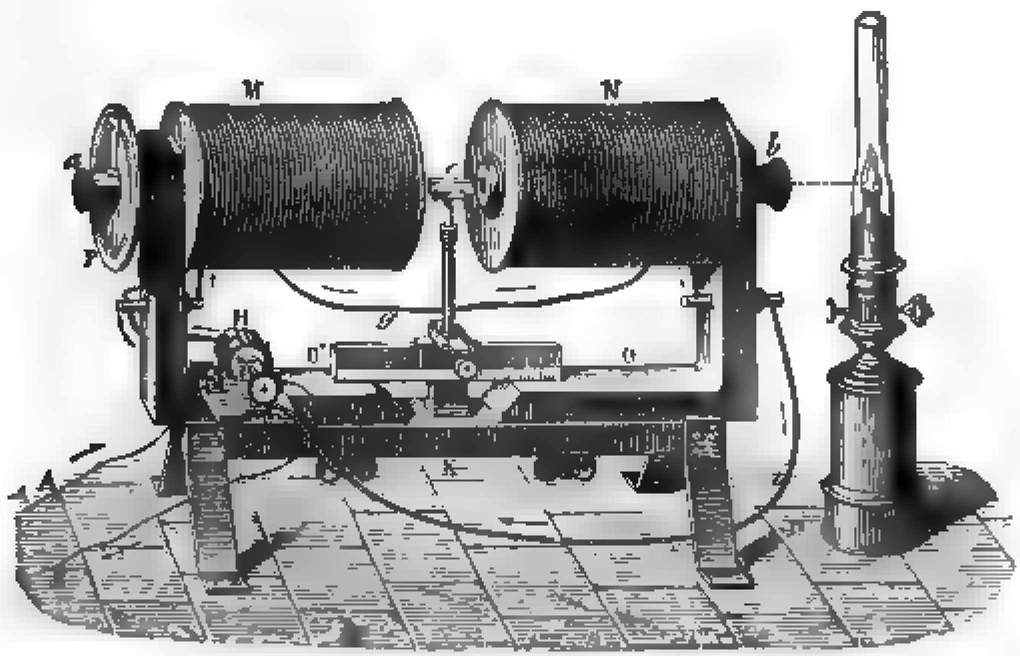


FIG. i.

plane, and if this medium be placed in a powerful magnetic field, then the plane of polarisation of the ray is in general slightly rotated in its passage through the medium. It was Faraday who discovered this. The accompanying figure indicates how the

'electro-magnetic rotation of a plane polarised ray' may be demonstrated. If more exact results are desired the necessary experimental arrangements are less simple. M and N are powerful electro-magnets, provided with hollow iron cores, their unlike poles being opposed so as to give a powerful field at *c*. Here is placed the piece of glass or other transparent body, whose rotating powers, when subject to a powerful magnetic field, are to be observed. At *b* is the *polarising* Nicol, and at *a* the *analyser*.

If (as we here suppose) the body at *c* has no innate rotating power as has (*e.g.*) quartz, then before the current passes the plane of polarisation of the ray is not rotated; and no light can be seen through *a* when the principal planes of the polariser *b* and of the analyser *a* are at right angles to one another. But when the current passes so that a powerful field is produced at *c*, then some light is received through *a*, and we must turn the analyser *a* about the line *a b* as axis, in order to recover the initial darkness. When monochromatic light is used we can then readily measure the amount and direction of the rotation by the amount and direction of the rotation that has been given to *a*. If the light be not monochromatic, *e.g.* if it be white light, we can never recover the white light again; this showing that the rays of different wave-lengths have been rotated to a different amount.

By experiments that we have not space enough to give here, Verdet and others established several results. These results will be

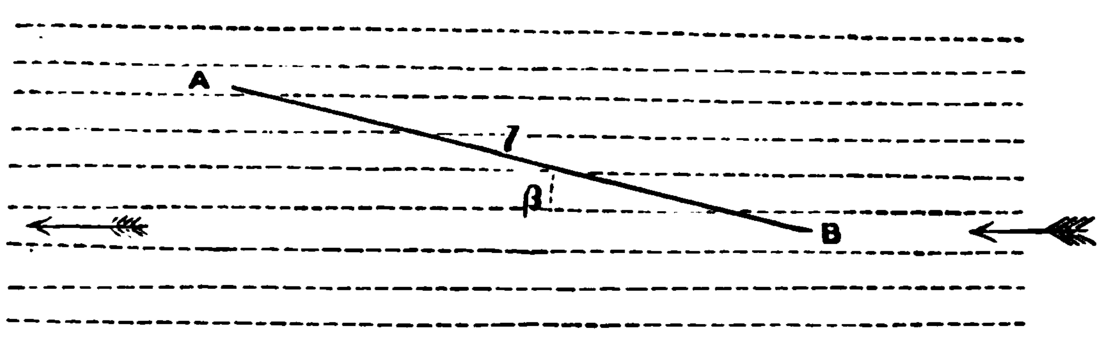


Fig. ii.

made easier to understand by the simple diagram here given. The dotted lines represent a magnetic field as usual; A and B are two points in it at which the magnetic potentials are V_A and V_B respectively, the former being [in the case represented] at the lower potential; A B or B A is the direction of the ray; β is the angle that this direction makes with the lines of force of the field; I is the field-strength; l is the

:e in the medium traversed by the ray; and θ is the angle
h which the plane of polarisation is rotated.
vas found that . . . . not
θ depended upon the field-strength, being directly proportional tried

θ is directly proportional to *cos* β.
) θ is directly proportional to l.
) θ depends upon the wave-length of the radiation in question.
The sign of θ, or the direction of rotation, is not the same in
lia. If we call the rotation positive when it is as a right-handed
vould rotate as it advanced down the lines of force, then we
ıy that in vacuo there is zero rotation (as far as experiment
es), in diamagnetic media the rotation is +, and in magnetic
(such as a solution of ferric chloride) it is −.
) This rotating power seems to be innate in the molecules pro-
that they are in a magnetic field. A substance preserves its
when in solution or when mixed with other bodies, and so we
th certainty prepare mixtures of the two classes of solutions
shall give zero rotation.
.) The direction and magnitude of rotation are the same
:r the ray pass from A to B, or in the reverse direction. Hence
ation can be multiplied by causing the ray to be reflected to
› many times up and down the field, this being equivalent to
lying l. It would not be practicable to make l great in any
manner, if we wished still to preserve a strong field.
i.) With reference to (v.) we may add that all wave-lengths
ıe same direction of rotation in the same medium.
eory suggested by the above.—The above phenomena suggest
ırcibly that in a magnetic field the ether may be eddying about
ıes of force. For, were this so, the wave would naturally be
l in azimuth during its passage.
membering how the lines of force pass down the axis of a
id or spiral, it would seem as though a spiral current caused the
o eddy about the axis of the spiral. It has even been suggested
:rhaps electricity is itself ether, so that the eddy is produced by
l current of ether through stagnant ether.
present, however, there is in this direction little but conjecture.
day, no doubt, electro-optical phenomena such as the above will
› the establishment of very important results as to the real
of magnetic, electrical, and radiant phenomena.
: may observe that the opposite directions of rotation in different
indicate a real difference between the two classes of bodies, or
› show that diamagnetism is not merely a relative phenomenon.

§ 17. Other Electro-Optical Phenomena.

There are other phenomena that show how the propagation of a wave of radiant energy is affected by the magnetic or electric condition of the medium through which the ray passes.

Kerr discovered that when a plane polarised ray is reflected from the polished surface of the pole of a magnet, the plane of polarisation is rotated. He believes that this is due, at any rate in part, to the action at the reflecting surface, and is not simply due to the air or other gas that fills the magnetic field.

He also discovered that when a simply refracting transparent dielectric is subjected to electric stress, as *e.g.* is the glass of a charged Leyden jar, it becomes doubly refracting. The effect was on the whole similar to that which was produced if it was subjected to compression along the direction of the lines of force. It seemed, in fact, as if there was *tension* along the lines of electric force, tending to compress the glass.

§ 18. The Electro-Magnetic Theory of Light.

The genius of Clerk Maxwell founded a theory which should connect all these phenomena that are usually classed under the heads of magnetism, electrostatics, and electrodynamics, on the one hand, with the phenomena of light, heat, and of radiant energy in general on the other. But before this theory could grow into a complete structure the author of it died, and the work has never been completed.

In what follows we have tried to indicate in an elementary manner the nature of the theory, and also to show how much uncertainty still exists with respect to it. We have made use of the suggestions put forward by Dr. Lodge, Professor S. P. Thompson, and others, as such suggestions show the directions in which Clerk Maxwell's fundamental views tend.

(*a*) *The ether.*—By the use of the expression *the ether* is implied a belief, now general among men of science, in a universal medium pervading all space and penetrating solids and liquids.

The ether is not matter, but possibly all atoms of matter are merely indestructible vortex-rings of ether, so that this latter is the parent of matter. Again, the ether is not energy; but yet it is only through the ether that energy can be transmitted from one group of matter to another.

The ether penetrates all bodies, but it may be that in some cases the matter has, to a greater or lesser degree, a 'hold' upon the ether that pervades its intermolecular spaces, and gives to it something of the properties of a rigid solid, while in other cases the ether may be left quite free.

(β) *Radiant energy; heat and light.*—We may here remind the reader that the transmission of heat and light is believed to be effected by means of waves propagated in *the ether*; the vibrations being at right angles to the direction of propagation, somewhat as in the case of a shaken rope. Further, it depends upon the wave-length of the radiation whether it give the impression of *light* when received upon the retina, or in what respect its action is most remarkable. *Heat* and *light* are partial expressions, *radiant energy* is the more general term for transmissions of all wave-lengths.

(γ) *Nature of electricity.*—It has been suggested that if anything can rightly be called 'electricity,' this must be *the ether* itself; and that all electrical and magnetic phenomena are simply due to changes, strains, and motions in *the ether.* Perhaps *negative electrification* (as we believe has been suggested) means an excess of ether, and *positive electrification* a defect of ether, as compared with the normal density.

(δ) *Electrostatic phenomena.*—It is possible that all electrostatic phenomena follow from strains in the ether. These strains (or deformations) can only occur where the ether possesses some degree of rigidity. On this view then it would appear that *dielectrics* are bodies which have what we may style so much 'hold' upon the ether that pervades their intermolecular interstices, that this is given, to a certain degree, the properties of an elastic solid. Thus the phenomena of the stored-up energy of a charged Leyden jar may imply that the ether pervading the glass, and probably the glass itself, is in a state of elastic strain analogous to that of a bent spring. *Conductors*, on the other hand, would be bodies in which the ether is free and, to speak from analogy, fluid, or in which it is incapable of retaining a strain.

(ϵ) *Dielectrics are transparent, conductors are opaque.*—Now from mechanical principles we should say that through bodies in which the ether has some rigidity, waves of transverse vibration can be propagated; while through those in which the ether is free and fluid, only waves of longitudinal vibrations can be propagated. From (β) and (δ) we should therefore predict that dielectrics would be transparent to the transverse waves of radiant energy (*i.e.* to rays such as those of heat and light), while conductors would be opaque to the same. If we take into account such disturbing influences as those of internal and irregular reflexion, it can be fairly said that dielectrics are, as a rule, transparent; while the conducting metals are certainly the most opaque of bodies. Here, then, is one fact that tends to confirm directly the theory.

(ζ) *Electric currents.*—There are reasons that would lead us to assume that when 'an electric current flows' through a conductor, there is either a direct translation of the ether along the conductor, or

a propagation of longitudinal vibrations similar to those of sound in air. This view is consistent with the hypothesis that in conductors the ether is free and not rigid, and therefore not capable of transmitting waves of transverse vibrations.

(η) *Electro-motive force.*—We might then suppose that electromotive force may be a kind of 'ether pressure' due to un-uniformity of distribution, or to be, at least, of a nature analogous to what is suggested by such a term.

(θ) *Electro-magnetic induction.*—Let us suppose a 'current' to be sent along a wire, this wire being surrounded by a dielectric such as air. Now we shall see in Chapter XXI. that when any change in current-strength occurs, all neighbouring conductors are affected; temporary currents being 'induced' in them. Since the possibility of action at a distance is now denied, we must suppose that this action is due to some kind of transmission of energy through the dielectric; and it is most consistent with the views expressed above, and with the observed facts of induction, to imagine that something of the nature of a wave in which the vibrations are transverse is propagated at right-angles to the wire carrying the current. If this is the case we should expect to find the velocity of propagation of electro-magnetic inductive action to be the same as the (average) velocity of radiant energy, since both are supposed to be waves of transverse vibrations, propagated in the ether. Now this is actually the case; many measurements having shown that the rate at which electro-magnetic inductive action is propagated is nearly the same as the mean velocity of light. In this agreement we have what is probably the strongest corroboration of the theory that we are discussing.

(ι) There is another piece of confirmatory evidence in the relation that certainly exists between the specific inductive capacity of a dielectric and its refractive index.

(κ) *Lines of magnetic force.*—In § 16 we pointed out how the phenomena there described suggested to us that lines of magnetic force are in some way axes of eddies formed in the ether. Here we may add that the eddy may be one in which the ether moves continuously round, or may be formed of circular vibrations in which there is no final displacement of the ether.

Considering the wire bearing a current, the circular lines of magnetic force about it, and the radiating lines in which electro-magnetic inductive action is propagated, we may say that probably the following relation holds. 'When there is a discharge of electricity in one direction the lines of electro-magnetic induction are at right-angles to this direction and radiate from it, while the lines of magnetic force are at right-angles to both these directions.'

CHAPTER XXI.

ELECTRO-MAGNETIC INDUCTION.

§ 1. **General Account of Induction Phenomena.**—Faraday discovered that when a conductor moves in a magnetic field, or when the field in which a conductor is situated is caused to vary in strength, then there is in general a current *induced* in the conductor, this current lasting only as long as the movement or variation lasts, and ceasing when conditions are again stationary.

Since a current implies an electro-motive force, we may say that such movements or changes give rise to *induced* E.M.F.s in the conductors. The direction of the E.M.F., and of the consequent current, depends upon the nature of the movement or change. If there be an E.M.F. (and current) in the conductor already, the induced E.M.F. if in one direction will be added to the initial E.M.F., and in the other direction will decrease, reduce to zero, or even overpower and 'reverse' the initial E.M.F., according to the relative strengths of the two.

It is soon noticed that the cases in which an E.M.F. is induced in a conductor are mainly as follows.

(*a*) When a conductor cuts across the lines of force of a magnetic field. If a complete circuit so cut across the lines, it may happen that the E.M.F.s induced in the opposite sides of the circuit are equal and opposed, giving a zero resultant E.M.F. in the complete circuit.

(*b*) When a magnetic field is created in the midst of a circuit, or is caused to cease. This case is shown in Experiment (i.).

(*c*) When the field enclosed by a circuit is varied in strength. This case is shown in Experiment (iii.).

The changes in question can be brought about in various ways: by the continuous movement of a simple rectilinear wire across a magnetic field; by a movement of translation of a circuit

across a non-uniform field; by rotation of a circuit in any field; by making and unmaking an electro-magnet (or solenoid) in the midst of a circuit; by the approach of a current, solenoid, or magnet towards a circuit, or its withdrawal from the same. And in many other ways. In the following experiments, which are here given without further comment, we use *coils* instead of simple circuits in order to obtain more remarkable results.

Experiments.—(i.) A coil is made of a quantity of fairly stout insulated wire, and the ends of this wire are connected with the terminals *c* and *d*, so that a current can be sent through it when desired. This coil A is concealed in fig. i., but is seen in fig. ii. A second coil B is made; it is hollow and encloses the coil A, but is entirely insulated from it. The ends of this latter coil are connected with the terminals *a* and *b*. A galvanometer is also connected

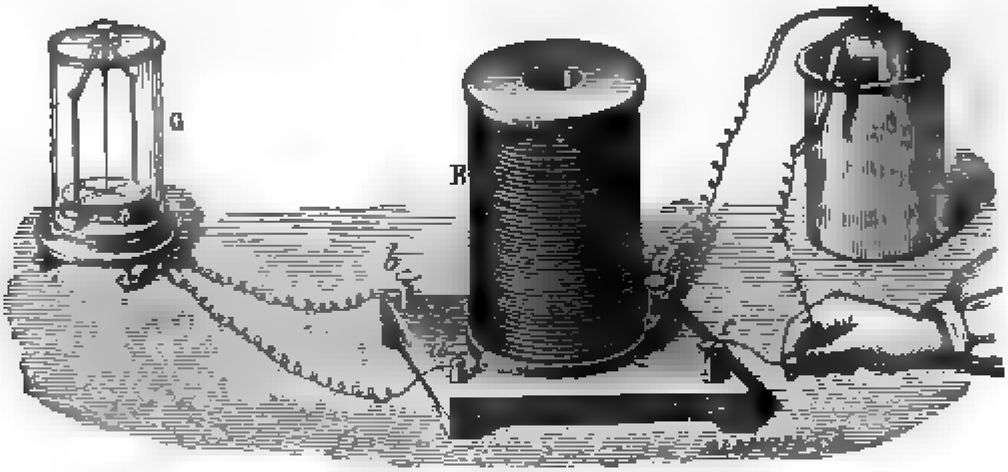


FIG. i.

with these terminals, so that any current in B will be detected. The effects are more marked if the wire of which B is made be very long. It, therefore, must be of fine wire, to admit of great length and many turns within a reasonable compass. The coil A is called the *primary*, and the coil B the *secondary*.

When a current is sent through the primary, the galvanometer indicates a sudden current in the secondary; this current lasting only until the primary current is established and steady. When the current is broken again we observe another current induced in B, this time in the other direction.

Thus the creation or destruction of a field within B produces currents in B, the currents being in opposite directions respectively. There is no current in B, while the current through A is steady; that is, while the field within B is constant.

(ii.) In the next figure the arrangements and connections are the same, but now A can be thrust into B or withdrawn from B. It is found that by thrusting A, while a current is flowing in it, into B, we induce in this latter a current in the same direction as would have been induced had we left A within B and had then sent the current through A. So also the withdrawal of A

answers to stopping the current in it. We find, also, that the currents induced are more or less violent according as we move A more or less rapidly respectively.

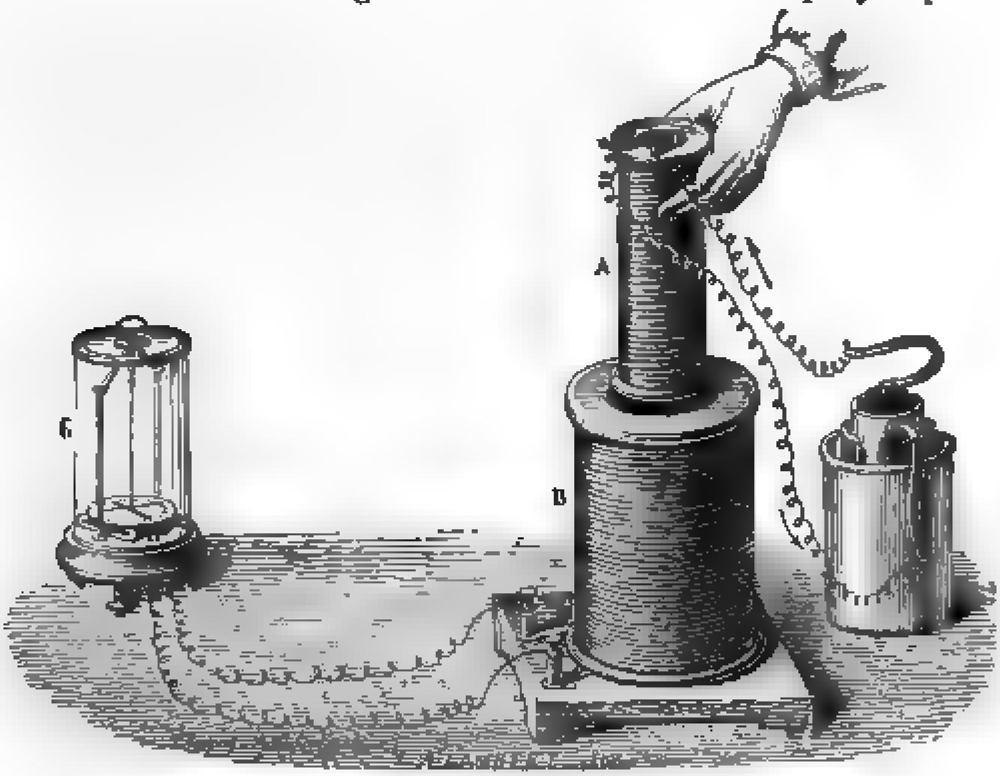


FIG. ii.

(iii.) Similar effects are produced if we thrust a magnet into B, or withdraw it again. We should expect this, having seen that a coil bearing a current acts as a weak magnet

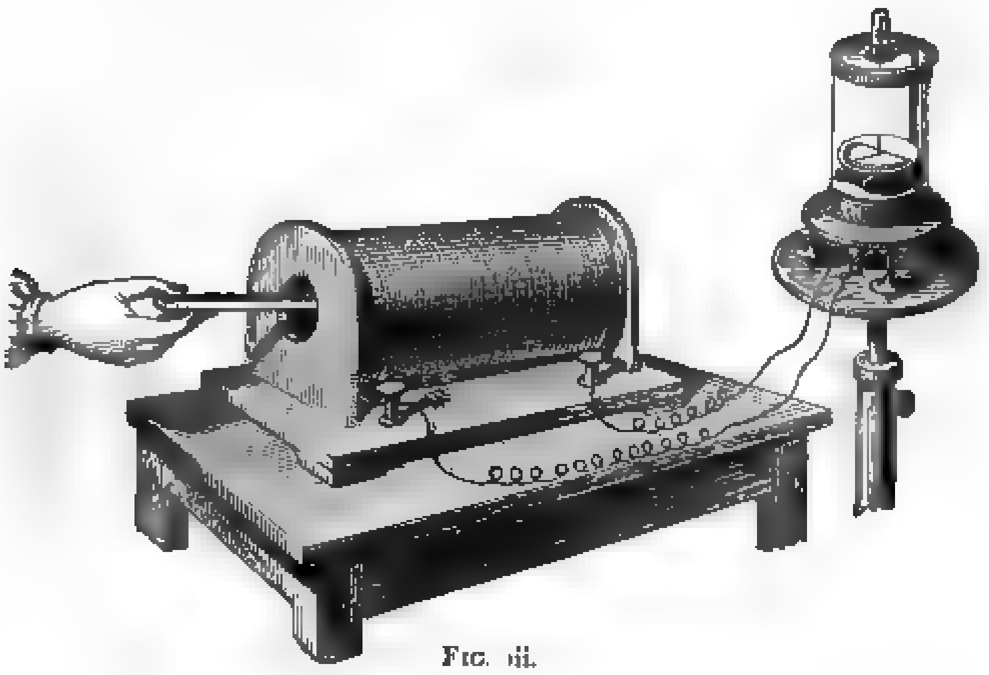


FIG. iii.

As regards the relation of the direction of the induced currents to the direction of the primary currents, or to the *sign* of the

magnetic pole which is presented to the coil B, enough will be said later on.

§ 2. **General Reason for 'Induced Currents.'**—The actual reason why such movements or changes in field-strength as those described should have the effect of inducing E.M.F.s in the conductors concerned, cannot be said to be truly known. For it must depend upon the real nature of electric currents and of magnetic fields, and upon their relation to 'the medium that pervades all space.'

But we have certain great laws established by experiment, which have included and bound together in one theory all the phenomena that have been noticed up to the present point. If we can show that these new phenomena of 'induction' come also under these laws and could be predicted from them, we shall in a sense have shown the 'reason for' induction. The laws to which we refer are *Conservation of energy*, *Faraday's laws* (Chapter XII. § 7), and *Joule's law* (Chapter XV. § 4).

Let us consider the case of a circuit A which includes a battery-cell, opposite to which is a magnetic pole N. (Instead of a single pole we might consider the one end of a very long magnetic bar whose other end is far away and need not be considered.)

This circuit will attract or repel the pole N, according to their relative position; since the circuit acts as a magnetic shell, and produces a magnetic field. If it attracts N, then work can be done by N as it moves up towards A; and this work comes out of the system of the two bodies A and N, so that there must be an equivalent of energy lost from this system by '*Conservation of energy*.'

How is this energy supplied?

If the two bodies A and N were a *gravitation system*, *i.e.* merely two masses attracting one another by the action of gravitation alone, we know that we should have lost an equivalent of potential energy; that is, we could not restore N to its initial position save by doing upon it just as much work as we previously got out of it. In moving towards A it moves with the lines of force, in moving from A it moves against them.

But in our *electro-magnetic system* of the circuit A and the pole N matters are very different. We can, without any work, make or break the current, *i.e.* create or destroy the magnetic field at

all. Hence we could turn on the current when we are pulling N towards A, and thus get work done; and could turn it off while restoring N to its initial position, and thus require to do no work on N. Thus, in a *gravitation system* the law of conservation of energy demands only that we consider the change in relative positions of the bodies, or 'the configuration of the system'; while in an electro-magnetic system we must, if the law is to hold, seek for some source of energy other than that depending on relative positions; for we have shown how we could get unlimited work done and yet end up in the initial positions.

We are driven to conclude then that the energy must come out of the circuit A, and shall be able (*see* § 3) to see how the requisite energy can be supplied if we suppose E.M.F.s to be induced in the circuit A whenever work is being done on or by the pole N; the E.M.F.s being in opposite directions in the two cases respectively.

§ 3. **More Exact Reasoning, in a Simple Case.**—Let us consider a vertical rectilinear current A, and a single pole N pivoted so as to be capable of movement about A. (We cannot in practice have a single pole, but we can contrive an arrangement in which a current acts on one pole only of a magnet, so as to produce continuous rotation.) When the current passes, the pole will be urged round and round the wire A continuously. Hence we can have unlimited work done, while the pole continually returns to its initial position. As argued in the last section, there must be an equivalent of energy lost from the circuit of which A is part.

We will employ a notation similar to that used in Chapter XV. Let E be the E.M.F. of the battery in the circuit; R the total resistance of the circuit; C_0 the current that flows when the pole N is stationary, *i.e.* when no work is being done external to the circuit; C the current when the pole N is moving, *i.e.*, when external work is being done. [We here use absolute units.]

(In order to understand better what follows, the student is recommended to read Chapter XV. §§ 3, 4, 8, and 9 again.)

Initially, *i.e.* before the pole N begins to move and to do work, we have a current $C_0 = \frac{E}{R}$, energy lost in the cell at the rate of $E\,C_0$ per second, and an exact equivalent of heat energy appearing in the circuit at the rate of $C_0^2 R$ or $E\,C_0$ per second.

Now let the pole move and external work be done. How can the circuit supply the necessary energy? At first sight it seems simplest to suppose that C_0 remains unaltered, while less heat is evolved; the diminution of heat evolved per second being the exact equivalent of the mechanical work per second done by the pole. In this way the current C_0 would be unaffected, only it would give us less heat with an equivalent of mechanical work. Such an hypothesis would save the law of '*Conservation of energy*' from being broken; but it is neither in accordance with experiment, nor with Joule's law which is supported by experiment. For, by Joule's law, as long as the current is C_0 so long is there heat $C_0^2 R$ given out per second. And since $C_0 = \frac{E}{R}$, this heat is equivalent to the whole energy $E\,C_0$ given out by the battery per second; so that there is none left for the external work.

Hence, when the pole moves we conclude that the current cannot remain the same. Let us suppose, then, that it decreases and becomes C instead of C_0; and let us see if such an alteration will give us the energy needed for the external work done in moving the pole. The total activity of the cell is now $E\,C$, and the heat activity is $C^2 R$. Since $E = C_0 R$, and since C is less than C_0, it follows that $C^2 R$ is less than $E\,C$; or we have less heat activity evolved than is the equivalent of the cell's activity. Thus, if the current decrease from C_0 to C while E remains the same, we have a balance of activity left to account for the mechanical work done on the pole.

Now, since the battery and its E.M.F. E remain unaltered, and since R is itself unaltered also, it follows from Ohm's law that the current cannot fall from C_0 to C unless an E.M.F. e arise *contrary* to E.

If such an E.M.F. e be supposed to arise, we have (*see* Chapter XV. § 9). . . . .

$$\left\{\begin{array}{l} E\,C = \text{energy per second expended by battery.} \\ C = \dfrac{E - e}{R}, \text{ and is less than } C_0 \text{ which } = \dfrac{E}{R}. \\ C^2 R, \text{ or } (E - e)\,C, = \text{energy per second appearing as heat.} \\ E\,C - C^2 R, \text{ or } e\,C, = \text{energy per second available to do the} \\ \qquad\qquad \text{external work on the pole.} \end{array}\right.$$

The reader will notice at once the close resemblance of the above distribution of energy to that which occurs when a battery is employed to electrolyse any body, *i.e.* to do chemical work. In both cases there is a reverse E.M.F. e; and we have total energy is E C, heat energy is $(E - e)$ C, and energy expended upon the chemical or mechanical work is e C. (All these reckoned per second.)

If the pole N be caused to revolve in the other direction, we must do external work upon it, since we move it against the lines of force. Thus we keep doing work on the system without gaining energy of position. So we conclude that we must in some way be giving energy to the circuit. But we cannot, by Joule's law, give heat to the circuit so long as C_0 remains constant. As before, then, we conclude that C_0 must alter; and, this time, must *increase* to some value C'. This implies an E.M.F. e' in the *same* direction as E. And we have energy of battery E C', heat energy $(E + e')$ C', energy given to the circuit from outside e' C' (all reckoned per second). In fact, we merely change the sign of the induced E.M.F.

Thus we have shown how, by reasoning founded upon the laws quoted above, we should predict that E.M.F.s would be induced in conductors whenever movements or changes are made in which + or — work is done by the system. The case above was chosen for its simplicity; but the same reasoning applies to all cases. The main facts of 'induction' were discovered and investigated by experimental methods. But it is satisfactory to see how readily they fall into place under the same few great laws as are followed by all the electrical phenomena with which we were previously acquainted.

§ 4. **General Expression for Induced E.M.F.**—More generally, let there be any system of currents and poles, or of currents alone. Let + or — work be done by movements occurring in this 'electro-magnetic system'; the work being due to the existence of the currents. Then the same arguments as those given in §§ 2 and 3 show us . . . .

(i.) That the equivalent of this + or — work is not to be sought in the change in potential energy due to the changed relative positions of the bodies forming the system, as would be

the case with a gravitation system, but is found as a diminution of or addition to the electric energy of the system.

(ii.) That, in consequence of Faraday's and Joule's law, this loss or gain of the electrical energy is possible only if an E.M.F. be induced in one or more of the circuits.

(iii.) That if this induced E.M.F. be e, and there be a current C in the circuit, then the loss or gain of electrical energy per second, answering to the work done per second in consequence of the movements referred to, will be measured by e C.

In what has preceded we have for simplicity spoken only of cases where the work done per second is constant.

When this is the case the induced E.M.F. e will be constant, as will also the current C which $= \frac{E \pm e}{R}$ (*see* § 3); and we have a constant gain or loss of electrical energy C e per second, the equivalent of the work done per second.

In the case of a circuit moved in a magnetic field, the work done between any two positions is measured in *ergs* by C $(N_1 - N_2)$, as explained in Chapter XIX. § 3; the symbols having the meanings there given to them.

Now let $(n_1 - n_2)$ be the change per second in the number of marked lines of force piercing the circuit. Then the work per second due to the movement is measured by $C(n_1 - n_2)$. But we have seen that the equivalent of this is e C.

Whence it follows that . . . .

$$e = (n_1 - n_2).$$

Or, *the E.M.F. induced in a simple circuit is measured by the change per second in the number of marked lines of force piercing the circuit.*

Here e is in absolute units of *E.M.F.*, not in *volts*. (In fact, whenever we do not state to the contrary the reader may take it for granted that in this Chapter absolute C.G.S. units are intended.)

Note.—We must remark that where a pole of strength μ threads through a circuit and round again any number of times, returning to its initial position, the above formula is not sufficient by itself. This was shown in Chapter XIX. § 11.

Thus for each complete circle made about the circuit by the pole μ we must consider $4\pi\mu$ marked lines of force to have been added to those piercing the circuit (*see* Chapter XIX. § 11). As a particular case, when the pole μ

revolves a times per second about a rectilinear wire, it is as though there had been added $4 a \pi \mu$ marked lines per second to those piercing the infinite plane circuit of which we may consider the wire to form the edge. In this case, therefore, we have $e = 4 a \pi \mu$, this being the form of the 'general expression' that applies to this particular case.

If the circuit be made of m turns of wire, all of them pierced by the lines of force, then the *E.M.F.s* induced in them all severally are added, and give a resultant E.M.F. in the circuit that is m times as great as that in one turn. Hence if a change $(N_1 - N_2)$ in the number of marked lines piercing the coil occur in t seconds of time (where t may be a small fraction), we have . . . . .

$$\textit{Resultant E.M.F. } e = \frac{(N_1 - N_2) \times m}{t}.$$

§ 5. **Induction where there is no Initial Current.**

(I.) If we consider the case of a conducting circuit, in which there is *no current* flowing, moving across a magnetic field so as to change the number of lines of force piercing the circuit, there is no reason from the point of view of 'conservation of energy' why there should be an induced current at all; for the circuit when stationary in any part of the field has no potential at all, not being subject to any force.

(II.) Or again, let us consider a circuit in which there is an initial *E.M.F.* E; and let us suppose that, by making the rate of change in the number of lines of force piercing the circuit great enough, we induce an opposed E.M.F. e exactly equal to E. In such a case the current will first cease, since the total E.M.F., $E - e$, is *zero*. There is no reason, from the conservation of energy point of view, why there should be any further induction if we still further increase the rate of change of the lines. For we might now have any velocity of change without work being done, since the current C is zero, and therefore $C(n_1 - n_2)$ is also zero.

But we may fairly conjecture that, as no doubt the true cause of induction lies in the reaction between the conductor that forms the circuit, and the magnetic field existing in the surrounding medium, therefore there will in case (I.) be induction, and in case (II.) further induction.

If there is such induction, so that there is a current C, then since the work per second due to the induced E.M.F. e is

measured by e C and is also measured by C . $(n_1 - n_2)$, it follows that as before . . . .

$$e = n_1 - n_2,$$

where $(n_1 - n_2)$ is the change per second in number of lines of force piercing the circuit.

Thus in case (I.) we should have an induced E.M.F. $e = (n_1 - n_2)$, and a current $C = \frac{e}{R}$. We should do work per second on the circuit measured by C $(n_1 - n_2)$, and should gain equivalent electrical energy measured by e C. In case (II.) we should do negative work e C as long as the current flowed against the E.M.F. e, *i.e.* as long as E was $> e$; zero work as long as $E = e$ and C was zero ; and positive work e C when C had changed sign in consequence of e being now $>$ E. [All reckoned per second.]

Experiment fully confirms this conjecture, and proves the formula $e = (n_1 - n_2)$ to be universally true for conducting circuits ; e being added to or subtracted from the initial *E.M.F.* E if there be such, or appearing alone if there be no initial E.

This occurrence of induced E.M.F.s in all cases, whatever be the initial *E.M.F.* E, must be regarded as a phenomenon due to the 'nature of things' ; it could not have been predicted from the laws of 'Conservation of energy,' &c., alone.

Note.—In the simple case of § 3, where a pole revolves about a wire that now we will suppose to bear no current, there will be an induced E.M.F. measured by $4 a \pi \mu$; the symbols having the same meaning as in § 4, note.

§ 6. **Direction of the Induced Currents ; Lenz's Law.**—We have stated that the direction of the induced E.M.F.s depends upon the nature of the movement. It is not hard to see how we can, from the law of 'Conservation of energy,' predict this direction.

There are three main cases to be considered.

(I.) *Where the electro-magnetic system does positive mechanical work.*—Under this head come all cases in which magnets, or wires bearing currents, are caused to move in consequence of electro-magnetic attractions or repulsions. We will take as a typical case that of a pole urged (by the electro-magnetic field) round a wire carrying a current ; the case, in fact, of § 3. Here there is mechanical work done upon the pole, since this latter is either

gaining kinetic energy or perhaps is set to work some machine at a constant speed. According to the argument of § 3, there is an induced E.M.F. e opposed to E. That is, there is an E.M.F. induced in such a direction as would alone give a current in the opposite direction to that which is actually flowing, and which is causing the pole to move. We may say then that the induced E.M.F. *opposes the movement* that induces it.

In all similar cases where there is work done by the system and electrical energy expended, the induced E.M.F. e is opposed to the initial E.M.F. E ; and would, if acting alone, drive a current opposed to that actually running. But in electro-dynamics, when the current changes its direction, then attractions and repulsions are interchanged. Hence, in all cases coming under this head, the induced E.M.F. *opposes* the inducing movement.

If, in the simple case considered, the pole be absolutely free to move, then it will move with increasing velocity until the induced E.M.F. e, which (*see* § 4, note) is measured by $4 a \pi \mu$, becomes equal to E. As e approaches E in value the current C becomes smaller ; since $C = \frac{E - e}{R}$. Hence the force acting on the pole becomes smaller and smaller. For this reason it will take an infinite time for e to become actually equal to E, though it may not take long for it to become nearly equal. Supposing it to have become exactly equal, then the current ceases ; there is no energy expended by the battery, and, since the pole is by hypothesis perfectly 'free' to move, it will continue revolving with constant velocity, and there will be no work done. Things, therefore, will have come to a standstill, from the conservation of energy point of view, but there is all the time an electro-magnetic inductive action, since there is an induced E.M.F. e equal to E.

(II.) *Where positive mechanical work is done by an external agent upon the electro-magnetic system.*—Where we alter the relative positions of poles or circuits so as to do positive work against the electro-magnetic forces, there must be given to the electro-magnetic system electrical energy equivalent to the work done. This can only be, as was argued in § 3, by the induced E.M.F. e being in the same direction as the original *E.M.F.* E. Hence, the induced E.M.F. tends to drive a current in the same direction as that originally flowing. But it is owing to this original current

that we are (by hypothesis) doing work *against* forces that oppose the movement. Hence the induced E.M.F. is in such a direction as to *oppose* the inducing movement.

Thus in the simple case of § 3, if we whirl a pole about a wire bearing a current, in the direction opposed to that in which it is urged by the field due to the current, we do mechanical work per second on the system; and this appears as electrical energy eC gained per second by the circuit. This induced E.M.F. e adds to the current which is opposing the movement of the pole. There is now given out in the circuit per second *heat* to the amount of EC due to the battery, and also of eC due to the work done on the system from outside. We feel resistance in moving the pole; and we can regard eC as a kind of equivalent 'heat of friction' given out per second in the circuit.

(III.) *Where there was originally no current.*—We have seen in § 5 how there are E.M F.s induced by movements even when there are initially no currents.

Considering the simple case of § 3, it is easy to see in what direction the induced E.M.F. must be.

Let us whirl the pole about the wire, and let the induced E.M.F. e, and current arising from it, be in such a direction as to oppose the movement. We then do work on the system, and gain electrical energy in the system; or conservation of energy is maintained. But if we were to suppose the E.M.F. and current to be induced in the opposite direction, *i.e.* so as to urge the pole in the direction in which it is moving, we should then have electrical energy both giving out heat in the circuit and doing external work in urging the pole, and we should have created this energy out of nothing; since now no work is being done on the system. And this is against conservation of energy.

Hence we conclude that the induced E.M.F. is such as to oppose the inducing movement.

From considerations such as those given above we should predict that . . . .

The currents which are induced in consequence of movements or other changes in an electro-magnetic system are invariably in such a direction that they tend to oppose these movements or changes. This law was observed experimentally and enunciated by Lenz. It is

called *Lenz's law.* The experiments that illustrate this law are endless in number. We give here only a few.

Experiments to illustrate Lenz's law.—In *experiment* (i.) of § 1 we can, without much difficulty, show that when the current in the primary A is started there is induced a current in the secondary B in the opposite direction; while, when the current in A is broken, there is induced in B a current in the same direction. In both cases, therefore, the induced current opposes the change, for it will repel that in the primary while the latter is rising from zero to a maximum, and will attract it while falling from a maximum to zero. Or, again, the creation of a current in A has the effect of thrusting lines of force into B in one or other direction, and the inverse induced current in B opposes this by thrusting out lines in the opposite direction, while, when the current in A ceases and the lines are withdrawn, the direct induced current in B creates lines in the same direction, tending (as it were) to pull back those due to A.

(ii.) So, again, it can be shown that in *experiments* (ii.) and (iii.) of § 1, the induced current in B is such that the coil A or the magnet is repelled by B when advanced, and attracted by it when withdrawn, respectively. Regarding the coils as solenoids, and remembering the rule of Chapter XX. § 3, it is easy to verify this statement. Another way of stating the observed result is to say that when more lines are thrust into the circuit B, the current induced is such as to thrust opposed lines out; while, when lines are withdrawn, the current induced opposes the withdrawal by creating lines in the same direction.

(iii.) A circuit is arranged comprising a battery, resistances for regulating the current, a single rectilinear wire B, and a delicate galvanometer. When a wire A bearing a current is brought up to B, the decrease or increase in the deflexion of the galvanometer will indicate a current induced in B in the opposite direction to that flowing in A; when A is withdrawn the galvanometer's increase or decrease in deflexion indicates an induced current in the same direction as in A. So, by Ampère's laws, the induced current is such as to oppose the movement.

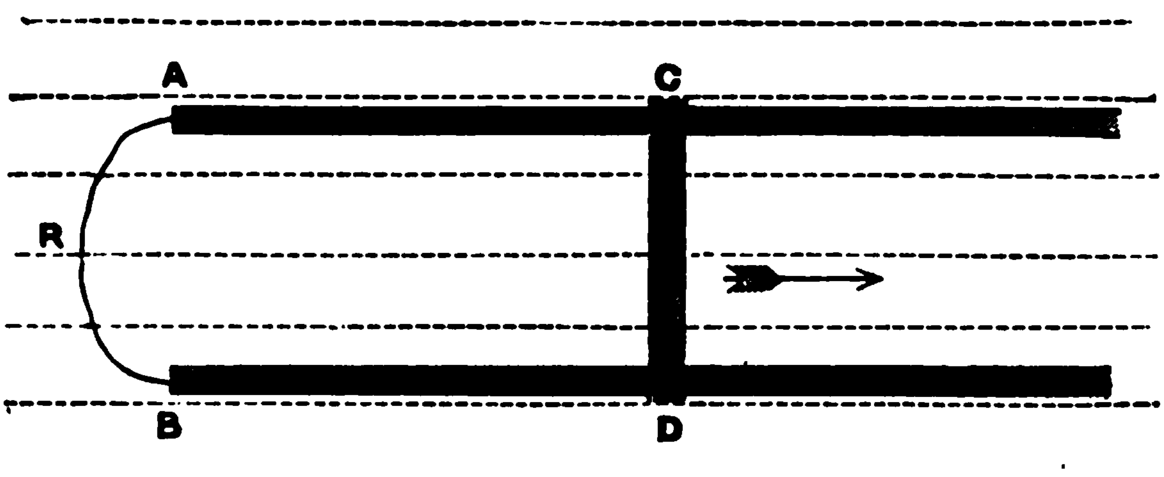
(iv.) With the same apparatus we may rapidly pass a powerful magnet pole across the wire B, without touching it. The current induced can be shown to be such as tends, according to the law given in Chapter XVIII. §§ 2 and 12, and Chapter XIX. § 7, to oppose the inducing movement.

§ 7. **Constant Induced Currents.**—The question as to how we can obtain a constant induced current is of great importance. We shall, later on, discuss those machines by means of which we can induce currents for the purpose of electric lighting, &c.; and we shall show how a practical constancy in the current induced can be obtained.

In this place we propose only to show how an absolutely constant induced current could theoretically be obtained.

(I.) *By revolution of a pole about a wire.*—When a pole μ revolves with constant velocity a times per second about a wire, the induced E.M.F. e is, as we have seen, constantly equal to $4 a \pi \mu$. An arrangement in which the wire revolves with uniform velocity about a pole, equivalent to the converse movement, is practicable, but is not a very serviceable machine for obtaining induced currents. Since the resistance R of the wire is constant, it follows that the current is constant, being equal to $\frac{e}{R}$, or $\frac{4 a \pi \mu}{R}$.

(II.) *By the rail-and-slider arrangement.*—In the figure A R B represents a conductor of resistance R, including a galvanometer. A C and B D are thick strips of copper of no resistance that is



appreciable. C D is a cross piece, also of no resistance, sliding in perfect contact with the two strips. The dots represent a uniform magnetic field in which we suppose the lines to run from above perpendicularly down into the plane of the paper, *i.e.* into the plane of the apparatus.

If the slider move with uniform velocity in the direction of the arrow, we add to the circuit A C D B A a constant number of marked lines of force per second; say a number n. Then the induced E.M.F. is measured by $e = n$; and, since we suppose the resistance to lie in the part A R B only, the induced current is equal to $\frac{e}{R}$ or $\frac{n}{R}$, and is constant. In the case given it will be such as to oppose the introduction of more lines down into the circuit. That is, it will be such as to thrust lines up out of the circuit; *i.e.* the circuit will have a N polarity presented upwards, or the current will flow in the direction C A B D.

If, other things remaining the same, we incline the field so that the lines make an angle θ with the perpendicular to the

plane of the circuit, it is clear that now we add only $n \cos \theta$ marked lines per second to the circuit.

Resistance expressed as a velocity.—We have seen in Chapter XVIII. § 5 that, in the electro-magnetic system, R is of the dimensions of a velocity. It happens that the above simple apparatus enables us to show how R can in fact be measured by a velocity. (Compare also Chapter XVIII. § 4, *note* on method (ii.) of determining resistance.)

Let us assume for simplicity that the other quantities concerned are each unity; *i.e.* that the field is of unit strength and is perpendicular to the plane of the apparatus, so that we have one line of force piercing each 1 *sq. cm.*; that the rail is of 1 *cm.* length, and that the current is of unit strength.

The rail must move with such a velocity that, in the formula given above, n may be numerically equal to R. This means that we must add R *sq. cms.* to the area of the circuit each second; or, since the rail is 1 *cm.* long, this must move with a velocity expressed by the number R, or with R *cms.* per second.

Hence, under the above 'unit conditions' we can measure R by the velocity of the rail required to give unit current.

The velocity that would thus measure the absolute unit of resistance is that of 1 *cm.* per second, while 10^9 *cms.* per second measures 1 *ohm.*

§ 8. **Changes that give Induced Currents.**—In considering the question as to whether in any particular case there will be an induced current, we have to remember two facts.

(*a*) That when a conductor cuts across lines of force there is always an induced E.M.F.

(*b*) That when there is a change in the number of lines piercing a circuit there is a resultant E.M.F. induced in the circuit.

We will consider several cases.

(I.) *A circuit moving in a uniform field.*—If the circuit move parallel to itself (in such a way that the direction of movement causes the circuit to cut the lines of force), the number of marked lines embraced will not alter; and there will be no resultant E.M.F., or current, induced. The two sides of the circuit, it is true, do cut the lines; and so, by principle (*a*), there is an E.M.F. induced in each side. But these E.M.F.s are equal and opposed. There is no resultant E.M.F. in the circuit, but the top and bottom of the circuit, where the wire does not *cut* the lines of force, will be maintained at different potentials, as could be demonstrated connecting them with a quadrant electrometer.

If, however, there be any movement of rotation, then the number of marked lines embraced is altered, and there is an E.M.F. induced. It is, in fact, a very usual way of obtaining induced currents to rotate a circuit or coil in a field that is more or less uniform.

(II.) *Movement of a circuit in a non-uniform field.*—Where the field is not uniform, movements of a circuit will in general produce a change in the number of marked lines embraced, and so there will be an E.M.F. induced. In certain cases the two movements of translation and rotation respectively *may* give as a result no change in the number of lines embraced; in which case there is no E.M.F. induced in the circuit as a whole.

(III.) *Movements of an incomplete circuit; e.g. of a rectilinear wire.*—In considering the movements of an incomplete circuit, such, *e.g.*, as a rectilinear piece of wire, we may adopt two courses. We may consider it to form part of a closed circuit, the rest of which is at an infinite distance, and is indefinitely remote from the magnetic field in question; or we may consider the piece of wire by itself. It may be stated as a general law that there will or will not be an E.M.F. induced according as the wire cuts or does not cut the lines of force, as strings are cut by a knife.

Note.—If this were absolutely true we should never have an E.M.F. induced when the wire either moved in its own direction, piercing the field endforemost as a needle, or if it moved in any way in a plane in which lay the lines of force. Now, it is certainly true that in the latter case we never have an induced current. But in the former case we might have a current; for the wire might, so to speak, 'tunnel its way' end-on along the axis into a cylindrical system of lines of force; the self-induction in a rectilinear current, mentioned in the note to § 10, being a case of this nature.

While, therefore, the statement that . . . .

When a conductor moves in a magnetic field there will or will not be an induced current according as it does or does not cut the lines of force . . .

is a good general rule, covering all cases of importance, still it is well to seek for a rule that shall cover all cases. Now, such a rule can be found from Lenz's law, if we remember the nature of the field given by a rectilinear current, and so by a current of any shape. We may say that . . . .

When a conductor moves in a magnetic field there will or will not be an induced E.M.F. in it according as the field due to a current produced by such an E.M.F. can or can not oppose the change in field due to the movement.

(IV.) *Cases where the conductor (or circuit) is stationary, the field being altered.*—In such cases there is no obvious cutting of lines

of force. But both theory and experiment tell us that all such cases of alteration in the field about a conductor, or circuit, can be regarded as equivalent to cases where the same change has been produced by movements on the part of the conductor from a weaker to a stronger part of the field, or conversely. We may, therefore, consider that all cases included under this head have been already discussed above.

§ 9. **Coefficient of Mutual Induction, or of Mutual Potential.** —Let us consider two simple circuits, A and A′, carrying currents C and C′ respectively. Each gives a magnetic field, and each is placed in the field due to the other. If n and n' are the number of marked lines due to A′ embraced by A, and due to A embraced by A′, respectively, then the potentials on each circuit due to the other are Cn and $C'n'$, respectively. That is, it would require Cn *ergs* to bring up A from infinity to its present position, A′ remaining stationary; or $C'n'$ *ergs* to bring up A′, A remaining stationary. Now, if A be moved to infinity, we do $-$ Cn *ergs* work; and if A′ be moved after it into the same relative position as initially, we do $+$ $C'n'$ *ergs* work. But, on the whole, since things are exactly as they were, we must, by 'Conservation of energy,' have done no work. Hence it follows that Cn and $C'n'$ must be equal; and, therefore, two circuits exert on one another a *mutual potential.*

Again, the number n of marked lines of force due to A′ that pierce A are, *cæteris paribus*, directly proportional to the current C′ of A′; this following from the fact that in our system of units we measure currents by the field-strength produced at constant distance. And the number n' of marked lines that pierce A′ are in like manner directly proportional to C.

Hence, since we have shown the potential to be 'mutual,' this 'mutual potential' must be measured by some expression of the form $CC'M$; where M depends upon the shapes and positions of, and distance between, the two circuits, and not on the current-strengths.

Thus we have $Cn = C'n' = CC'M$.

If we make C and C′ both unity, we find that $M = n$ or n'. Hence we see that when unit current flows in each circuit, the number of marked lines due to the other, piercing each circuit respectively, is the same.

And we see, further, that the symbol M, whose meaning we had not given exactly, represents this number. The reader will notice that, for unit current, this number is, as we said, something depending upon the shapes and positions of, and distance between, the two circuits.

It only remains to state that *M* is called the ***coefficient of mutual induction*** or ***of mutual potential.***

Hence, ***the coefficient M of mutual induction, or of mutual potential, between two circuits is measured by the number of marked lines due to either that are embraced by the other when the currents are both unity. And when the currents are C and C′ respectively, then the mutual potential will be expressed by CC′ M.***

If we consider a current C sent through the one circuit, it will in consequence send C M marked lines of force through the other. If it take *t seconds* to establish the current (where *t* may be a small fraction of a second) then there are $\frac{C\,M}{t}$ lines added per second to the second circuit. Hence the induced E.M.F. $e = \frac{C\,M}{t}$. This shows us (1) the reason for M being called the coefficient of ***mutual induction*** as well as that of *mutual potential*; and (2) how, in such an arrangement, the magnitude of the induced E.M.F. *e* is inversely proportional to the time taken to establish or destroy the primary or inducing current.

§ 10. **Self-Induction. The 'Extra Current.'**—Let us consider a circuit comprising a coil of many turns and a battery; this circuit being so arranged that it can be made or broken at will.

When the circuit is made the current does not rise to a maximum at once. If we consider only the positive current, or the flow of + electricity from the + pole of the battery to the — pole, we find that it takes time to rise to a maximum; and that in so rising the turns of wire that are more remote from the + pole of the battery always lag behind those that are nearer to it. Each turn of wire, as the current in it increases, thrusts an increasing number of marked lines through the adjacent turns of wire. It, therefore, acts inductively on these, and, by Lenz's law, the induced E.M.F. is in such a direction as to oppose the rise in current-strength. There is, in fact, an *inverse induced current.* Thus the coil, as a whole, offers an inductive obstruction to the rise in current that is quite distinct from resistance; and that will, as far as it is

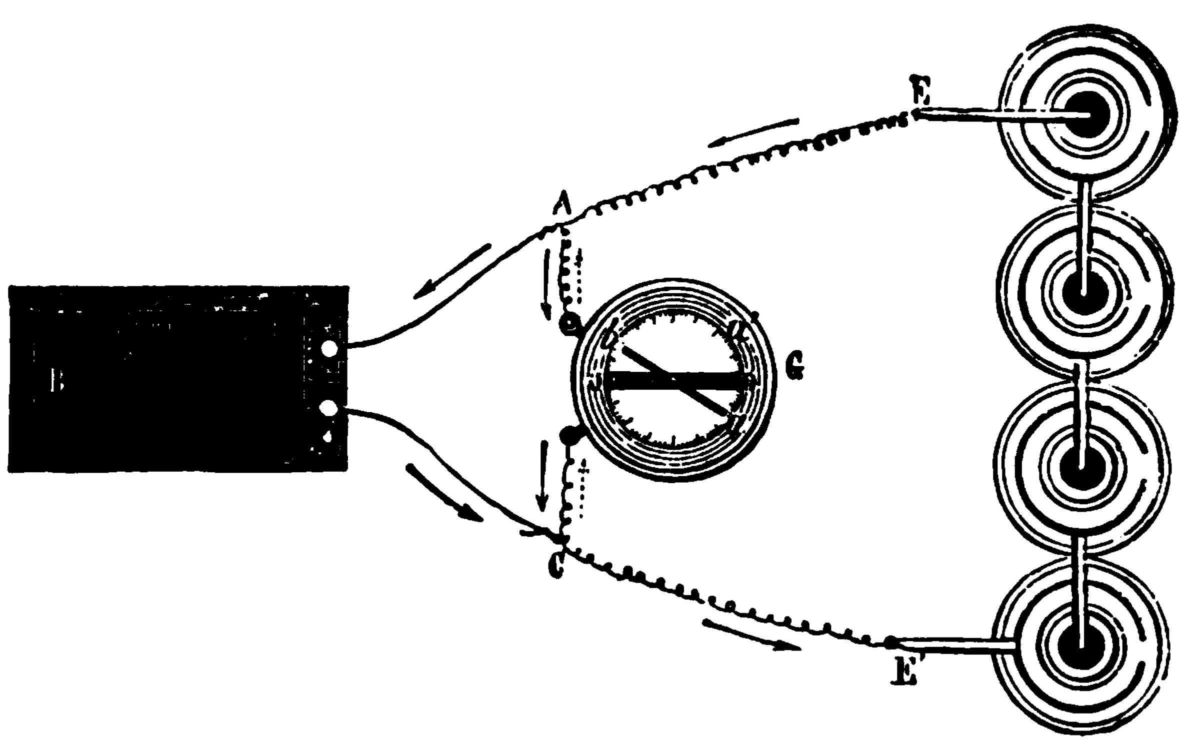
due to the above given cause, disappear if the coil be unwound and laid out as a straight wire.

Note.—In a straight wire there is also induction, but to a less degree. In this case each bit of wire gives a field of circular lines of force, not only about itself, but also, to a much smaller degree, about the wire in front and behind; and any increase in this field is also opposed by induction (*see* § 8 (III.), *note*).

When the circuit is broken, induction in the inverse direction, *i.e.* so as to oppose the cessation of the current, ensues; and there is an induced current *added* to the original current just as the latter is ceasing. Now, the primary current can be broken much more abruptly than it can be made; and hence, since *e* is inversely proportional to the time taken to effect the change in field, the E.M.F. of this *direct induced current* is much greater than is that of the inverse induced current; it may, indeed, be made very great indeed.

This direct induced current that occurs in the coil when the circuit is broken is called the *extra current.*

Experiments.—(i.) In the figure B is a coil in circuit with a battery whose poles are E and E′; this circuit can be broken at E. The two points A and C are connected by a short circuit that includes a galvanometer G.



When this circuit is complete the current flows as shown, and the galvanometer is deflected as indicated. When the circuit is broken at E, it is easy to see that a direct extra current in the coil would now pass round the galvanometer in the contrary direction to that in which it before passed, although in the coil B it flows in the same direction as before. Hence the extra current

would be indicated by an opposite direction of deflexion in G. In order to distinguish this reverse swing from the fall back to zero, which would ensue merely from the fact that the battery circuit is broken, it is necessary to keep the needle at zero by means of stops placed so as to prevent the former deflexion, but so as to permit of the second deflexion if such there be. Such a deflexion is, in fact, observed; and it demonstrates the existence of the extra current. (The present writer has found that very marked results may be obtained with two Leclanché cells as battery, an ordinary rough astatic galvanometer at G, and a powerful electro-magnet instead of the simple coil B.)

(ii.) If there be in circuit a powerful electro-magnet and a sufficient battery, and if the circuit be broken by a person holding in his moistened hands the two ends of the circuit, a shock will be perceived. The high *E.M.F.* of the induced extra current is sufficient to drive some current through the human body that has suddenly been interposed in the circuit.

We see, then, that when a circuit is made, the current rises slowly, and research has shown that its rise is oscillatory. It then continues uniform as long as the battery is constant. When the circuit is broken, there is a sudden leap in the magnitude owing to the extra current; and finally it ceases again in an oscillatory manner, much more abruptly than it began. All this is very readily exhibited by means of a curve, the abscissæ measuring time and the ordinates magnitude of current.

§ 11. Induced Currents of Higher Orders.

It is found, as might indeed have been predicted, that induced currents will themselves act as inducing currents.

We may arrange a series of coils somewhat as follows. First, a primary A, and round it a secondary B. B may then be in circuit with a coil B′ at a distance from A, so that currents induced in B will circulate also in B′; and round B′ is placed another coil C upon which B′ can act inductively, while A is too remote to have any direct influence. We will use the words 'direct' and 'inverse' when the currents are in the same direction as, or in the opposite direction to, the original current in A respectively.

We find that when the current is *made* in A, we have an *inverse* current induced in B and therefore passing in B′. This current, as it rises in strength from zero to a maximum, induces in C an opposed current, which will therefore be *direct*; and, as it falls again to zero, it induces in C a current in the same direction, which will therefore be *inverse*.

So, when the current in A is *broken*, we have a *direct* current induced in B; and in C, an *inverse* followed by a *direct* current.

CHAPTER XXII.

ARAGO'S DISC, RUHMKORFF'S COIL, AND OTHER CASES OF INDUCTION.

§ 1. **Induced Currents ('Eddy Currents') in Solid Metallic Masses moving in a Magnetic Field.**—When a conducting mass moves in a magnetic field currents are induced. Here, as always, the currents are such as to oppose the movement; work must be done on the masses in order to move them, and we have developed in the mass equivalent electric energy, which finally runs down into the form of an equivalent of heat.

Such currents will, as a rule, run in *eddies*; but if we connect two points in the mass by a conducting wire, these two points being so chosen as to be at different potentials, we thereby modify a portion, at least, of the induced 'eddy currents' into a current running round a definite circuit.

Experiments.—(i.) If a copper cube be caused to rotate between the poles of a powerful electro-magnet while this latter is as yet 'un-made,' and if we then 'make' the electro-magnet by sending a current round it, the cube will be visibly retarded or stopped in its movement.

(ii.) A disc caused to rotate in the field will get perceptibly heated.

(iii.) In § 3 we shall see how a current may be 'collected' from the disc.

(iv.) When a person cuts through the field between the poles of a very powerful electro-magnet with a copper knife, it will appear as though he were cutting through soft cheese, so strong is the opposition due to the induced currents.

§ 2. **Arago's Disc and Magnetic Needle.**—There is one case of the above that is of especial historic interest, and though in no way peculiar, will be described at some length. In 1824 Arago discovered the 'damping' effect produced by the presence of copper and of other conducting masses on magnetic needles oscillating near them. If a needle oscillate very close over a copper disc, and still more if it oscillate between two copper discs,

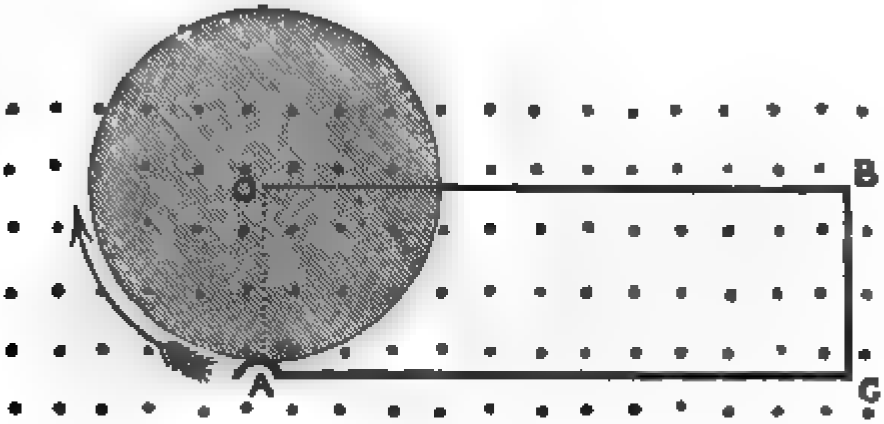
it will very soon come to rest. This is due to the induction of currents in the copper, these currents being such as to oppose that motion of the needle which is the origin of the induction.

In the figure we have a copper disc caused to rotate with great velocity under a magnetic needle; a sheet of glass between the two obviates any disturbance due to air-eddies caused by the rotating disc. The needle is deflected in the direction of rotation of the disc, and will, if the velocity be great enough, finally rotate also. This motion of the needle is not difficult to explain. In consequence of the rotation of the disc in the magnetic field due to the needle, currents are induced in the former. These currents are in such a direction that they oppose the relative motion of disc

and needle. They would be induced equally if the disc were stationary and the needle rotated. We have thus a reaction between needle and disc that tends to stop the relative motion; and so, if the needle be free to move, it will be urged round in the same direction as the disc. If slits be cut radially these interfere with the induced currents, and therefore with the actions described; but if they be cut in circles whose centres lie on the common axis of rotation of disc and of needle, their presence makes much less difference.

§ 3. **Continuous Current Collected from Barlow's Wheel.**— In the figure the dots represent a field of force, supposed to be running down into the plane of the diagram. The circle is a copper disc, revolving in the direction of the arrow. O B C A O

in a circuit, of which O C A is a wire connected at O with the axis of the disc, and having sliding contact with the edge of the disc at A; this circuit is completed by whatever radius of the disc happens to lie between O and A. If a current from an external source be sent in the direction A O B C A, the disc will rotate in the direction of the arrow. We can consider the field as acting always on the moveable radius O A, urging it (by the law given in Chapter XIX. § 7, and elsewhere) to the left. When so used, the disc is called '*Barlow's wheel.*' If there be no external source of current, but the disc be forcibly turned with the arrow, there will be a current induced in the circuit in such a direction as to oppose



the motion, *i.e.* in the direction O A C B O; the seat of the induced E.M.F. being in the shifting radius O A, which is always cutting the lines of force. The current thus induced gives lines of force rising perpendicularly upward from the plane of the diagram, opposed to those of the inducing field.

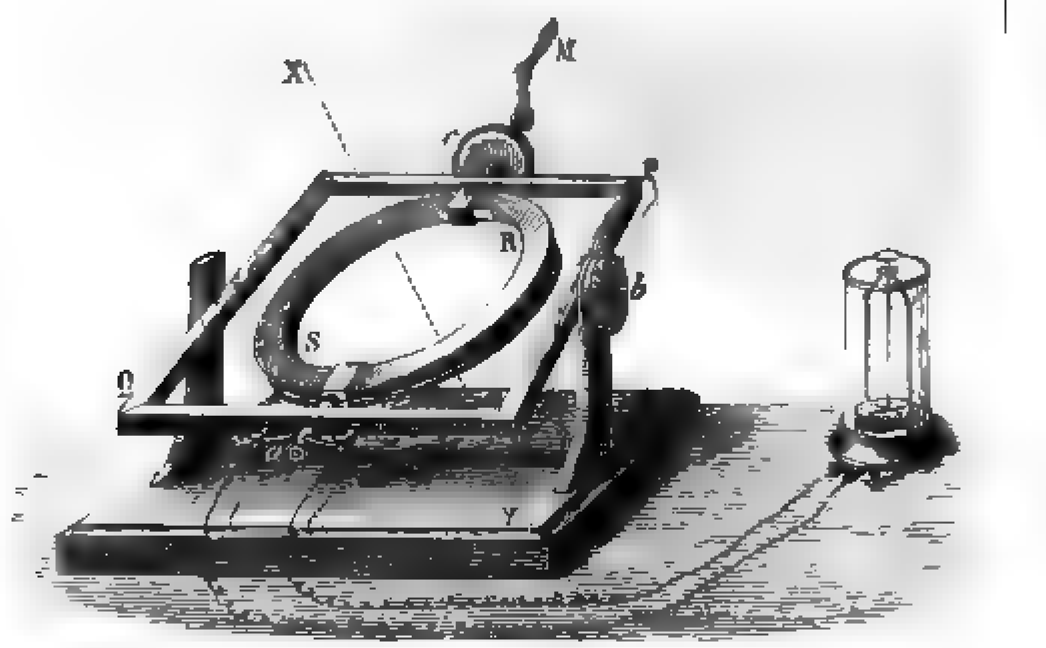
With a powerful electro-magnet this experiment is very easily performed.

If the rotation be performed in the opposite direction, a very remarkable result follows. The current is of course, in any case, in opposite directions in the portions O A and B C, which we may for simplicity suppose to be parallel. Hence the motion of the disc (when this turns in the contrary direction to that of the arrow), tending as it does always to move O A towards B C, will be against the electro-dynamic repulsion of the parallel and opposed currents in O A and B C respectively. This will give rise to induction opposing the motion; *i.e.* the opposed currents in O A

and B C, or the current of the circuit, will increase in strength. Thus, if a current be started in the circuit, and if then the external field be caused to vanish, induction will continue, owing to the action between the portions O A and B C. We thus have the phenomenon of a current maintained solely by work done on a system of copper conductors; there being no external magnetic field, no battery, and no electricity due to friction.[1]

If the disc turn as in the figure, the inductive action between B C and O A tends again to oppose the motion; and in this case the effect will be to lessen the current in the portions O A and B C, *i.e.* the current of the circuit.

§ 4. **Induction in the Earth's Field.**—We can obtain induced currents by rotation of a coil in the earth's field. In general, when a coil so rotates the number of lines of force piercing it is varied, and there will be E.M.F.s induced.



In the figure, S R is a coil of many turns of insulated wire, an XY represents the direction of the earth's lines of force. Th coil is turned by means of a handle M. The whole is mounte upon a stand in such a way that the axis of rotation may lie any direction whatever. If the coil be initially in the positio shown, viz. perpendicular to the lines of force, then as it is turn

[1] The apparatus as so used is called '*Sir W. Thomson's electric curr accumulator.*'

the number of lines piercing it will be diminished, becoming zero when the coil presents its edge to the lines of force. As it is further turned the lines will begin to pierce the other face, so that, between the initial position shown and that which is 180° from it, the number of lines piercing the one face of the coil will pass from a + maximum, through zero, to a − maximum. All this will induce an E.M.F. and current in one direction. Thus, if the lines of force run from X to Y, this rotation through 180° causes, first, a withdrawal of the + lines piercing the face that is initially uppermost, and then an introduction of − lines into this face. Both these changes, which may, indeed, be regarded algebraically as a continuous diminution in the + lines piercing this face, are opposed by the face in question becoming S in polarity; for this polarity opposes the weakening of the + field piercing the face, and also opposes the introduction of − lines. Hence, in this case, the current will run clockwise. But as the coil continues to rotate through the remaining 180° back to its initial position, similar reasoning shows us that an opposite current will be induced. Hence, as the coil rotates steadily, there are equal and opposite currents induced each half-turn. To collect these into one current that may be caused to deflect a galvanometer, &c., a *commutator* is arranged at *a*. This is a simple contrivance, to be described later in Chapter XXIII. § 4, by means of which the two ends of the coil interchange their connection with the two ends of the external circuit exactly at the time when the current changes its direction; *i.e.* just when the coil is passing the position shown in the figure.

Experiments.—(i.) It can be shown that the current is greatest when the change per second in number of marked lines embraced, for the same speed of rotation, is greatest. This will be when the axis of rotation is perpendicular to the lines of force; since then the lines pass from the *maximum* possible to the *minimum* possible.

(ii.) When the axis lies along the lines of force, there is no current. For now no lines pierce the face, and there is no change produced by rotation, the lines lying always in the plane of the coils.

(iii.) The current is greater as the velocity of rotation is greater.

This apparatus would, theoretically at least, enable us to determine the earth's elements at a place. For that direction of the axis which gives us zero induced current on rotation, is the direc-

tion of the earth's lines. And if the axis be now placed at right angles to these, the magnitude of the induced current for a given velocity of rotation would, if the 'constants' of the instrument are known, give us the earth's field-strength.

§ 5. **Induction Coils; General Plan.**—We have seen in Chapter XXI. § 1 how currents may be induced in a 'secondary' coil by the making and breaking of a current in another coil, called the 'primary,' placed inside the former. We have further seen how the magnitude of the induced E.M.F. depends mainly upon three conditions (*see* Chapter XXI. § 4), viz. . . . . .

(i.) The field-strength given by the primary circuit when the current flows through it.

(ii.) The number of turns of wire in the secondary that are pierced by the lines of force due to the primary.

(iii.) The suddenness with which the primary current is made or broken.

In order to fulfil condition (i.), the primary is provided with a soft iron core, and the current sent through the primary is strong. Thus we make or destroy a powerful field each time we make or break the primary current.

For condition (ii.), it is necessary to have very many turns of wire in the secondary. The wire, therefore, has to be very thin, and we get a high E.M.F. but a great resistance.

For condition (iii.), there are special arrangements devised. These will be given in § 8 and elsewhere.

By paying proper attention to these three conditions, we may obtain induced currents of very high E.M.F. but of very short duration. Such currents give sparks, produce shocks, and produce other phenomena not obtainable from the primary current. For certain purposes this modification of the energy of the primary battery is desirable.

An arrangement consisting of a primary coil with its own iron core, a secondary coil of many turns of wire, and a contrivance for making and breaking the primary circuit, forms what is called an *induction coil.*

If N be the total number of marked lines of force due to the primary that pierce the m turns of the secondary, and if it take t_1 and t_2 seconds respectively to break and to make the primary, then we have . . . .

$$\begin{cases} \text{On breaking, the direct induced E.M.F. } e_1 = \dfrac{N \times m}{t_1}. \\ \text{On making, the inverse induced E.M.F. } e_2 = \dfrac{N \times m}{t_2}. \end{cases}$$

By 'making' or 'breaking' we mean the increase from zero to the maximum, and decrease from the maximum to zero, respectively. The times t_1 and t_2 will be some small fraction of a second of time.

§ 6. **Practical Difficulties to be Overcome.**—The main difficulty encountered in the contriving of an induction coil which shall give induced currents of high E.M.F., lies in fulfilling the last of the above three conditions.

When the current is *made* there is an inverse current induced in the primary itself, and this causes the rise to a maximum to be relatively gradual. Hence the E.M.F. e_2 of the inverse current induced in the secondary will be relatively small, since the time t_2 (*see* § 5) is relatively great. (We may here remind the reader that when a coil has an iron core the inverse and direct currents of self-induction are far greater than when there is no iron core, since the field due to the core is added to that due to the current.) This difficulty cannot be overcome; and hence the devisers of induction coils have turned their attention entirely to rendering the *break* of current very abrupt, or t_1 very small, and so obtaining in the secondary a direct induced current of high E.M.F.

When the primary current is *broken*, there is in the primary a direct extra current due to self-induction, the presence of the iron core making this effect greater. This direct extra current has a high E.M.F., so high, indeed, that the current breaks disruptively across the break just made in the circuit. Thus, when we break the primary the extra current gives a spark at the break; and this both prolongs the current, or increases t_1, and therefore diminishes e_1, and also may fuse and injure the metal at the key by which the circuit is made and broken.

This difficulty is met and overcome by means of the *condenser* described in § 8.

Another obstacle in the way of abrupt *break* is the induction of eddy current in the iron core; these currents, of course, opposing the cessation of the primary current. This is obviated by

slitting the core longitudinally or by making it of a bundle of soft iron wires insulated from one another.

Again, the iron core takes a certain time to become magnetised and demagnetised. This is not such an important obstacle; but even this is nearly entirely obviated by means of the condenser, as will be explained in § 8.

Again, so great is the E.M.F. induced in the secondary that there is danger of a spark penetrating the insulating coating of this wire. This danger is obviated by a special method of winding.

§ 7. **Ruhmkorff's Coil.**—Fig. i. represents a form of coil called Ruhmkorff's coil, after the inventor. Here N and P are

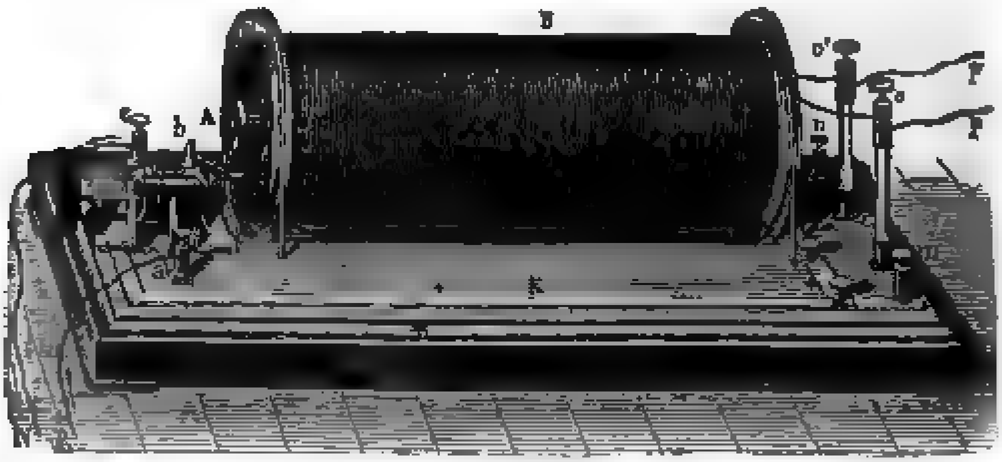


FIG. i.

the terminals of the primary connected with the battery; the primary coil is not shown, but its core A, composed of a bundle of soft iron wires insulated from each other so as to obviate eddy currents, is seen slightly projecting from the coil; C is an arrangement for turning the current off, or on, permanently; *b* is a piece of metal called the *hammer*, which alternately is attracted up to the iron core A, thereby *breaking* the current, and falls back by its own weight, thus *making* the current again; B is the secondary coil; *p* and *p'* are its terminals; inside the stand K is the condenser, whose function will be described later.

As a special instance we may quote the dimensions of Spottiswoode's large coil. This was constructed on the same principles as the above but the arrangement for making the circuit was worked separately and was not the automatic hammer contrivance shown in the figure

In this large coil the primary was a wire of about $\frac{1}{10}$ inch diameter and 660 *yards* in length, wound in about 1,350 turns. The secondary was a wire of about $\frac{1}{100}$ inch diameter, and 280 *miles* in length. The whole coil was 4 feet in length and 20 inches in diameter.

The maximum spark obtained was about 42 inches long. This coil is the largest yet made.

In fig. ii. we have a section of a coil very similar to that of fig. i., though the two do not correspond in all their details. Initially the hammer *b* lies on M, and the primary current from the battery X passes by P into the primary coil D, out of this again to the metal piece L, and so by the hammer *b* through M and N to the battery X again. By the passage of the current the core A is magnetised, increasing many-fold the field due to the primary coil alone. This creation of a field inside the secondary coil B induces an inverse E.M.F. in this coil; and, if the terminals *p p'* be in contact or very near to one another, an inverse current will pass. This inverse current is, as we have explained, of no very great E.M.F.

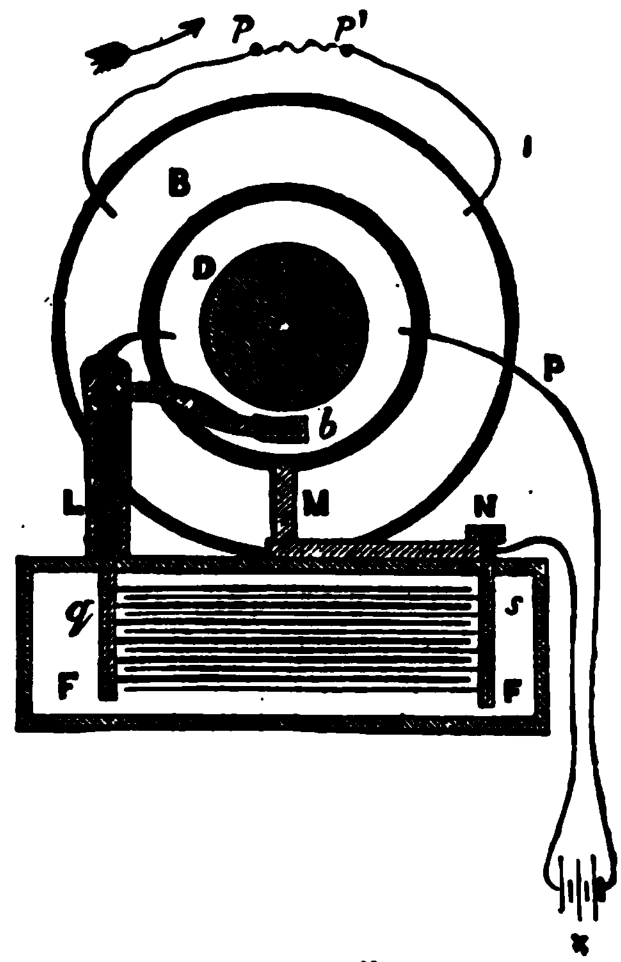


FIG. ii.

The moment that A is magnetised, the hammer *b* is attracted upward, and thus the primary circuit is broken. This causes the core A to be unmagnetised; and, in consequence of the withdrawal of the field, there is a direct E.M.F. induced in the secondary coil B. This E.M.F. is high, since the break of the primary is very abrupt, and t_1 (*see* § 5) is very small.

We have not yet described the part played by the condenser F F, as this matter deserves a section to itself. We may here remark that L and N are connected by the route N M *b* L only, there being no connection through *q* and *s*.

Caution.—It may be well to warn the student that exposure to shocks from the induced currents of an induction coil may be very

painful or even fatal. He should not take shocks from any but [illegible] coils, say those giving ¼-inch sparks as their maximum.

As regards the winding of the secondary coil, we may say that [illegible] wire is wound in sections, separated from one another by eb[illegible] partitions; these sections being such that any two portions of the [illegible] which are at a great difference of potential at the make or break, [illegible] separated from each other by one of these partitions. In fact, g[illegible] care is taken to prevent the coil being ruined by internal discharge[illegible]

The primary may be made and broken in many ways; the 'h[illegible]mer' arrangement is only one of many forms. Thus the 'ma[illegible]and-break' is sometimes effected between a platinum point and [illegible] alloy of mercury and platinum, under the surface of alcohol; alte[illegible]tion of the surfaces due to the extra-current spark being thus obviat[illegible] The speed of the make-and-break may be regulated by hand, by cloc[illegible]work, or in other ways. Thus the primary currents are more unde[illegible] control, and the inverse and direct induced currents may be observed separately.

§ 8. **The Part Played by the Condenser.**—Connected with the metal piece L*q* are a series of sheets of tin-foil; and connected with N*s* are another series. These two sets lie as shown in the fig. ii. of the last section, being separated by sheets of waxed paper or of other insulating material. The whole thus forms a condenser of very large surface, the set connected with L forming one plate, and that connected with N forming the other. Now when the primary current is broken by the rise of *b*, an extra current is self-induced in the primary. Were there no condenser, this would leap across the space between *b* and M in disruptive discharge; thus both prolonging the time t_1 of break, and injuring the surfaces of contact. But with the condenser the case is different. Before the extra current can give a spark across the gap between *b* and M, it must raise the two large condenser plates connected with *b* and M, to the necessary difference of potential. These plates have a very large capacity, however. Hence the extra current is employed in charging the condenser, and does not give a spark across M *b* at all.

This is the main use of the condenser; it prevents 'sparking,' and thus permits of an abrupt *break* to the primary.

Another use is as follows. While *b* is still in mid air between M and A, the extra current not only charges the condenser, but again rebounds (as it were); and, traversing the primary in the

ary direction, helps to reduce A more abruptly to the neutral

A further effect of the condenser is to lower the E.M.F. of inverse current that is induced in B on 'making' the primary. the current has to charge the condenser; and hence its rise maximum is delayed. We have said that it is on the direct uced current that we must depend for a high E.M.F.; and we ll show in § 11 how it is an advantage to reduce as far as sible the E.M.F. of the neglected inverse current. The conser, therefore, will have been of service in a third way.

§ 9. **Condition of the Secondary Circuit when Closed.**—ferring to the formula of § 5, we see that in the coil just scribed the number N is the same for both induced currents, t the time t_2 is greater than t_1.

From this it follows that the inverse induced E.M.F. e_2 is less than the direct E.M.F. e_1, but exists for a longer time. Therefore the inverse current is weaker than the direct, but lasts longer.

Now suppose the secondary circuit to be closed; no air-space left between the terminals p and p' of § 7. We should predict that for each make-and-break the total quantities of electricity flowing in the two directions respectively will be equal. Various experiments may be tried with the closed secondary circuit which will illustrate the conditions of things that then obtains.

Experiments.—(i.) When we include in the closed secondary circuit a cell consisting of a copper solution and two copper plates sufficiently large to obviate polarisation and to give good conduction, there is zero resultant action. Here the action depends upon quantity, not E.M.F.; and the result indicates that the quantities in the two currents are equal, and flow in opposite directions.

If the plates are small, so that there is high resistance and polarisation, there will be some slight action showing a predominance of the direct current which has a higher *E.M.F.*

(ii.) If we use a cell of acidulated water and platinum electrodes, we have results which vary with the size of these electrodes and with the conductivity of the dilute acid. In general we have liberated at each electrode a mixture of the two gases; these will be in proportions which are nearly equivalent at each pole, if the electrodes are large and the liquid very conducting, but will show a predominance of the direct current if the converse conditions hold. This is again due to the higher *E.M.F.* of the direct current.

(iii.) If we include in the circuit a galvanometer of sufficiently long coil, we have a feeble action in favour of the direct current; but this action is very

faint compared with that which obtains when (as will be explained in § 10) we cut off the inverse current altogether.

(iv.) A person included in the circuit experiences shocks from both currents, but mainly from the direct with its higher *E.M.F.* (*see* § 7, *caution*).

§ 10. **Secondary Circuit with Air-Break.**—If now we separate the terminals of the secondary circuit, gradually increasing the distance between them, we find that less and less of the inverse current breaks across the air-space ; until at last, by interposing a sufficient interval, we have left only the direct current, which from its high E.M.F. is able still to bridge over the gap with a disruptive discharge.

Experiments.—(i.) Interposing such an air-space in the circuit, we find that electrolytic cells in the circuit exhibit decompositions that indicate a powerful direct current.

(ii.) A galvanometer included in the broken circuit now shows a steady deflexion due to the direct currents that rapidly succeed one another.

§ 11. **Electrostatic Condition of the Open Secondary Terminals. The Charging of Leyden Jars.**—Now let us suppose the interval between the secondary terminals to be so great that not even the direct induced current can strike across.

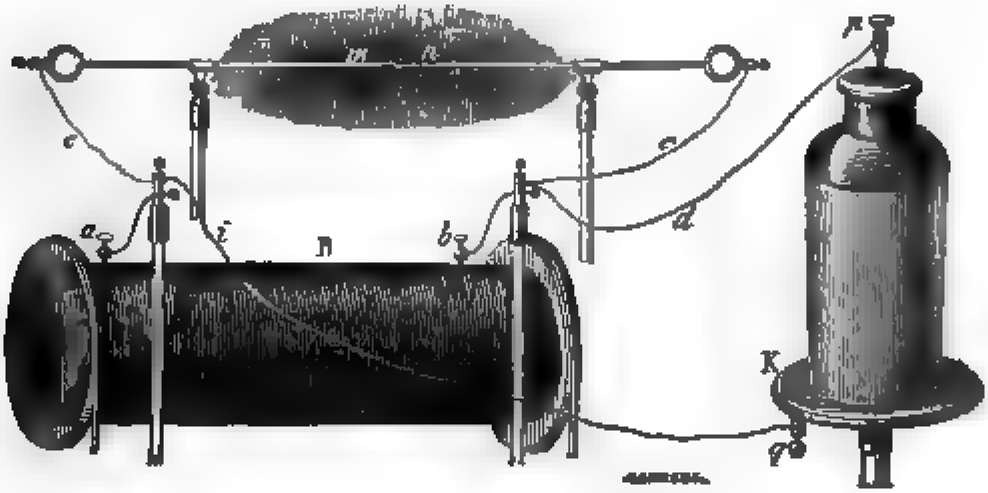
When the current in the primary is 'made,' the secondary terminals pp' become of positive and negative potentials respectively ; we may, *e.g.*, suppose p to become + and p' to become − But, as the E.M.F. of the inverse current is not great, so the electrostatic difference of potentials observed at p and p' will not be great.

When the primary current is 'broken,' the direct induced current will cause p to become —, and p' to become + ; and the electrostatic difference of potential between p and p' will now be far greater than it was.

When, as with the vibrator or other high speed interruptor, the direct and inverse currents follow one another rapidly, the terminals p and p' will show a permanent potential difference, owing to the difference in the E.M.F.s of the two currents. We shall find p to be −, and p' to be +. This Δ V is of course not really constant, but is continually rising and falling. When, however, the currents follow one another with sufficient rapidity, the needle of a quadrant electrometer will give a deflexion indicative of the average Δ V between p and p'.

We saw in § 8 that the *condenser* acted not only to increase the E.M.F. of the direct, but also to decrease the E.M.F. of the inverse, induced current. Hence the condenser acts so as to increase the permanent Δ V between the secondary terminals.

Coil used with Leyden jar.—If the two coatings of a Leyden jar are connected with *m* and *n* in the manner shown in the figure,



the discharge is modified in form. We do not now, as is the case when no jar is used, have a discharge each time that the 'break' is abrupt enough to give the E.M.F. required to overcome the air-space *m n*. On the contrary, the jar is charged only in virtue of the difference in E.M.F.s between the two currents; and the discharge across *m n* takes place only when the jar is sufficiently charged; so that the period between two discharges depends upon the capacity of the jar. The striking distance is very much smaller than when no jar is used; and, for some reason which the present writer believes has not yet been made quite clear, it depends further upon the capacity of the jar.

Charging a Leyden jar battery.—The above given method may be employed to charge a very large jar or battery. But where it is desired to charge such a battery and to leave it charged, another arrangement is adopted. Here the battery is charged by means of the direct current alone, an air-space being interposed. When the inner coating of the jars rises to a certain potential, sparks will cease to strike across the air-space. The battery may then be removed and its charge utilised.

The use of a 'secondary condenser,' as it is called, causes the

spark to be much denser; the noise also becomes very much greater.

§ 12. **Various Phenomena of the Secondary Discharge.**

(I.) *Luminous effects.*—When the discharge takes place under ordinary atmospheric pressures, and without the use of a 'secondary condenser' (*i.e.* a Leyden jar connected with the secondary terminals), it generally takes the form of a zigzag bluish-white spark, a peculiar sharp cracking sound accompanying it. Under certain conditions this may be seen to be surrounded by a luminous haze. Indeed, there appear to be two modes of discharge that occur at the same time; viz. the true spark, and a quieter form resembling the 'brush discharge' spoken of in Chapter VI. § 15. When the space is too great to admit of a spark passing, there may be observed a phenomenon resembling the brush, or silent, discharge that occurs under similar conditions with electric machines.

The luminous effects occurring in high, and in low, vacua will be discussed in § 14, &c.

(II.) *Chemical effects.*—When, by means of an air-break, the inverse current is cut off and the direct only passes, we may obtain all the electrolytical phenomena described in Chapter XII. But there is another effect produced by the spark of the induction coil, as also by the continued spark of the Holtz or other similar machine. If the spark be passed for a considerable time through mixtures of gases, there will in certain cases occur combinations that cannot be obtained *directly* by any other means. Thus we may in this way cause nitrogen to combine directly with oxygen.

(III.) *Heating effects.*—The temperature of the spark, especially when a secondary condenser is used, is very high. It is, however, rather difficult to distinguish clearly between effects due to the high temperature of the spark, and those due to the violent mechanical disturbance that accompanies the disruptive electric discharge. If, for example, we find the material of the terminal carried off and vaporised, we cannot say that this is wholly due to heat; it may be partly, at least, what we may more fairly call a mechanical action.

(IV.) *Use with the spectroscope.*—It is found that the discharge has the effect of carrying off and vaporising part of the material of the terminals; this being especially the case when the secon-

dary condenser is used. When the spark is observed with the aid of the spectroscope, there will be observed the spectra peculiar to the materials composing the terminals.

§ 13. **High and Low Vacua.**

In the present Course there is no space to give an account of the kinetic theory of gases. The student is therefore recommended to read elsewhere enough on the subject to understand the following points.

(i.) The general nature of a gas.

(ii.) How, though the motion of a single molecule cannot be followed, certain general statements may be made as to the average condition of the molecules.

(iii.) What is meant by the *mean velocity* and the *mean free path* of the molecules.

When we say that there is in a vessel an *ordinary*, or *low*, *vacuum*, we imply that the mean free path of the molecules is still of inconsiderable length as compared with the dimensions of the vessel, though, of course, much longer than under ordinary atmospheric pressure. Thus, in low vacua, no group of molecules can move across the tube, or from end to end of it, as a whole; if such a group were to start together in one direction, it would be broken up, and the molecules composing it would be scattered and dispersed before any considerable distance had been traversed. In fact, with respect to the dimensions of the vessel, we may say that the rare gas that fills it when there is a low vacuum presents the same perfect confusion of movement on the part of the molecules that is presented by gas under ordinary pressures, though, of course, there are much fewer molecules per unit volume.

When we say that there is in a tube a *high vacuum*, sometimes called a *Crookes' vacuum*, we imply that the mean free path is now of a length equal to, or exceeding, the dimensions of the vessel. When this is the case, a group of molecules started from one side or end of the vessel will as a whole reach and impinge against the other side.

§ 14. **Discharge in Low Vacua.**

(*a*) *Vacuum tubes.*—For the purpose of exhibiting the phenomena of discharge in low or high vacua, tubes or other vessels are prepared in which the air or other gas has been reduced to the required degree of rarity, a true vacuum being necessarily unattainable. These tubes are provided with platinum terminals that pass through and are fused into the glass. Usually there are two such terminals, one connected with the +, and the other with the −, pole of the source of current.

(*β*) Almost all that is stated in this and in the next section is true

whatever be the source of the current, whether an induction coil, a dynamo, a Holtz machine, or a voltaic battery. The E.M.F. of the source must, however, be very great. But in what follows we shall, unless the contrary be stated, assume that the source is an *induction coil.*

(γ) Consider now that we begin to exhaust the air from one of these tubes, the platinum terminals being connected with the secondary terminals of the coil. When the pressure has fallen to half an atmosphere or so, the discharge will have greatly altered its character. Instead of the comparatively short loud-sounding spark, we shall have a silent apparently continuous illumination of the whole tube, and the distance across which the discharge will take place will have become much extended.

At a pressure of 1 mm. or so the brilliant soft illumination is very remarkable.

The difference between this and the ordinary spark reminds one of, and is doubtless analogous to, the difference between the aurora borealis and the lightning spark.

This glow, when examined, gives us the spectrum peculiar to the rare gas that fills the tube.

(δ) *Fluorescence phenomena.*—This discharge appears to be peculiarly rich in the rays (mainly the invisible short ultra-violet waves) that excite *fluorescence.* When the glass of the tube contains *quinia sulphate*, *uranium oxide*, or other suitable body, the characteristic fluorescent glow will be observed, in addition to the light due to the gas in the tube.

(ϵ) *The discharge is disruptive.*—When two points are separated by an air-space and sparks due to any source of current are striking across, there is a constant average ΔV between the points. This is the case in disruptive discharges. But when the points are separated by a *conductor*, whether good or bad, the ΔV between the points varies according to the E.M.F. of the source, and the distribution of resistances in the circuit, as explained in Chapter XIII. § 11. In the present case the discharge is thus shown to be disruptive.

(ζ) *Striæ or strata.*—Under certain conditions the discharge is not continuous in appearance, but shows striæ or strata of greater luminosity separated by darker intervals. The figure here given indicates the appearance presented by these.

The existence of these, their form, and their movements, depend upon the rapidity of the *make*-and-*break* of the primary, the steadiness of the primary battery, and upon other conditions as yet not fully understood.

All striæ seem to have their origin at the + terminal.

(η) *Behaviour in a magnetic field.* — When a long luminous discharge in a wide vacuum vessel is exposed to a magnetic field, it behaves as would a flexible and extensible conductor carrying a

FIG. i.

current. Thus, in fig. ii. we see how a horseshoe magnet acts upon such a discharge; the discharge is *inflected*.

By means of a more complex piece of apparatus the discharge can be caused to rotate about a magnetic pole.

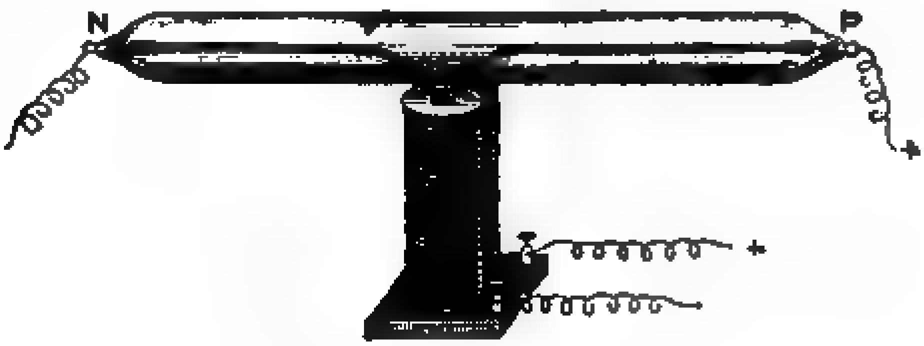


FIG. ii.

(θ) *Action of parallel discharges upon one another.*

Dr. Crookes has recently verified experimentally that two parallel discharges in a low vacuum obey Ampère's laws of parallel currents given in Chapter XVIII. § 16.

§ 15. **Discharge in High Vacua.**

The remarks made in (a) and (ε) of § 14 apply also to the present section. We now proceed to mention briefly the phenomena peculiar to high vacua. To Dr. Crookes is due our knowledge of these interesting and beautiful phenomena.

(α) Let us suppose that the tube in the figure has a negative pole at the centre, and a positive pole at each end.

When the exhaustion is what is usually called 'good' for a low vacuum, say, for example, that the pressure is only half a millimètre of mercury or about $\frac{1}{1500}$ of an atmosphere, there is observed on each side of the negative electrode a dark space, this space being bounded by a brilliant luminosity. Many experiments tend to show that the extent of this dark space measures the mean free path of those mole

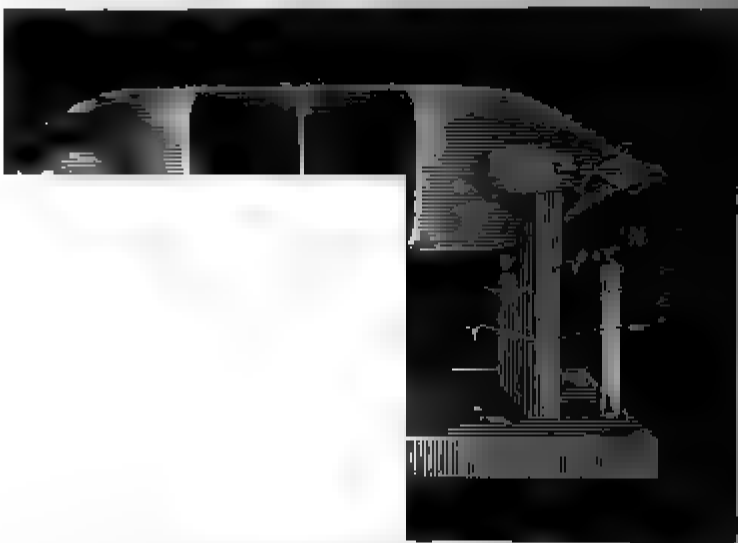


FIG. 1.

cules which are charged by contact with the – pole and are then repelled. It seems that such molecules are projected from the – pole normally to its surface; they beat back the other molecules, and at the surface over which this conflict is most energetic there is the brilliant light as evidence of the energy. There is no dark space at the positive pole.

(β) As the exhaustion proceeds, the dark space (or mean free path of the molecules) increases, and at last the dark space extends right across the vessel, and the molecules projected normally from the – pole bombard the glass on the opposite side. It appears that all ordinary glass thus bombarded exhibits a peculiar greenish phosphorescence. Many precious stones and minerals are rendered beautifully luminous by this molecular bombardment. This projection takes place from the – pole only, and it proceeds straight from it, no matter where the + pole may be. In the figure we see to the left the appearance in a *low* vacuum, the discharge taking place between the poles. To the right we see the phenomenon of a high vacuum; the direction of discharge from the – pole being independent of the position of the + pole.

(γ) *Deflexion by a magnet.*—It appears that the molecular stream is deflected by a magnet, but not in the same way as was the

ischarge in a low vacuum. In the first place the stream is permanently *de*flected, not merely *in*flected as was the low vacuum

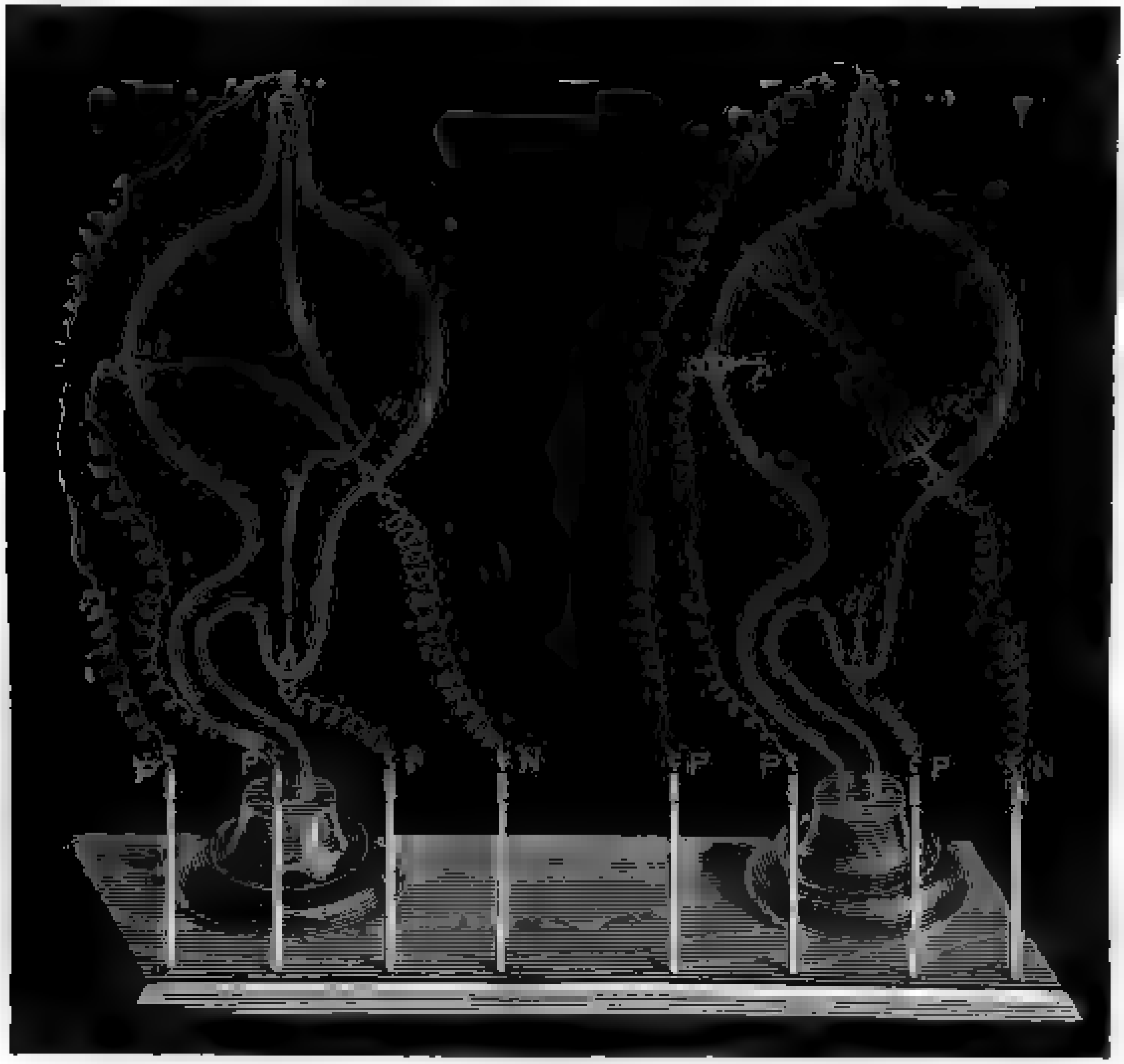


FIG. ii.

discharge of § 14 (η), fig. ii. This is shown here in fig. iii. In the second place the deflexion is independent of the direction in which

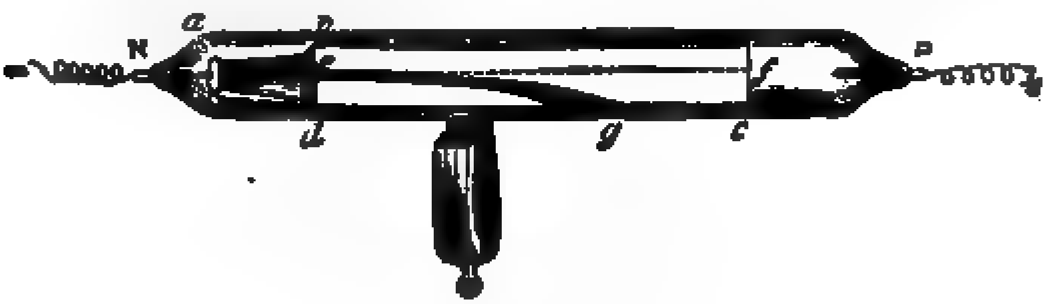


FIG. iii.

the molecules are moving. To use Dr. Crookes' simile, the deflexion may be *compared to the* action of a high wind on a stream of

mitrailleuse bullets, being constant for both directions of movement of the stream. This is not the case with low vacuum discharges.

(δ) *Rotation about a pole.*—So also we can obtain rotations about the pole of a magnet; but, unlike the case of low vacuum discharges, this is independent of the direction of movement of the molecules.

(ϵ) *Action of molecular streams upon one another.*—Dr. Crookes found that two streams running in the same direction side by side repel one another; they appear to behave as mutually repelling similarly charged particles, not as parallel currents.

Note.—It has been supposed that two similarly charged particles moving side by side have two actions on one another, the one being an electrostatic repulsion, the other being an electro-magnetic attraction. When they have the 'ratio velocity' (*see* Chapter XVIII. § 5), or about the velocity of light, it is supposed that these two actions balance one another; when a smaller velocity, the electrostatic repulsion predominates; when a greater velocity, the electro-magnetic attraction predominates. But from the extreme difficulty there is in conducting such experiments with any certain data as to velocities, &c., there is still much that is uncertain and demanding further investigation.

CHAPTER XXIII.

DYNAMO-ELECTRIC MACHINES.

In the brief sketch of 'Dynamos' here given, the author has been guided mainly by the writings of Prof. S. P. Thompson. To these writings the student is referred for fuller information on this subject.

To the same authority the author is indebted also for the principle of the diagram given in Chapter XXIV. § 7; and to the courtesy of the publishers, Messrs. Spon & Co., for the use of the diagrams given in §§ 16, 18, 19, 21 of the present Chapter.

§ 1. **General Nature of a 'Dynamo.'**—We have seen in Chapter XXI. that when a conductor cuts lines of force in a magnetic field, there is an E.M.F. induced in this conductor; and that when a coil is moved in a magnetic field in such a way as to alter the number of lines of force piercing it, there is a resultant E.M.F. induced in this coil. At the end of § 4 in Chapter XXI. we indicated the manner in which this induced E.M.F. may be calculated.

Those machines on which we expend mechanical work in causing the necessary movements, and from which obtain electrical energy, are called *dynamo-electric machines*, or more briefly *dynamos*.

§ 2. **General Account of Induced Currents.**— Let the symbols e, N_1, N_2, m, and t have the same meaning as in Chapter XXI. § 4, end. Then we have in general . . . .

$$e = \frac{(N_1 - N_2) \times m}{t}$$

If the resistance of each turn of wire be R, then the resistance of the whole coil is m R. Hence, if the external resistance be r we have by Ohm's law an induced current C measured by . . .

$$C = \frac{e}{mR + r} = \frac{\dfrac{(N_1 - N_2) \times m}{t}}{mR + r}.$$

We have then a case similar to that of a series of voltaic cells arranged in series. By the principles of Chapter XIII. §§ 6 and 13, we can determine whether it is more advantageous to arrange the turns of wire in 'series,' or to arrange them 'parallel' so as to be equivalent to one turn of thicker wire, or to arrange them partly in 'series' and partly 'parallel.' It all depends upon the relative magnitude of R and *r*.

As an example we may consider the simple case of Chapter XXII. § 4. Let the area of the coil, measured in sq. cms., be S; let the number of turns of wire be *m*; let the strength of the earth's field be I; and let the coil be completely rotated *a* times per second. When the coil has its plane perpendicular to the lines of force, there are *m* I S marked lines of force piercing the circuit; this is because there are *m* turns of wire, and because the 'number of lines of force' piercing each sq. cm. measure the field-strength, as is explained in Chapter X. §§ 13 and 14, and in Chapter XVII. § 1. When the coil is edgeways to the lines, no lines pierce it. When it has turned through 180°, so as to be again perpendicular to the lines of force, then *m* I S lines pierce it in the opposite direction. Hence there is a total change of 2 *m* I S lines. When it is turned through the remaining 180°, the same change occurs. For one complete revolution there is therefore a change of 4 *m* I S lines. And in one second there will be a change of 4 *a m* I S lines. Hence, if by means of a commutator the contrary E.M.F.s induced in the two halves of a revolution be caused to give a current in a constant direction (*see* Chapter XXII. § 4, and § 4 of the present Chapter), we have . . . .

$$e = 4\,a\,m\,\text{I}\,\text{S}$$

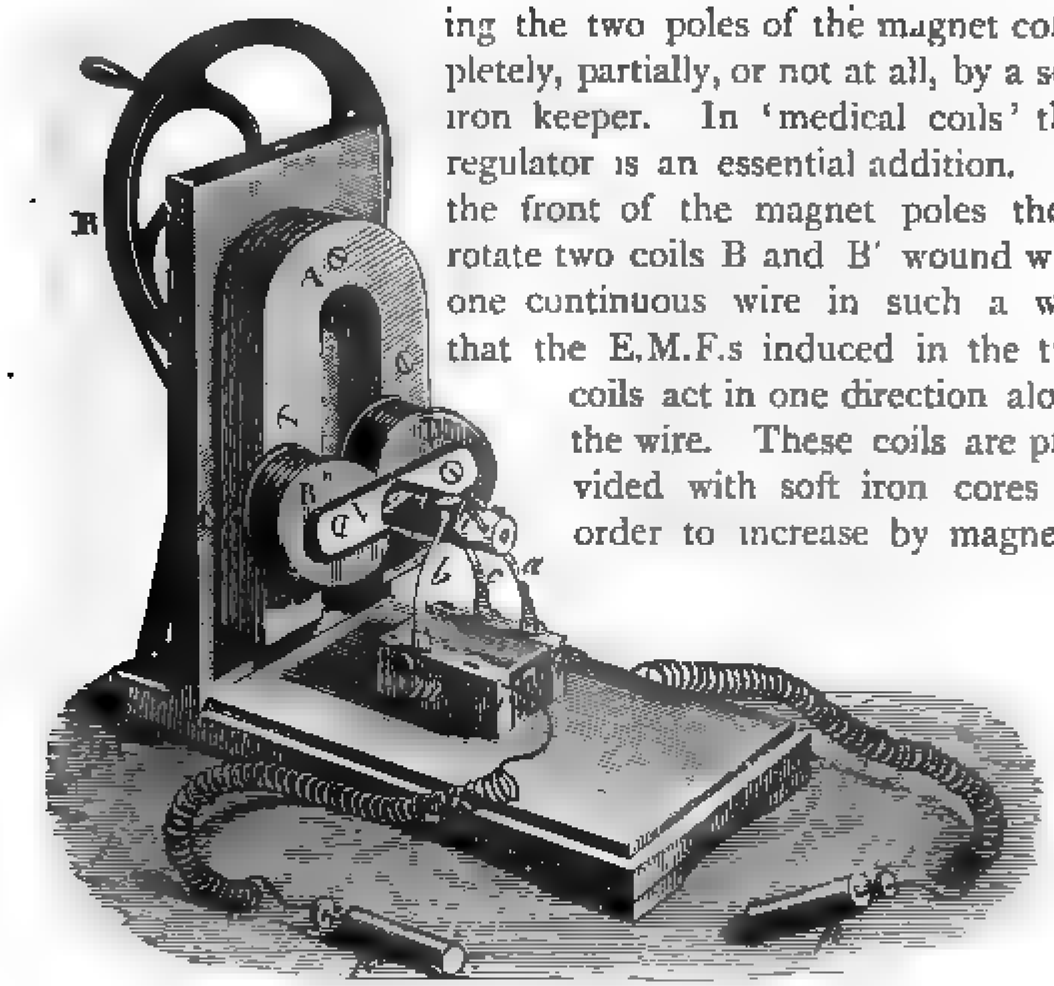
$$\text{and } C = \frac{4\,a\,m\,\text{I}\,\text{S}}{mR + r}.$$

The reader is recommended to return to the general remarks of this and the last section when he has read the accounts of several *dynamos* described in what follows.

We may add that though large E.M.F.s may be obtained by means of dynamos, we cannot approach in magnitude the E.M.F.s obtainable by means of induction coils.

§ 3. **Clark's Machine.**—The simplest form of *dynamo* is that constructed on the pattern of the 'Clark's machine.' This has a field given by permanent magnets, and is therefore often called a *magneto* machine.

In Clark's machine there is a permanent magnet A that gives the field; the strength of this field can be regulated by connecting the two poles of the magnet completely, partially, or not at all, by a soft iron keeper. In 'medical coils' this regulator is an essential addition. In the front of the magnet poles there rotate two coils B and B′ wound with one continuous wire in such a way that the E.M.F.s induced in the two coils act in one direction along the wire. These coils are provided with soft iron cores in order to increase by magnetic induction the number of marked lines of force piercing the coils.



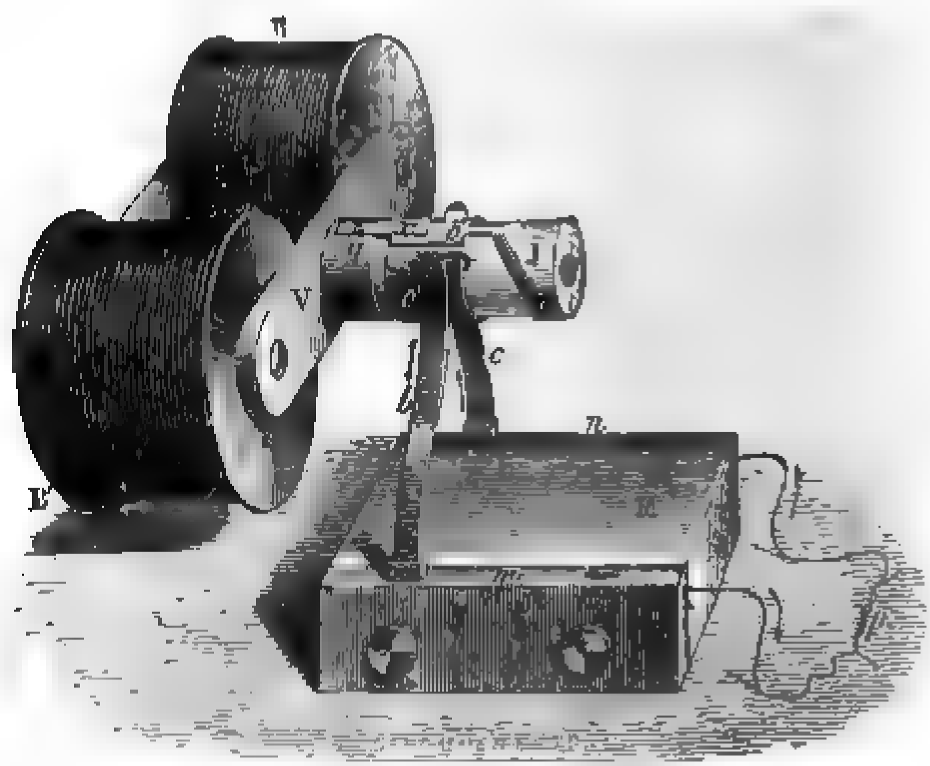
Let B and B′ start from the position shown. Then, as we turn them through 180°, we first withdraw the lines that pierce them, and then introduce lines in the opposite direction. Hence the E.M.F. in each coil acts in one direction through this 180° of rotation; and the coils are so wound that the E.M.F.s of B and B′ act in one direction. Through the second 180° of rotation the E.M.F.s are reversed; but a commutator, described in § 4, maintains the current in a constant direction in the external circuit.

The direction of current can easily be predicted from Lenz's law; for, as B leaves the N pole of the magnet and approaches

the S pole, it will acquire on the side turned towards the magnet a S polarity, or the current will there run 'clock-wise,' since this will oppose the motion.

By giving a rapid motion and having many turns of wire we increase the *a* and the *m* of § 2, or increase the induced E.M.F. We can thus get an E.M.F. large enough to give shocks and small sparks.

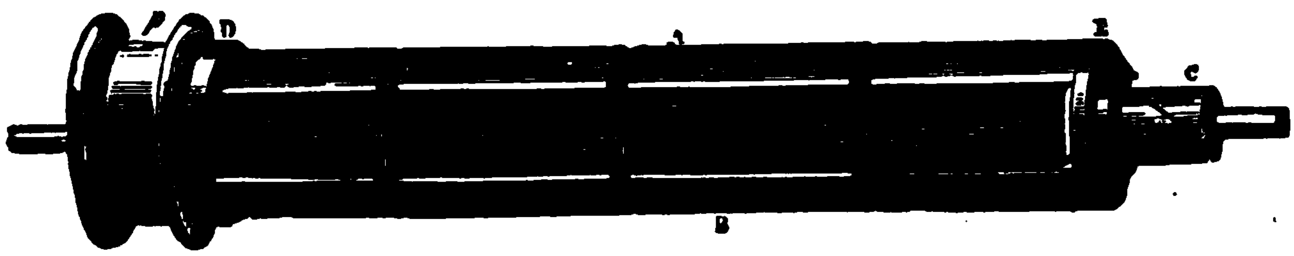
§ 4. **The Simple Commutator.**—The figure here given represents a simple form of commutator. The one end of the wire is soldered to the metal piece *o*, and the other to *o'*; these pieces



pass each nearly half-way round the axis J, but are insulated from one another by two slits, one of which is seen in the figure. The axis itself is of ivory, ebonite, or some other insulating material. The circuit is completed by means of the metal springs *b* and *c*. Just as the coils are passing the position in which their E.M.F.s change direction, the metal springs *b* and *c* cross the two slits and come into contact each with that metal piece (*o* or *o'*) that was just before in contact with the other spring. Thus the external current is maintained constant in direction.

The other metal piece, shown in the figure, is for another purpose that we need not describe.

§ 5. **Siemens's Armature.**—We may here mention that the system of core and coil, in which the *E.M.F.* is induced, is called an *armature*. In order to get the coil into a powerful field close to the magnets, Siemens invented a very compact form of armature.



In this form the core is a long cylinder of soft iron in which are two deep grooves cut longitudinally on opposite sides. The wire is then wound longitudinally in the grooves. The general appearance of this armature is shown in the figure.

§ 6. **The Self-Exciting Principle.**—It soon occurred to makers that electro-magnets would give far more powerful fields than could be obtained from permanent steel magnets. (We may here mention that the magnets giving the field in which the armature moves are called *field-magnets.*)

These electro-magnets could be excited by the whole or part of the current given by the armature of the machine itself.

It is found that the (so-called) soft iron cores of the electro-magnets always retain enough magnetism to give a current in the armature when rotated. This current, or part of it, passing round the field-magnet coils, increases the magnetism of the cores, and therefore the strength of the field. In consequence of this the current induced in the armatures becomes stronger. The reciprocal action proceeds until the field-magnets acquire some maximum strength depending on various conditions not specified here.

Mr. Ladd was the first to construct a 'self-exciting' machine of this sort. In his machine there were two armatures; one served to excite the field-magnets, and the other to give the external current.

§ 7. **'Continuous Current' Machines.**—In the machines described above we do not obtain a continuous current. In the coils there is in one half-revolution induced an *E.M.F.* that begins, rises to a maximum, and then ceases; in the other half-revolution an E.M.F of similar rise and fall, but contrary to the former in direction. This gives in the coils two currents in opposite directions, separated by an instant of zero current; and, if there be a

commutator, this gives in the external circuit two currents for each revolution, these currents being in the same direction, each rising to a maximum and falling again, separated from each other by an instant of zero current.

For many purposes it is desirable to have a current more continuous in nature. This can be effected in more than one way; we will describe very briefly one of these ways, and more at length another.

(α) The first method is to have an armature in which there are more than one pair of opposite coils, these pairs being arranged round the circumference of a circle. Thus we may conceive of a Clark's machine in which there are four, six, eight, or more pairs of coils; each pair is wound with one continuous wire whose extremities terminate in two metallic *segments* fixed to opposite sides of the axle. All these segments are insulated from one another; and therefore the circuit of each pair of coils is closed only when they come into contact with the two metallic springs or 'brushes' by means of which (as in the simple case of § 4) the circuit is completed through the external circuit. Each pair of coils thus feeds the external circuit in turn; and, by so arranging the springs that they come into contact with one pair of *segments* before they have quite broken contact with the preceding pair, we can insure that the current will never fall to zero. It is, however, of an undulatory character.

(β) The second method cannot be understood until some description has been given of quite a different form of armature. As this method is of great interest, we shall describe at some length one form of machine that is constructed on this principle: the machine, viz., that is called the *Gramme*, after the name of its inventor. His armature, however, is not quite original in construction, being similar to an earlier form, invented in 1862 by Prof. Pacinotti.

§ 8. **The 'Gramme.' Construction of Armature.**—In fig. i. we give a general view of a hand-model of the *Gramme machine*. Here we can see the ring-shaped armature turning between the pole-pieces *a* and *b* of the steel magnet A; the arrangements for giving a rapid rotation to this armature; and the collecting springs or *brushes c* and *i*, in contact with the axle on opposite sides of it, by means of which a current is sent through the external circuit.

Fig. ii. shows us the construction of this armature. There is a circular core composed of soft iron wires ; these are shown in

Fig. i.

section. From the way in which the armature rotates these wires do not cut the lines of force ; and thus induced 'eddy-currents,' a cause of waste of energy, do not occur as would be the case were the core solid.

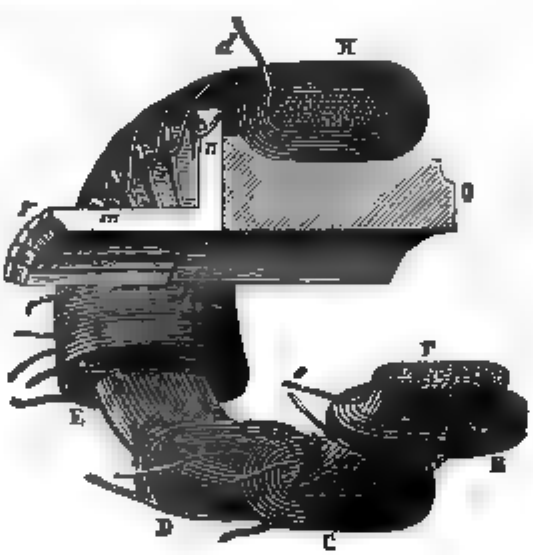


Fig. ii.

Round this core is wound the insulated copper wire that forms the coil in which the *E.M.F.s* are induced. *This wire*—it must be carefully noted—*is continuous*; and herein we have a great difference between this kind of armature and that of Clark or Siemens. In the upper part of fig. ii. we see this wire

as it is actually wound; and in the lower part of the figure there are shown a few sections of the coil, to indicate the mode of winding.

This wire is, as we said, continuous. But at regular and close intervals the wire is brought out, is laid bare of insulating material, is soldered to an insulated copper segment *m n* (a few only of these copper segments are given in the figure), and is then led back to continue the winding. From what we have said it is clear that we have a continuous wire with which we can, by means of the insulated segments *m n*, &c., make metallic contact at regular and frequent intervals all round the ring. These segments are arranged all round the axle, and it is with them that the brushes of the external circuit make contact. In fig. ii. O represents the solid body of the axle; it is made of hard wood or other insulating material, there being an inner core of metal, insulated from the segments *m n*, to give necessary strength.

§ 9. **The Gramme. The E.M.F.s Induced in the Ring.**—Let us now consider the condition of this ring-armature, with its continuous wire, when it is rotated between the poles *a b* of the magnet; and let us at present suppose that there are no springs or brushes in contact with the segments. Were it not for the core, the magnetic field would consist of lines of force running nearly straight across from one pole-piece to the other. These lines would pierce the coils of the armature, the number piercing each coil depending upon the position of that coil. Thus, a coil which is at the top of the ring depicted in § 8, fig. i., or which is situated *equatorially*, embraces the maximum of lines, supposing the field to be uniform. As the ring turns, this coil embraces fewer and fewer, until, when it lies edgeways to the lines or is situated 'axially,' it embraces none. (If the field be not uniform this statement must be modified; but, in any case, the coil embraces zero lines when situated axially.) As the ring turns still further, our coil begins to embrace lines *in the other direction*; and these reach a maximum when it is at the bottom of the ring, *i.e.* when it is again situated equatorially, 180° from its initial position. All this will give an induced E.M.F. in one direction, as has been already explained in Chapter XXII. § 4, and elsewhere. As the coil turns through the other 180° back to its initial position, there will, it is clear, be induced an E.M.F. in the opposite direction.

Thus, as the ring rotates, all the coils that are to the one side of the equatorial diameter give an E.M.F. in the one direction, and all those lying to the other side give an equal E.M.F. in the opposite direction. The result will be that no current will flow, but that the different parts of the wire, and so also the copper segments connected with them, respectively, will be maintained at different potentials. Thus, in the figure here given we shall have

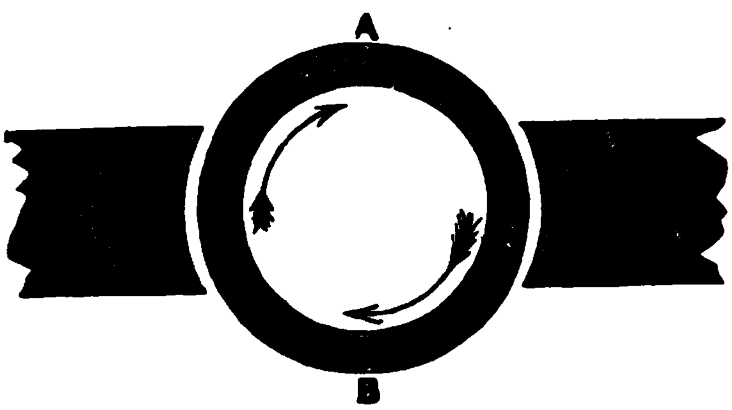


FIG. i.

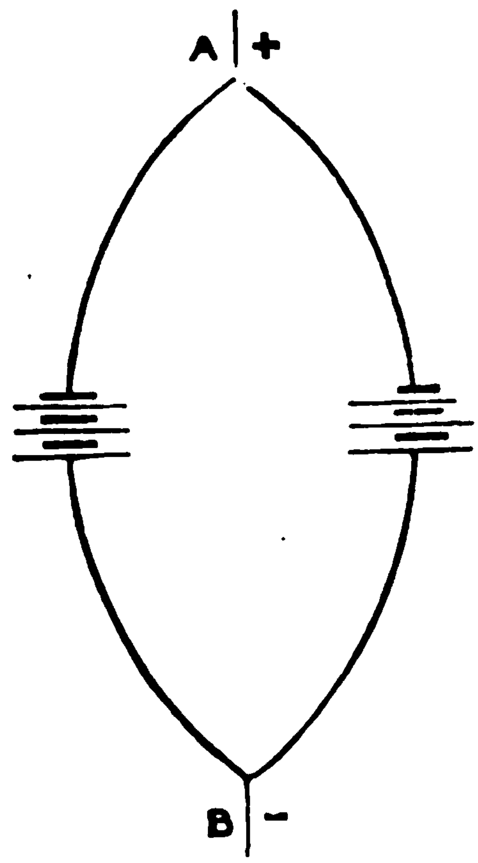


FIG. ii.

the wire at A and B, *i.e.* at the extremities of the equatorial diameter, maintained at the greatest ΔV, the potential falling from (say) A symmetrically along the wire, through each route, down to B.

Thus, though the *individual* coils assume all positions in turn, the ring, *as a whole*, maintains a character that is constant as long as there is constant field-strength and velocity of rotation. The revolving ring has been not inaptly compared to a stationary system of two equal batteries (*see* fig. ii.) set against one another; these batteries giving no current in their circuit, but maintaining the poles A and B at a constant ΔV. We can evidently obtain a current in an external circuit by connecting A and B.

The core.—We have hardly mentioned the core in what we have said. The fact is that the core only modifies the field; its presence does not affect the *general* theory of the ring as given above. Its action is mainly to concentrate the field upon the coils. The lines of force cannot now, to any considerable extent,

get across the space inside the ring; they follow the core through the coils. The presence of the core does not affect the two main facts: (1) that when the coils lie axially they cut no lines of force; and (2) that the change in direction of the E.M.F. occurs as the coils pass the equatorial position.

It is found that the soft iron wire changes its magnetism in a time that is practically inappreciable; and hence we can, if we like, regard the core as stationary, and the coils as slipping round upon it.

(For a modification of the sense in which *axially* and *equatorially* must be used when a current is running, *see* § 12.)

§ 10. **The Gramme. The Collecting Brushes.**—As in the analogous case of the two opposed batteries, represented in § 9, fig. ii., so, in the case of the gramme-ring represented in fig. i., we can obtain a current if we connect the parts A and B.

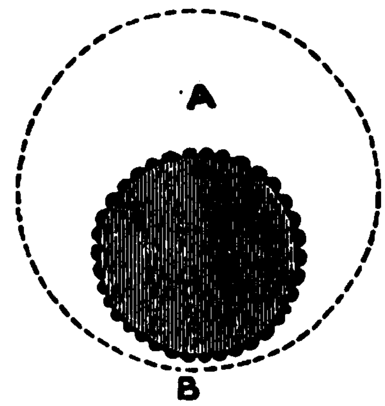
We have explained in § 8 how the wire that forms the coil is, at a series of points all round the ring, connected with insulated copper segments that are fixed in the axis on which the ring turns. If, then, there be metallic springs or brushes formed of wire, pressing against the axis at the extremities of the equatorial diameter of the same, these will practically be in contact with each segment of the wire coil in turn as it arrives at the positions A or B. In the Gramme machine these collectors are *brushes* of metallic wire. However great the speed of revolution, these brushes will never be jerked away from contact, as might a single spring; and with them it is easier to have contact always with two consecutive segments at once, and thus insure a current that is continuous, even though undulatory. The wires of the external circuit are, of course, attached to these collectors. The two halves of the ring then combine, as would the two batteries acting 'parallel,' to send a current through the external circuit.

§ 11. **Curve of Potential Round the Collecting Axis.**—Let us suppose the brushes to be detached, and the potential at different positions round the axis to be examined. This may be done by employing a quadrant electrometer, of which one pair of quadrants are to earth, and the other pair connected with an insulated wire brush; this brush is applied to the axis at different points in succession.

We find a fall or rise in potential from segment to segment

as we move from A to B or from B to A; this fall, in properly constructed machines, occurring symmetrically down both halves of the axis.

This fall in potential is, of course, discontinuous, since the segments are limited in number. But when the segments are very numerous the fall in potential round the collecting axis can be represented approximately by a continuous curve.



In the accompanying diagram, the shaded circle represents a section of the axis; the dark dots on this circle represent sections of the insulated copper segments; and the larger dotted curve represents the fall in potential from the upper extremity A of the equatorial diameter, both ways, round to the lower extremity B of the same. If we draw a radius from the centre of the axis, through the dot representing any particular segment, to meet the outer curve, then the intercept between this dot and the outer curve represents in magnitude the relative potential of the segment in question. (This will indicate to the reader how the outer 'curve of potential' is plotted out.)

In a good machine this fall of potential should be regular The brushes should be in contact with the axis at the points of maximum and minimum potential respectively. We shall find, however, that, as soon as a current runs, these positions of maximum and minimum potential shift round the axis.

§ 12. **The 'Lead' that Occurs when a Current is Running.**—*First*, let us suppose that the ring is revolving, but that, the external circuit being not yet completed, there is no current flowing. The two halves of the ring act merely to maintain two points, *e.g.* A and B, at a certain maximum ΔV. If the lines of force (*see* § 9, fig. i.) run straight across in what we have called an *axial* direction, then A and B will lie *equatorially* as shown. If, however, the core of the ring take an appreciable time to gain and lose its magnetism, the lines of force, and so also the line A B joining the points of maximum ΔV, will be shifted round to a greater or less extent. This question can be tested directly by experiment. A brush makes contact with the axis at different points in succession round its circumference. An insulated wire

connects this brush with one pair of quadrants of an electrometer, the other pair of quadrants being to earth. It is stated that when the positions A and B of maximum and minimum potential are thus tested, the line A B is found to be practically equatorial, or lies at right angles to the line joining the poles of the field-magnets, whatever be the speed of revolution of the ring. Hence we conclude that the core of the ring does not take any appreciable time to change its magnetism as it rotates.

This view is supported by S. P. Thompson and others; but there are some who are not satisfied of its truth.

Secondly, let us suppose that the brushes are attached and that an external current is running. If the direction of the current be followed, according to the principles explained in Chapter XXI., it will be seen that (*see* § 9, fig. i.) the currents in the two halves of the ring both act to make the core of a N polarity at B, and of a S polarity at A. Now the field-magnets tend to make the core of a N polarity at N′ opposite to S, and of a S polarity at S′ opposite to N. Hence, on the whole the core will acquire a N polarity at some point between N′ and B, and a S polarity at some point between S′ and A, opposite to the former. The effect will be as if the polarity of the core were shifted on in the direction of movement of the ring. Hence the line A B, of the points of highest and lowest potential, will also be shifted on through an equal angle. And therefore the brushes must also be shifted on.

This shifting on of the brushes in the direction of rotation is called '*Lead.*'

According to the above view, which is supported by experiments on the part of S. P. Thompson and others, the amount of lead depends upon the ratio between the field-strength due to the field-magnets, and the inductive action upon the core of the current in the ring. It does not depend upon velocity of rotation, provided that the current is constant; whereas, were '*lead*' due to residual magnetism in the core, the amount of this lead would depend upon the velocity of rotation though the current were constant.

§ 13. **Armatures Wound 'for E.M.F.' and 'for Current.'**—As with batteries, so with armatures; we can construct them either to give a high E.M.F., regardless of the resistance thereby unavoidably introduced, or to give a low resistance, thereby sacri-

ing E.M.F. All depends upon the nature of the external circuit (*see* Chapter XIII. §§ 6 and 13).

For high E.M.F. we must have many turns of wire; and therefore it must be long and fine, or of high resistance.

For low internal resistance we must have few turns, and thick wire. Having few turns, we have low E.M.F. (*see* § 2).

Sometimes there are on the same ring, or on parallel rings, two coils, these being capable of being joined either in series or in parallel circuit.

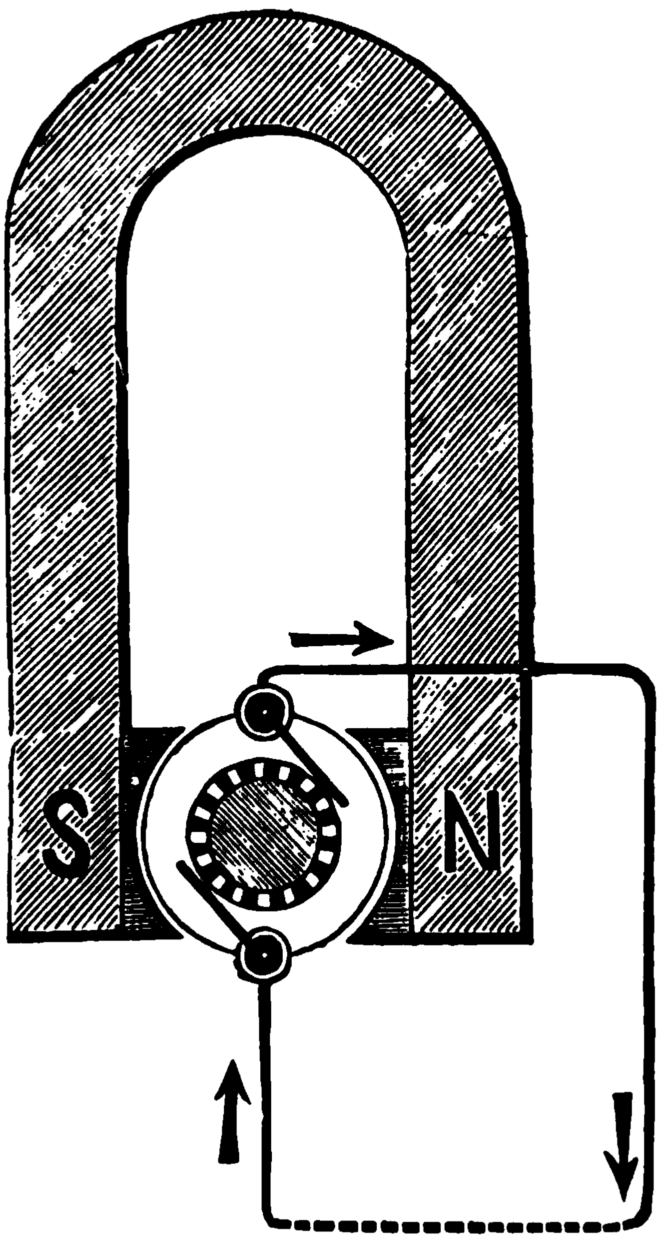
§ 14. **The Siemens-Alteneck Armature.**—As we have stated in Chapter XXI. § 8, the origin of the induced E.M.F. must doubtless be sought for in the cutting of the lines of force by the wire of the coil. Now since, as can be experimentally shown, there are hardly any lines of force that find their way across the inside of the ring, it follows that the wire lying on the inside of the ring is not cutting any lines, and is therefore passive as regards the productiveness of an *E.M.F.* It is simply a dead resistance. In the Siemens-Alteneck machine a *drum* replaces the ring, and the wire is wound over this, there being therefore no wire, saving at the ends of the drum, that does not cut lines of force. The long shape of the drum further renders smaller the proportion of the idle wire at the ends.

§ 15. **The Brush Machine.**—On a very different principle is constructed the 'Brush' machine; the name being after that of the inventors. The theory of this machine is so complicated that we cannot give it here clearly and yet briefly. We therefore refer the reader to S P. Thompson's work on 'Dynamo-Electric Machinery' (Spon & Co., London).

This machine gives very high E.M.F.s, and is therefore used where such are required.

§ 16. **Magneto Machines.**—We will now discuss very briefly the four most important types of machines, beginning with the '*magneto machine*' as the oldest and simplest form. The accompanying figure represents diagrammatically the typical 'magneto.' In this form, of which the machine shown in fig. i. of § 8 was an example, the field is given by a permanent steel magnet. Such fields are very constant, and therefore the E.M.F. of the current will vary simply with the velocity of rotation. For purposes of demonstration and explanation of the theory of dynamos, such a

property is very useful. Hence the *magneto* is much us teaching purposes. The same property, coupled with th regulation of the field by of a keeper (*see* § 3), mak *magneto* useful also for n purposes. But in the ar in commercial undertaki is never used, as being rel feeble for its mass as con with any of the electro-n forms.



Magnetos cannot be with certainty to be of the E.M.F. We can therefore them in series only, such a of magnetos acting like or chine of the sum of their *E.* and of the sum of their ances. We cannot couple parallel, or 'for current,' f machine of lower *E.M.F.* serve only as a branch through which the other m was driving a current. It act as a 'motor' (*see* Chapter XXIV.) and not as a driver of c

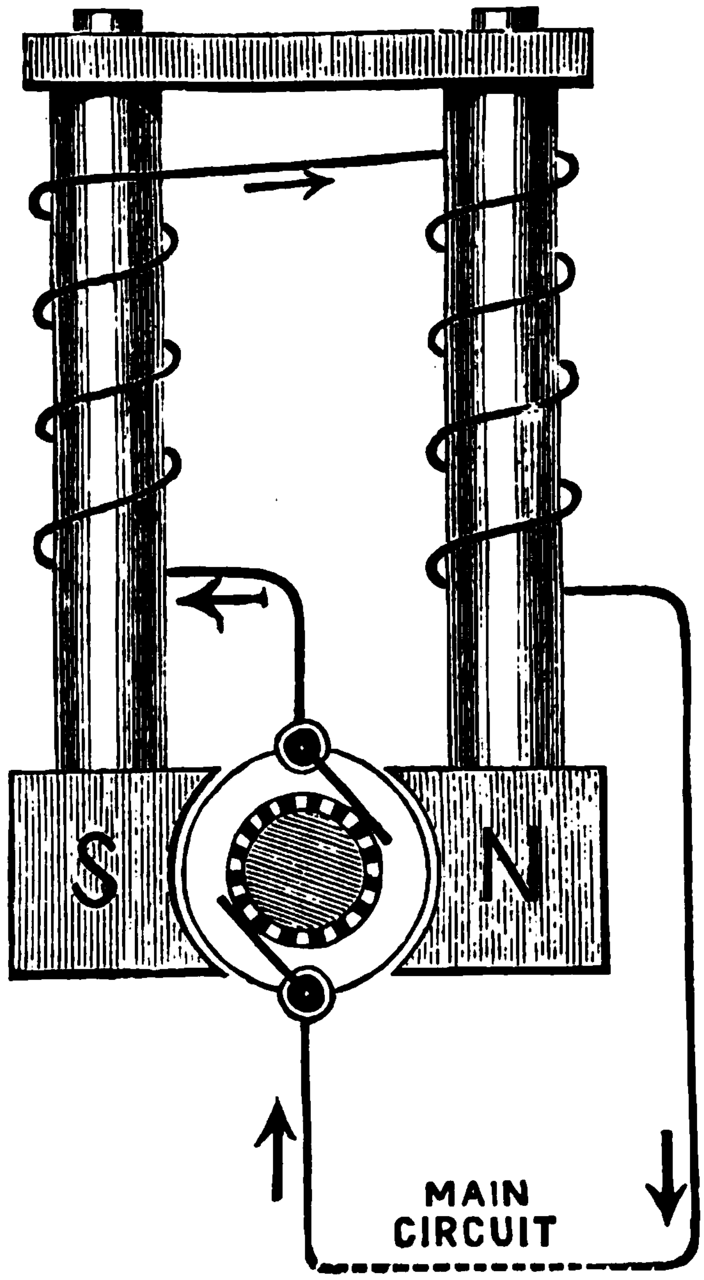
We may add that when a *magneto* is run at constant its *E.M.F.* is constant. Hence, in calculations we may de such as if they were voltaic batteries of constant *E.M.F.*

§ 17. **Separately-Excited Machines.**—When the field-m are *electro-magnets*, excited by a current from an indep source, we have what is called a *separately-excited machine.* form differs very little in principle from a *magneto*, but tl is much stronger, and we can vary its strength within ve limits. Hence we have great control over the E.M.F.

Such machines can be used in series or in parallel circu

An obvious disadvantage of this form is that we re separate source of current. Neither the magneto nor the rately-excited machine can be constructed so as to be sel lating, whereas we shall see that some other forms can constructed, these forms being compound.

§ 18. **Series-Excited Machines.**—In some machines, as was ated at the end of § 6, the current from the armature is led first and the field-magnets and en round the external circuit. machine thus constructed called a *series dynamo.*



Let us suppose that in ch a machine the external rcuit is completed, and that e armature is then run at certain velocity. There will e initially a weak field, due the residual magnetism in e cores of the field-magnets, nd this will give a slight itial current in the armature. his current, passing through ne coils of the field-magnets, ill make the field stronger; his will induce a stronger current in the armature, and o on until some final, limiting alue is attained. This limiting value of the current depends upon . . . . . .

(i.) The construction and resistance of the armature, and upon ts core.

(ii.) The construction and resistance of the field-magnet coils, and upon their cores.

(iii.) The velocity of rotation of the armature.

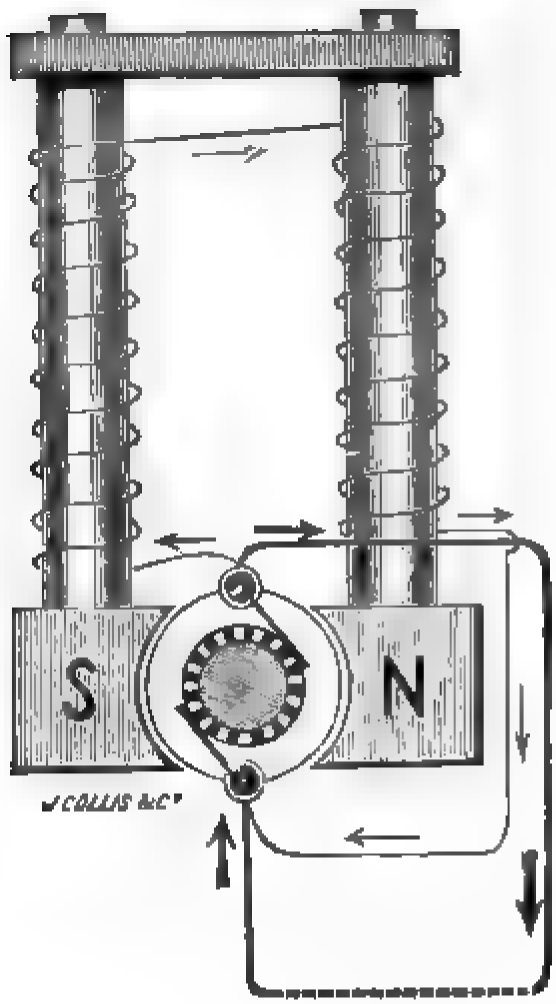
(iv.) The resistance of the external circuit.

Let us suppose that the resistance of the external circuit be ncreased. This will, of course, produce directly a decrease of current, and this in turn will weaken the field and cause a further lecrease. Finally, the current will settle down to some new value lepending upon the new conditions.

In the case, therefore, of a *series-dynamo*, any change in the external resistance produces a greater change in the current than would be the case with a magneto or separately-excited machine. It is very '*sensitive*' to variations in external resistance.

If the object be to drive a constant current, whate external resistance, the series-dynamo must certainly *not* be for (since the E.M.F. depends upon the current) the E.M. just when it should become greater in order to drive the against an increased resistance, and rises just when, o decreased resistance, a smaller E.M.F. is desired.

The *series dynamo* is particularly suited to such cases of Chapter XXV. § 18, where there is an external circuit c *parallel* branches, of which there may be at one time m at another time only one in use, and where it is desired the current through any one branch constant, whether th be in use or not. For such a purpose we must have a that is greater or less according as there are more or fewe branches in use. Now we know, by Chapter XIII. § 5, more branches there use the smaller is t resistance. And we h that the series-dynam than any other form chine, will respond creased resistance by of current. In such rangement, therefore respond to demand upon it.



§ 19. **Shunt-Dyn** In a very important machines there are t native paths for the to pursue when it le collecting brushes. path is through the the field-magnets, w made long and of high resistance ; the o is through the exte cuit. If there be in into the external circuit more resistance or an opposed this will throw a larger proportion of the current ro

field-magnet, and will increase the E.M.F. of the machine. Hence, this method of construction is particularly suited to cases where it is desired to keep a constant current under variations in the external circuit; for the E.M.F. of the machine rises as opposition in the external circuit increases, and falls as it decreases. Such a machine is suited to the case of, *e.g.*, electric lamps arranged *in series*, where it is desired to maintain a constant current, no matter whether one or more lamps be thrown in. It is entirely unsuited to the demands of the case mentioned in § 18, viz. that of an external circuit composed of many parallel branches. The converse is true of the series-dynamo.

§ 20. **Other Methods of Winding.**—In §§ 18 and 19 we have indicated for what purposes the series-, and shunt-, dynamos are respectively adapted. It is found, however, that neither the one nor the other give with sufficient exactness the result demanded. It is found necessary to combine different methods of winding. In some machines we have a combination of *series-excitation* with *separate-excitation*; in some, *series* with *shunt*; in others again, *shunt* with *separate*; and so on. The choice of method of winding is determined by the nature of the work that the machine is desired to perform, and upon the sort of 'constancy' that is demanded of it.

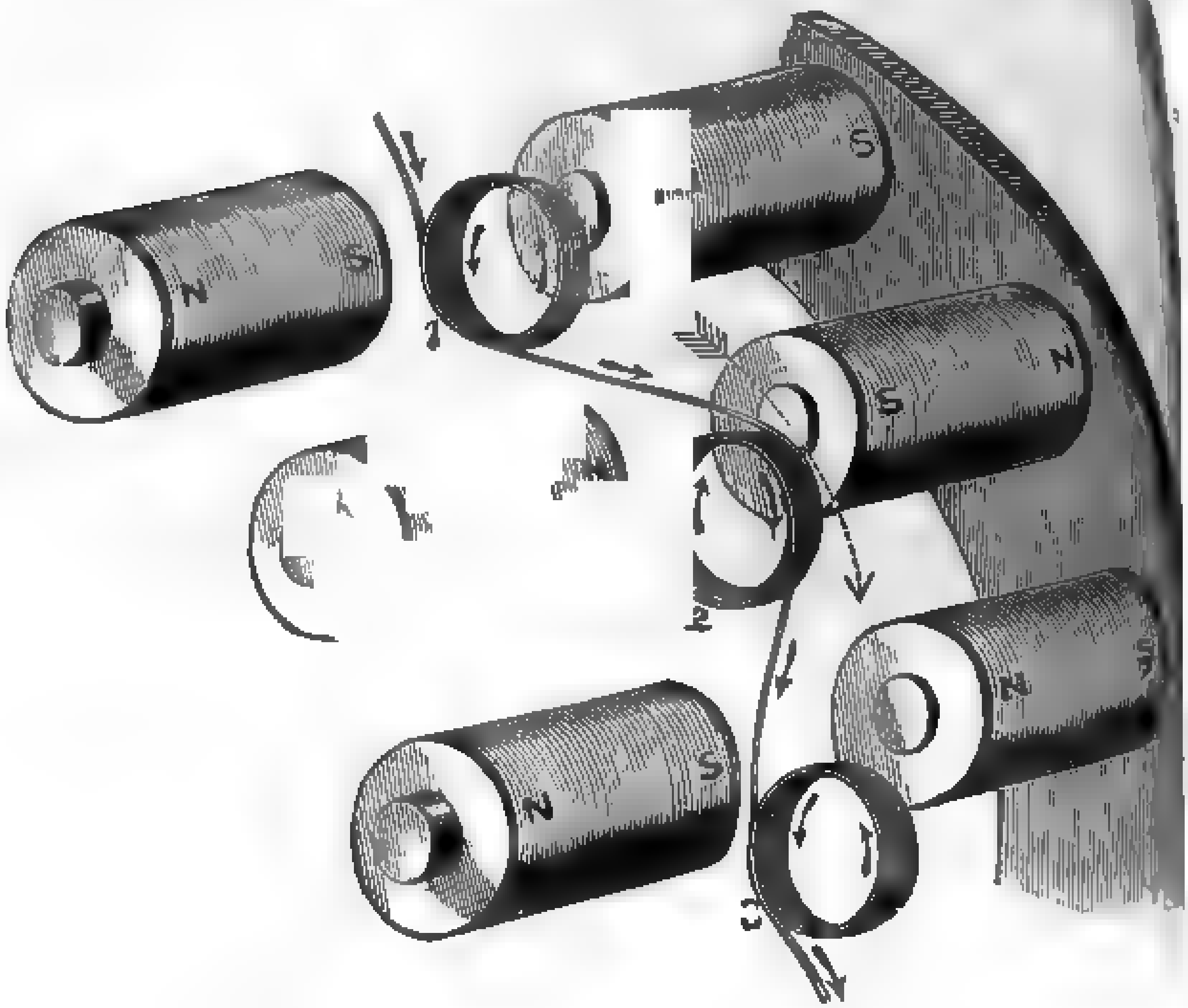
§ 21. **Alternate Current Machines.**—For certain purposes, chiefly for electric lighting (*see* Chapter XXV.), it is not necessary, nor even advisable, to obtain currents that flow in a constant direction. Machines that give alternating currents are called *alternate current machines.*

If in the Clark or Siemens's armature (*see* §§ 3 and 5) there be on the axis two metallic rings, insulated from one another, with which the collecting springs are continually in contact, and if the two ends of the armature coil be soldered to these respectively, then we shall have in the external circuit the same alternating currents that occur in the armature.

In fact, it may be stated as a general principle that in most machines, excepting always those in which the armature is constructed on the same principle as is the *Gramme* armature, the induced currents are naturally alternate, and require the addition of a commutator if it be desired to obtain in the external circuit currents in one constant direction. As a rule, such commutation introduces waste of energy from extra-currents and sparks produced.

It may also be stated, as generally true, that it is easier to construct alternate current machines which shall give a very high E.M.F. than it is to construct machines which shall give a current of high E.M.F. in a constant direction.

The accompanying figure illustrates the *principle* of more than one form of alternate current machines. A series of coils provided with soft iron cores is arranged on the circumference of a wheel,



their axes being parallel to the axis of rotation. Three of these coils are diagrammatically represented in the figure.

These coils rotate between the opposed poles of a series of electro-magnets, arranged round the circumferences of two other wheels, as indicated in the figure. The large dotted arrow represents the direction in which the circle of coils is rotating. The small arrows show the direction in which the current flows in each as it leaves the one pair of electro-magnets and approaches

the next pair. When the coils pass the next pairs of electro-magnets respectively, the current in each is reversed. The method of winding is such that the *E.M.F.s* of the coils act in one and the same direction along the wire. The final terminals of the wire are soldered to two insulated rings on the axis, against which the two collectors are pressed respectively. (Sometimes the current from a few coils is 'commutated,' and is used to excite the field-magnets; in such a case we have practically two machines in one.)

§ 22. **The Ferranti Alternate Current Machine.**

In this machine the armature is composed of flat 'strip' copper arranged in folds, so as to present an appearance somewhat similar to that of a star-fish.

Here we have no iron cores; but, owing to the extreme thinness of this flat, star-shaped armature, the two sets of field-magnets can be placed so close to one another that the field cut by the copper-strip of the armature is exceedingly strong.

Neither have we, in a strict sense, any 'coils;' the copper-strip being arranged in open folds. Hence we cannot here use the expression *alteration in number of lines of force embraced by the coils.* We must fall back upon the principle that, as we have already stated, underlies the above-quoted condition for induction; this more fundamental principle being that there is an *E.M.F.* induced in any conductor when it cuts lines of force.

In the present case things are so arranged that the *E.M.F.s*, induced in the several folds that are cutting lines of force, act in one and the same direction. These *E.M.F.s* alternate in direction as each fold approaches a pair of field-magnets, and leaves the same, respectively.

CHAPTER XXIV.

DYNAMOS AND MOTORS.

§ 1. **Magnetos, or Separately-Excited Machines, as Motors.**—Let us consider what will happen if a current from an external source be run through the armature of any machine in which the magnetic field is constant, *i.e.* through the armature of a magneto or of a separately-excited machine.

A reference to the figures of Chapter XXIII. §§ 3, 5, 8, and 9, shows us that action will take place between the field-magnets on the one hand, and the armatures (mainly the iron *cores* of these) on the other; this reaction causing rotation of the armatures, since they only are moveable.

Either by tracing carefully the direction of polarity given in each case to the armature by the external current, or by the application of the arguments given in Chapter XXI. § 6, and summed up in *Lenz's law*, we can see that the rotation will be of such a nature that an E.M.F. is induced in the armature *opposed* to the E.M.F. of the external current. If the armature move with little friction, and be not constrained to do any work in turning, it will move faster and faster as the action continues. The faster it rotates the greater will be the induced E.M.F. opposed to the exciting current, and the smaller will this current consequently become. The limit of velocity possible is reached when the induced E.M.F. equals that of the exciting current, so that this is reduced to zero; this limit can never be reached in practice owing to friction. If the exciting current be reversed in direction, so will the rotation. A dynamo thus caused to turn by the passage of an external current through its armature is called an *electro-motor*. Any dynamo can be employed as a motor, but as a rule the best form for a motor is not the same as the best form for a driver of current.

Two 'magnetos' coupled.—If we connect the terminals of two

magnetos, such as the Grammes drawn in § 8, and if one machine be worked, the other will turn also in consequence of the current driven through its armature. The two machines will act, at least as regards direction of rotation, as would two wheels connected by an endless strap passing round them. A galvanometer in the circuit will indicate the changes in the current that ensue when that machine which is for the time acting as *motor* is held fast, allowed to turn slowly against friction, or allowed to run freely, respectively. [Compare with § 4.]

Magneto and secondary cell.—If we connect a Gramme, or other magneto, with a Faure's 'accumulator' or 'storage cell' an interesting experiment may be tried. Let us charge the cell by means of the machine, and let us then leave off turning this latter. The Faure's cell will now drive a current reverse in direction to that by which it was charged. This will pass through the Gramme, and will cause it to turn as a *motor*; the direction of rotation being, of course, such as to oppose the current from the Faure's cell. Now such a direction of rotation must be the same as that in which the Gramme turned when it was charging the cell. Or we shall see the Gramme, acting now as a motor, continuing to turn in the same direction as that in which it was turned when it acted as a driver of current.

§ 2. **Series-Dynamos as Motors.**—Inasmuch as the magnetic field in the series-dynamo changes sign when the current through the armature does so—[for the field-magnets are here in one circuit with the armature]—it follows that a series-dynamo when driven as a motor will turn in one direction only. We can readily determine whether this direction will be *with the brushes*—*i.e.* in the direction in which the machine is intended to turn, that in which the segments on the axis do not meet the points of the collecting brushes; or whether the direction will be *against the brushes*—*i.e.* such that the segments on the axis meet the points of the brushes. Imagine the machine to be turning *with* the brushes as a driver of current, and now imagine an external current to be driven through the machine contrary in direction to that which was driven by the machine. Were the field constant in direction, the machine would turn in the same direction as before, since such rotation would oppose the external exciting current. But the field is reversed by this reverse exciting current.

Hence the dynamo, acting now as a motor, will be driven in a reverse direction *against the brushes.* If we reverse the exciting current, we reverse the current in the armature and also the field; and hence the machine runs still in the same direction, *i.e.* against the brushes.

Similar remarks apply to the *shunt-dynamo* when used as a motor.

§ 3. **General Remarks on Dynamos and Motors.**—In what follows we shall speak of that machine which is driving the current as the *dynamo*, and that which is being driven by the current as the electro-motor or *motor.* This latter is generally of a different construction from the former, though not differing in principle. We shall use the following symbols. $E =$ the E.M.F. of the dynamo ; $e =$ the reverse E.M.F. induced in the motor when this is turned ; $C =$ the current in the circuit ; R the resistance of this whole circuit, including the armatures of both dynamo and motor ; C_0 the current that runs when the motor is not allowed to turn, or when e is zero.

By Ohm's law it follows that $C_0 = \frac{E}{R}$, while $C = \frac{E-e}{R}$.

To give the above symbols a clear meaning, and to simplify the reasoning that will follow in §§ 4 and 5, we shall, unless the contrary be stated, assume . . . .

(i.) That the dynamo and motor are both either magnetos or separately-excited machines, so that in both of them the E.M.F. depends solely on the velocity of rotation of the armatures.

(ii.) That in the simple (*i.e.* not branched nor shunted) circuit composed of the two armatures and the external connecting wires, R is constant; the speed of rotation not affecting appreciably the resistance occurring where the brushes make contact with the segments on the axes.

(iii.) That the speed of rotation of the driving dynamo, and therefore (since we have assumed its field to be constant) the driving *E.M.F.* E, is constant.

When the driver is a series-dynamo, matters are not so simple. As the reverse E.M.F. of the motor increases in magnitude the current falls off, and, therefore, so will the field and E.M.F. of the driver. Any accidental retardation in the dynamo may now

so lower E that it becomes less than e. The current will in such a case be reversed, and so will the field and *E.M.F.* E of the dynamo. Thus, when a series-dynamo is used as driver, not only are calculations less simple on account of the variability of E, but also we may have to encounter sudden reversals of action.

In practice such reversals can be obviated by means of pieces of apparatus that interfere automatically to set matters right whenever the current begins to be reversed.

With a shunt-dynamo matters are still more complex. For here we have to consider not only the changes in E, but also the complex nature of R due to the branched circuit that is open to the current.

Hence, as stated above, we shall keep to the simple case where dynamos and motors both have constant fields.

Referring to Chapter XV. § 9, *mutatis mutandis*, it is plain that the following statements are true.

$$\begin{cases} \text{E C} = \text{work per second, or activity, expended on the dynamo.} \\ e\,\text{C} = \text{work per second, or activity, expended against the reverse E.M.F. of the motor.} \\ (\text{E} - e)\,\text{C} = \text{C}^2\,\text{R} = \text{work per second, or activity, expended in developing heat in the circuit.} \end{cases}$$

§ 4. **Formulæ for Activity, &c. Maximum Activity.**—Let us suppose that a dynamo of a constant *E.M.F.* E is driving a motor, and let the symbols have the same meaning as in § 3. Instead of work per second we shall use the proper expression, *activity*. If E and e be given in *volts*, and R in *ohms*, then the activity is given in *watts*. We refer the reader to Chapter XV. §§ 2 and 3 for the relations between *watts*, *ergs per second*, and English *horse-power*.

The steam-engine expends a certain activity on the dynamo. In practice a small portion of this has its equivalent in heat evolved per second owing to friction. We shall, however, disregard this at present, and shall assume that it has its equivalent entirely in the electrical activity E C.

We may, as we did before in the case of a voltaic battery, think of our dynamo as a machine in which the electricity is pumped up through a certain ΔV measured by E, and thus acquires potential energy. In the voltaic cell we expended chemical energy in order to gain this

electrical energy; in the case of the dynamo it is mechanical energy that is so expended.

This activity E C is again transformed; it appears as (*a*) heat-activity measured by $(E - e)\,C$, or C^2R, since $C = \frac{E-e}{R}$, together with (*b*) the activity $e\,C$ expended against the reverse E.M.F. of the motor.

This division of the expended activity E C is the only one which would agree with Joule's law, and with the fact that to drive a current C against an E.M.F. e must require activity to the amount of $e\,C$; conservation of energy being also satisfied.

From the nature of an electro-motor it is clear that the activity $e\,C$ can only appear as mechanical activity; either kinetic energy gained per second, or work done per second against some force. We here neglect *friction* in the motor.

Now let us consider what will happen if we start with the motor at rest, and let it gradually increase in velocity until e comes to be as nearly equal to E as possible.

(1) *Motor at rest, e is zero.*—We now have $C_0 = \frac{E}{R}$, or the current is at a maximum. Hence the activity $E\,C_0$ expended on the dynamo is at a maximum. But it all appears as heat-activity evolved in the circuit; it gives us $E\,C_0$, or $C_0^2\,R$, heat-activity (measured in watts).

(2) *Motor moving slowly, e is small.*—Now we have $C = \frac{E-e}{R}$, or $C < C_0$. The activity expended on the dynamo is also less than before, being now E C, which is $< E\,C_0$. Of this we expend $e\,C$ on the motor, while $(E - e)\,C$ appears as heat-activity.

(3) *Motor moving rapidly, e nearly equal to E.*—In this case C is very small, since it equals $\frac{E-e}{R}$. The activity E C expended on the dynamo is also very small, or the dynamo is 'easy to turn.' Of this activity the part $e\,C$ spent on the motor is the greater, and the part $(E - e)\,C$ wasted in heat is the smaller.

Thus by letting the motor run very fast, which would be effected by giving it very little work per second to do, we can convert into useful activity a very large percentage of that expended upon the dynamo, and waste in heat a relatively small amount; but at the

same time we must notice that the activity is very small altogether. We are doing work without much waste, but the work is 'light.'

(4) *Motor moving so that* $e = E$ *(approximately)*.—In the limit we can conceive of e becoming finally equal to E. Under these conditions the current is *zero*, since $\frac{E - e}{R}$ now equals *zero*. So that at the moment when we attain the result of having no waste of activity in heat, inasmuch as $E - e$ now is *zero*, and of converting all the activity expended upon the dynamo into activity used in the motor, at this limiting moment the current becomes zero and the activity itself is zero.

Thus in case (1) the useful activity $e\,C$ is *zero* because e is *zero*; while in case (4) the same is *zero* because C has become *zero*.

(5) It is of interest to discover at what value of e and C the useful activity $e\,C$ is at a maximum.

This is easily found by an ordinary algebraical device.

We wish to find when $e\,C$ is at a maximum.

Now $e\,C = \frac{e\,(E - e)}{R}$, and is at a maximum when $e\,(E - e)$ is so.

We may write $e\,(E - e)$ in the form . . . .

$$\left(\frac{E}{2}\right)^2 - \left(e - \frac{E}{2}\right)^2.$$

Here each term is positive in sign, since each is a square. Hence the whole is maximum when the last term is zero, *i.e.* when $e = \frac{E}{2}$.

We see then that the work per second done on the motor is greatest when this motor is allowed to run at such a rate that the E.M.F. e is equal to $\frac{1}{2}$ E. When this is so, we have . . . . .

$C_m = \frac{E - \frac{1}{2}E}{R} = \frac{1}{2} \cdot \frac{E}{R} = \frac{1}{2}\,C_o$. We have also $(E - e)\,C_m = e\,C_m$; or we use only half the total activity expended on the dynamo, the other half being wasted on heat.

If then we wish to get through some work at the greatest possible rate, we cannot do it economically; we must waste half our expended work.

The reader will observe that these results are, *mutatis mutandis*, the same as those of Chapter XV. § 7 (β). [See also Chapter XIII. § 13.]

§ 5. **Efficiency.**—The ratio that the useful activity bears to the total activity expended is called the *efficiency*.

Thus it follows from this definition that . . . .

$$\text{'The efficiency is measured by } \frac{e\,C}{E\,C}, \text{ or by } \frac{e}{E}.\text{'}$$

Thus in case (1) of § 4 the efficiency is zero. As e increases, so does the efficiency. When the work is maximum, this occurring when $e = \frac{1}{2}E$ as was shown, the efficiency is $\frac{1}{2}$. In the limit the efficiency reaches its maximum value of *unity* first when $e = E$, or when current and activity vanish together.

The reader must distinguish very carefully between *maximum activity* and *maximum efficiency*. We do most work per second when the efficiency is but $\frac{1}{2}$; and when this latter approaches the economic ideal of *unity*, then the former approaches zero.

§ 6. **Representative Curves.**—We have seen that as the motor increases in velocity from zero to its limiting value, so that e rises from zero to a value approximating to that of E, we have the following variations.

(i.) The total activity falls from its initial and maximum value $E\,C_0$ to its final *zero* value.

(ii.) The useful activity rises from zero to its maximum value of $\frac{1}{2}E \times \frac{1}{2}C_0$, or of $\frac{E\,C_0}{4}$, and then falls to zero again.

(iii.) The efficiency rises from zero to its limiting value of unity.

Now all these changes can be very conveniently presented to the eye in one view by means of the graphical method, of which we have already given examples in Chapters XIII. and XVI.

In researches on the behaviour of dynamos, Dr. Hopkinson, Marcel Deprez, and others have made much use of the graphic method. It has been found possible to construct for each machine a curve, determined by a few experimental data; and then from this curve to predict by geometric methods the behaviour of the said machine under varied conditions.

Such curves are called *characteristic curves*; their construction was first suggested by Dr. Hopkinson.

omit

§ 7. **S. P. Thompson's Diagrams.**—Professor S. P. Thompson has devised a very simple form of diagram in which the results of §§ 4 and 5 are presented to the eye.

In the diagrams here given we have a figure similar to that of Euclid, Book II. 4.

A B C D is a square ; the length A D of its side represents in magnitude the constant *E.M.F.* E of the dynamo ; the vari ble

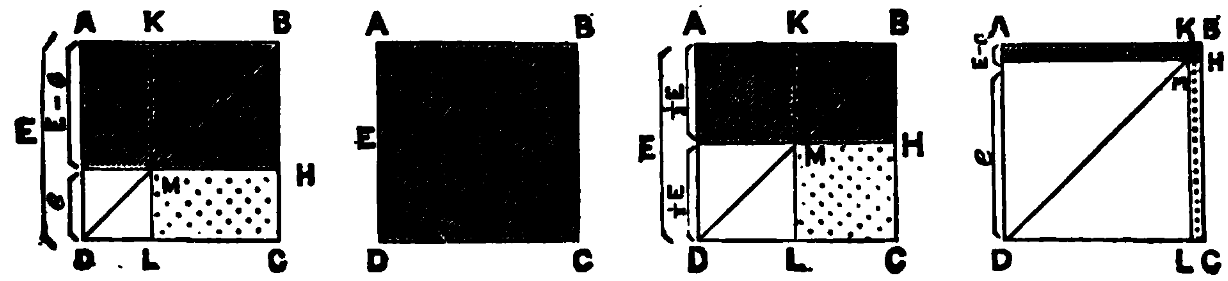


FIG. i. FIG. ii. FIG. iii. FIG. iv.

part A K represents on the same scale the variable reverse E.M.F. e of the motor, so that the remainder K B represents $(E - e)$.

Now we know that $C = \frac{E - e}{R}$, where R is constant. We may, therefore, for purposes of *comparison*, neglect the constant R ; and may say that the line K B represents in magnitude the current C also.

Hence, in the general case of fig. i. we have the magnitudes of the several activities represented by the areas of corresponding rectangles, viz. . . . .

(the total activity expended, *i.e.* E C) by the rectangle A H ;
(the useful activity, *i.e.* e C) by the rectangle A M or H L ;
(the activity wasted in heat, *i.e.* $C^2 R$ or $(E - e) . C$) by the square K H ;
$\left(\text{the efficiency, } i.e.\ \frac{e}{E}\right)$ by the ratio of areas $\frac{HL}{AH}$;
" " " or by the ratio of lines $\frac{AK}{AB}$.

The four diagrams correspond, fig. i. to the general case, fig. ii. to the case where e = zero, fig. iii. to the case of maximum work, and fig. iv. to the case where e is approaching E in magnitude respectively. Further explanation is not needed.

§ 8. **Transmission of 'Power' from a Distance.**—It often happens that there is a source of energy at one place, while it is desired to utilise energy at another. Such is the case, as a rule, where the source of energy is wind or water ; or where a powerful central engine is to be used for the work required over a range of

workshops. When activity is transmitted from the source to the place, more or less distant, at which it is to be utilised, we have what is called *transmission of power.*

For very short distances such transmission is effected by means of levers or toothed wheels; for longer distances by means of chains or flexible straps; for still longer distances by means of water, as in the gold-washing operations of California. Of late years such distant transmission has been effected by means of electric currents.

The question naturally arose, Would it not be necessary to have very thick, and therefore very expensive, conductors, if it were desired to transmit activity of any considerable magnitude? It was (the writer believes) Sir W. Thomson who first pointed out that this would not be necessary, if the transmission were effected under proper conditions.

Suppose, for example, it were desired to transmit activity from the Falls of Niagara to the city of New York. At the Falls we might have a number of dynamos worked by means of turbines or other contrivances, while at New York might be stationed the motors to be driven by the current. Now the activity utilised in New York would be measured by e C, while the current C is given by $\frac{E - e}{R}$. Hence we could have a large activity utilised, while the current to be carried by the conductor remained small, provided that E and e were nearly equal and were very large.

We must therefore have at the one end a dynamo that gives a large *E.M.F.* E, and at the other a motor running so fast as to give a reverse E.M.F. e that is nearly as large as E. That this may be the case, the motor must be connected by a 'geared-down' arrangement with the machine that it is intended to drive; so that the motor may run very fast while the machine moves at the usual more moderate speed.

In such an arrangement the current would be small, and hence the transmitting wire need not be of large diameter.

It is interesting to notice that, in the above case, if the motor were stopped the wire would be exposed to the large current $\frac{E}{R}$ and would very likely be fused by it; and if the dynamo were suddenly stopped, the wire might be fused by the current $\frac{e}{C}$ to which it would be for a

moment exposed, the motor maintaining for a moment its velocity owing to the inertia of the revolving masses.

§ 9. Electric Railways and Tram-Cars; Telpherage; &c.

Electro-motors can be used for locomotive purposes in more than one way; but at present such applications have been but little carried out into practice.

Electric tram-cars.—These are usually driven by means of Faure's accumulators that are carried in the car itself. These cells supply a current to an electro-motor, and this, through the intermediency of suitable geared-down connections, drives the car. The motor moves rapidly; the driving wheels of the car move with relative slowness.

Electric screw-launches.—Such trial boats as have been made as yet are driven either by means of accumulators, or by means of 'primary' bichromate batteries; these supplying a current to the motor that drives the screw.

Electric railways.—Lines of railway have been devised on which the train would be driven by motors, the current being conducted to them, from the centres at which the dynamos are being driven, by means of the rails themselves. Thus, stationary steam-engines supply mechanical energy to the train in all parts of its journey, through the intermediency of the current.

The whole distance can be so broken up into sections, and the electrical connections can be made in such a way, that the current is supplied to a train only when other trains are at a safe distance. In fact, collisions could thus be automatically prevented, breaks automatically applied, and risk of accidents reduced to the 'unavoidable minimum.'

Telpherage.—The idea of propelling light trains along single overhead rails was first conceived by Professor Fleeming Jenkin. Such trains might, *e.g.*, serve to transport material from mines or quarries over fields to the works for which such material might be destined, or for any similar purpose. This automatic transport has been called *Telpherage.* The reader will perceive that it differs only in detail, not at all in principle, from the railway system mentioned above.

The practical application of Telpherage is due to Professors Fleeming Jenkin, Ayrton, and Perry, who acted in concert in this matter.

In October 1885, there was opened in Sussex the first Telpher line instituted in this country. It serves to convey clay from the pits over meadows (where a tramway would have been inconvenient or injurious) to the railroad; and it was adopted, by the Cement Company who own it, as the most suitable method of transport.

(For further details as to this line the reader is referred to 'Nature, Vol. xxxiii. p. 13, whence this brief notice has been derived.)

§ 10. **Distribution of Potential in the Circuit of a Dynamo and Motor.**—In what follows we shall speak of a *dynamo* of constant *E.M.F.* E, and a motor of reverse E.M.F. *e*. The reader can easily apply the same methods to the similar case of voltaic piles and electrolytic cells (*see* Chapter XV. § 9).

As in Chapter XIII. so here, the ordinates represent potentials; the abscissæ represent resistances; and the whole diagram exhibits the fall of potential, that occurs in obedience to Ohm's law, throughout the circuit.

We have for convenience represented the *E.M.F.* E of the dynamo as a ΔV occurring abruptly at one point in the armature. This is not the case; for the *E.M.F.* E is the sum of a number of E.M.F.s occurring in the different turns of wire. But in what follows, this simple assumption will not lead us to any erroneous results; and without it we could have no such simple diagrams as those here given.

(i.) *Dynamo; open circuit.*—In fig. i. the points A and B represent the terminals (or brushes) of the dynamo; A x and y B together measure the internal resistance R_1 of the dynamo; the ordinate xy measures the *E.M.F.* E.

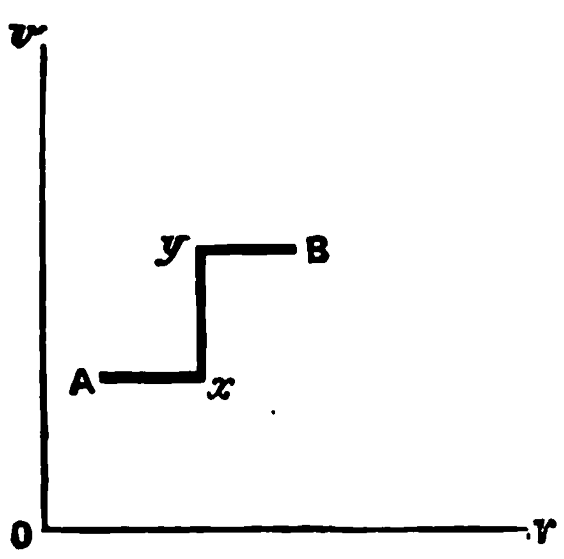


FIG. i.

Since there is no current, it is clear that A x and y B are horizontal, or that the points A and B have a difference of ordinate measured by xy. In other words, the *E.M.F.* E may be measured by the ΔV of the terminals A and B when the circuit is open.

(ii.) *Dynamo; closed circuit; no other E.M.F. in the circuit.*—Now let the circuit be closed through an external wire whose resistance is measured by the abscissa $b\,a'$ of fig. ii. Here A and A′ are really one point, the circuit being complete. The lines A x and y A′ make an angle ϕ with the axis O r such that C is proportional to *tan* ϕ, or to $\frac{x\,y}{a\,a'}$, as explained in Chapter XIII.

It is clear that in this case the points A and B have a difference

of ordinates that is *less* than the ordinate xy. This means that the ΔV between the terminals will not measure the total *E.M.F.* E of the dynamo, but will be less than this latter; there being some fall of potential down the internal resistance R_1, according

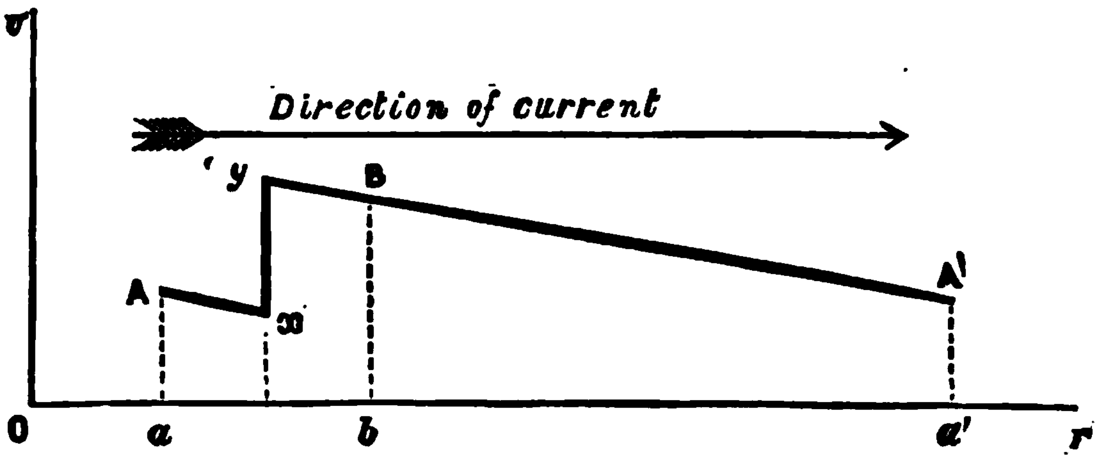


FIG. ii.

to Ohm's law. We may conveniently designate the ΔV between A and B by the symbol E_B^A. It will not be hard to find what relation E_B^A bears to the total *E.M.F.* E.

Let R_1 be the internal resistance ab, and R_2 the external resistance ba'.

Then, since there is in the whole circuit a total fall of potential measured by E, there will in the portion R_1 be a fall measured by $\frac{R_1}{R_1 + R_2} \cdot E$, by Ohm's law. This expression can be written also as $R_1 C$, since $C = \frac{E}{R_1 + R_2}$.

Hence, E_B^A falls short of E by the amount $R_1 C$; or . . . .

$$E = E_B^A + R_1 C.$$

We can therefore find out the value of the E.M.F. of a dynamo (supposing it not to be shunt-wound) when it is running and when the circuit is closed, by finding the ΔV between its terminals and by adding to this the product of its resistance into the current. If measures be made in *ampères*, *ohms*, and *volts*, there will be no 'constants' involved in these expressions.

(iii.) *Dynamo and motor; a reverse E.M.F.*—In fig. iii. we have the case of a circuit that comprises a dynamo and motor. A and B are the terminals of the dynamo, C and D those of the motor. The resistance R_1 of the former is measured by the

abscissa $a\,b$, and the resistance R_2 of the latter by $c\,a$. The remaining abscissæ $b\,c$ and $d\,a'$ together represent the resistance r of the connecting wires. $A\,x$, $y\,z$, and $w\,A'$, are of course parallel.

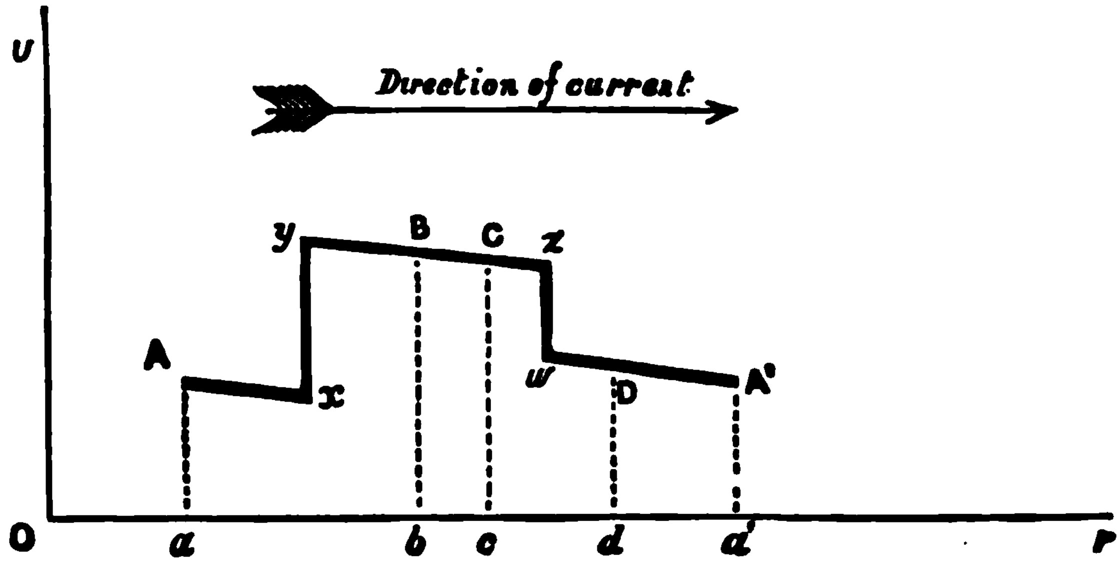


Fig. iii.

The ordinate $x\,y$ represents the *E.M.F.* F of the dynamo, while $z\,w$ represents the reverse E.M.F. e of the motor. The current is given by the expression $\dfrac{E - e}{R_1 + R_2 + r}$; or, in the diagram, it is proportional to $\dfrac{xy - zw}{a\,a'}$.

Using the same notation as in (ii.) above, we may say that the ΔV E_B^A between the terminals A B is related to E by the equation . . . .

$$E = E_B^A + R_1\,C.$$

In a similar way we have for the ΔV E_D^C between the terminals of the motor the relation . . . .

$$E_D^C = e + R_2\,C,$$

since here the fall of potential due to Ohm's law is *added* to the fall measured by e.

The reader should notice the fact, at first sight somewhat paradoxical, that in the dynamo the E.M.F. (represented for convenience by the abrupt rise $x\,y$) appears to be opposed to the current; while in the motor the E.M.F. (similarly represented by $z\,w$) appears to be acting with the current.

These apparent difficulties disappear upon a little consideration.

It is the work done on the dynamo that keeps, as it were, pumping

up the electricity from x to y; and it is the advantage thus gained that enables the current to flow in the direction indicated by the arrow. Hence the driving *E.M.F.* E is represented suitably in the diagram by the rise $x y$.

In the case of the motor we may observe that if the current is to do work, it must be by being, as it were, let down an 'electrical hill.' Hence $z w$ must occur in the opposite direction to that in which $x y$ occurs; the current must fall down $z w$. Moreover, such a fall as $z w$ rightly represents a reverse E.M.F., or one which opposes the current, for the following reason. The current is proportional to *tan* ϕ (*see* Chapter XIII.); and hence the current is diminished, if the inclination ϕ of the lines Ax, $y z$, and wA′, to the axis Or be diminished. Now the fall $z w$ does diminish the angle ϕ, as is easily seen if we compare fig. iii. with a figure in which $z w$ is removed, $x y$ and $a a'$ remaining unaltered.

Hence e is rightly represented by the fall $z w$.

§ 11. Work done per Second upon a Dynamo as Related to the Velocity v of Rotation.

The mechanical *work per second* or *activity* expended in driving a dynamo is, if we neglect friction, measured in *watts* by the product E C.

The manner in which the magnitude of this product depends upon the velocity v of rotation of the armature varies according to the nature of the dynamo.

(i.) *Case of magneto, or other constant-field, machine.*—Here we have, by the formula of Chapter XXIII. § 2, that E is proportional to v; and hence that, when there is no other E.M.F. in the circuit, C is also proportional to v.

Therefore the work is proportional to v^2.

(ii.) *Case of series, or other varying-field, machines.*—In these the E.M.F. is first affected directly by v, and then is affected also indirectly, inasmuch as increase of current increases the field-strength.

Hence the work is proportional to some higher power of v than the second power. Since however the increase in field-strength, corresponding to a given increase in the current, depends upon the degree to which the cores of the field-magnets are already saturated, it is not possible to express in any simple and yet exact manner the relation between the activity E C and the velocity of rotation v.

CHAPTER XXV.

VARIOUS APPLICATIONS OF ELECTRICITY; TELEGRAPHS, TELEPHONES, MICROPHONES, ELECTRIC LIGHTING.

§ 1. **Introductory.**—In the present Chapter we shall describe, but necessarily in a very brief manner, various applications of 'electricity'; these applications being for convenience divided into groups, as indicated in the heading of the Chapter. From the nature of the subject there will not be, as there was in most of the previous Chapters, any order or progress from group to group. The whole forms a somewhat miscellaneous collection, illustrative of various principles explained earlier in the Course.

TELEGRAPHS AND ELECTRIC SIGNALLING.

§ 2. **General Principle of Telegraphy.**—Let us consider two stations A and B, more or less remote from one another, connected by an insulated wire, the circuit being completed either through a return wire, or by means of large metallic plates buried in the soil, through the earth itself. It is clear that from either station can be sent currents that will pass through the other station. And if each station be provided, not only with a battery and with a suitable instrument for making or breaking current, but also with a receiving instrument on which the current acts to deflect a needle, sound a bell, or in other ways attract attention, it is clear that we have the means of exchanging signals and messages between the two stations.

Note on the 'return wire.'—We may remark that as a rule a return wire is not employed, the circuit being completed by means of large metallic plates buried in the earth, as is seen in the diagram to § 4. It is not necessary to suppose that the current does actually return through the earth. For the plates will be kept at what will be approximately *zero* potential (or the same

potential), since each has a large surface in contact with a practically unlimited conductor. And, as long as each end of the line is kept at *zero* potential (or the same potential), so long will the battery send a current through it.

(1) *The electric bell.*—The accompanying figure represents a very simple form of signalling instrument: it is similar in principle to the automatic *make-and-break* described in Chapter XXII. § 7.

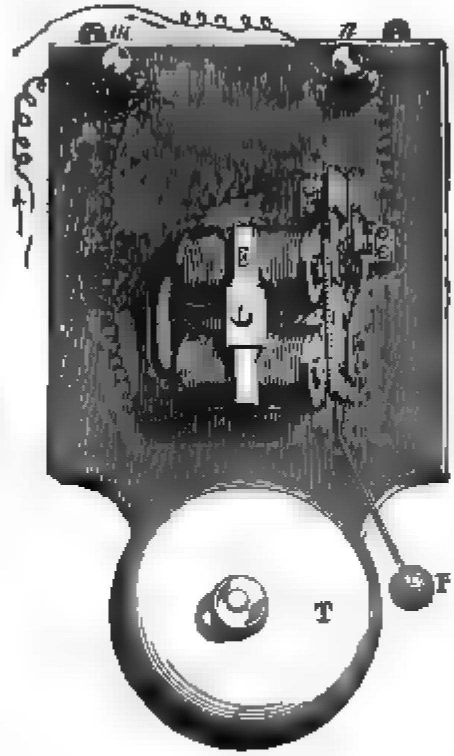


Fig. i.

When in the position shown here (*see* fig. i.) the circuit is complete. But as soon as a current passes, the electromagnet attracts the iron keeper *a*, causing the hammer to strike the bell, and the circuit is broken at the place where the spring C touched the keeper *a*. The current ceases, and with it the magnetism. Hence the keeper *a* is no longer attracted; and so, acted upon by the spring to which it is attached, it flies back again and makes contact once more with C. The process is then repeated. As long, therefore, as the current is kept on the line, so long will the hammer vibrate and the bell sound. This instrument is used only for attracting attention, not for messages.

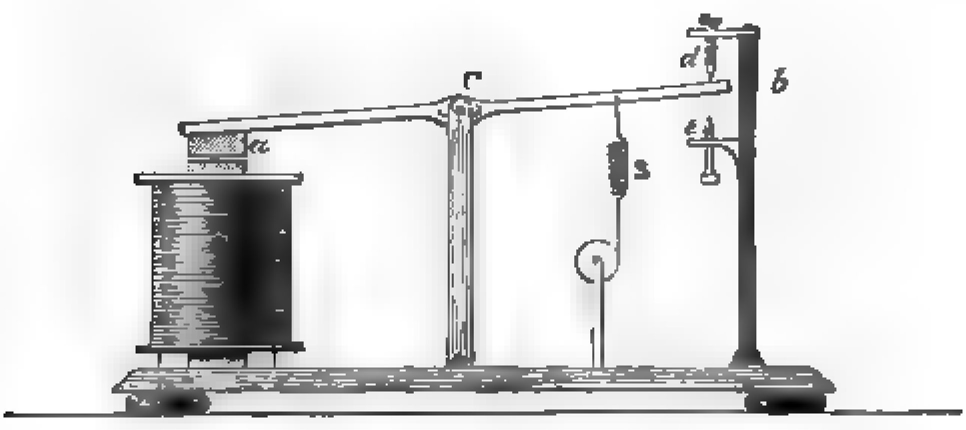


Fig. ii.

(2) *The electric sounder.*—This instrument (*see* fig. ii.) is very simple in construction. The current from the sending station passes

through the coils of the electro-magnet, and the iron keeper *a* is attracted downward. This causes the other end of the lever to hit with a sharp sound the screw *d*. When the current ceases, the spring S pulls the lever down again, and this hits the lower screw *e*. When the current is made and broken very rapidly, these two sounds occur almost simultaneously, as a sharp double 'click.' If the current be not broken again at once,

FIG. iii.

the two sounds are separated by a corresponding interval, and the sound is more deliberate. These two sounds can be readily distinguished; and by suitable combinations of these we can, as will be explained in § 3, represent all the letters of the alphabet, and so form a code for messages.

It is to be noticed that if there be passing between the point *d* and the lever a strip of paper, moved by clock-work at a uniform

rate, then each sharp make-and-break of current will cause a *dot* to be made on the strip; while each slower make-and-break will keep the point pressed against the paper sufficiently long to make a *dash* on it. When the signals are thus to be printed, the instrument is modified, the lever being provided with a pointed style, or with a small sharp-edged wheel, and *d* being replaced by a smooth roller.

FIG. iv.

(3) *The needle telegraph.*—By sending currents round any form of galvanometer in opposite directions, we can give to the needle deflexions in opposite directions respectively. On this principle is constructed the common 'needle telegraph,' of which one form is shown in fig. iii. Here the coils and the needle are both vertical, and the motion of the needle is limited by two ivory stops. In fig. iv. we see at the base of the instrument the commutating key by means of which the current can be made or broken, and sent in either direction. The current passes first through the sending instrument, deflecting its needle in one or the other direction. It passes thence through the line wire, and affects in a corresponding manner the needle of the receiving instrument.

By means of a suitable code both this and the last instrument are used to transmit verbal messages.

§ 3. **Telegraphic Alphabets.**—In all those instruments that are in extensive use there are only two elementary signals: the *dot*, to which we agree to consider a *deflexion to the left* to be the

corresponding signal in the needle instrument ; and the *dash*, to which corresponds the *deflexion to the right* of the needle.

The signals answering to the letters of the alphabet are various combinations of the above elements, the only principle observed being that simple signals shall represent constant recurring letters.

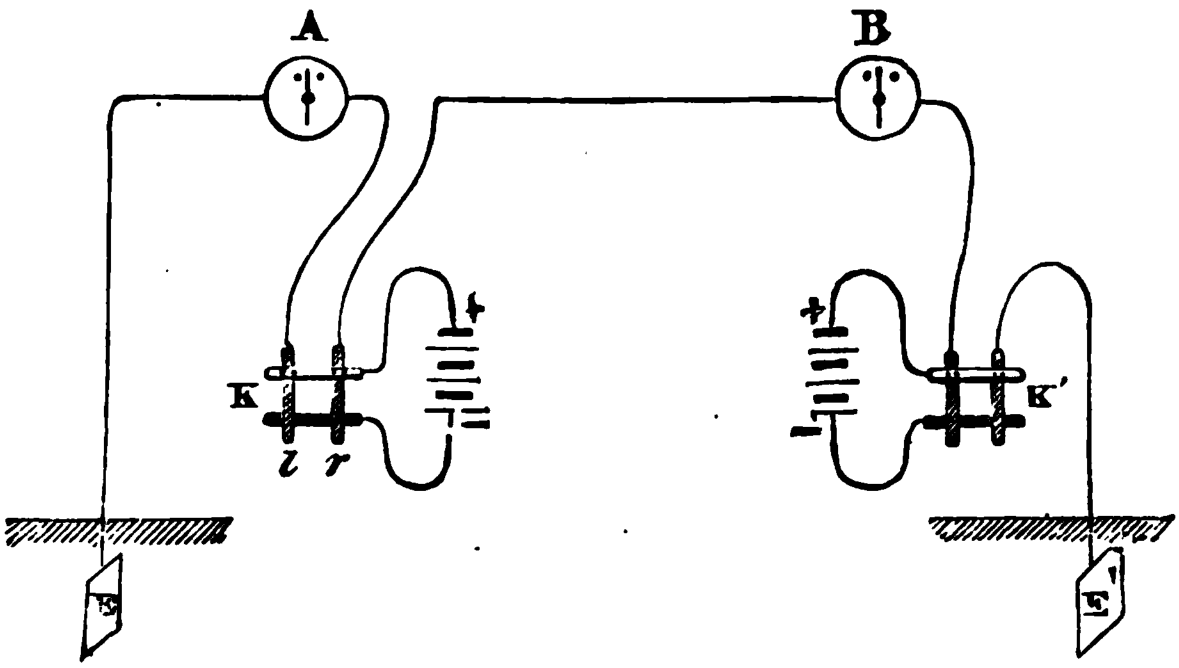
	PRINTING.	SINGLE NEEDLE.		PRINTING.	SINGLE NEEDLE.
A	- —	\/	N	— -	/\
B	— - - -	/\\\	O	— — —	///
C	— - — -	/\/\	P	- — — -	\//\
D	— - -	/\\	Q	— — - —	//\/
E	-	\	R	- — -	\/\
F	- - — -	\\/\	S	- - -	\\\
G	— — -	//\	T	—	/
H	- - - -	\\\\	U	- - —	\\/
I	- -	\\	V	- - - —	\\\/
J	- — — —	\///	W	- — —	\//
K	— - —	/\/	X	— - - —	/\\/
L	- — - -	\/\\	Y	— - — —	/\//
M	— —	//	Z	— — - -	//\\

This system is called, from its inventor, the *Morse code.* The reader will notice how readily this code, intended originally for dots and dashes in the *Morse instrument* described in § 5, adapts itself to other methods of signalling ; as by the needle instrument, by the voice, by flashes of light, by flags, &c.

§ 4. **The Needle System of Telegraphy.**—We will now describe briefly the simplest manner in which two stations may be united on the needle system. In the figure, A and B represent the two needle instruments (*see* § 2, fig. iii.) at the two stations respectively ; the batteries are represented as usual. E and E′ are large plates buried in the earth, serving to keep the two ends of the line at the zero potential ; K and K′ are the two commutating keys by means of which each station can send a current in either

direction through the other station ; the line wire (shortened out of all proportion, of course) is seen at the top of the diagram.

Action of the commutator key.—The terminals of the battery are connected with two strips of metal, marked *white* and *black* respectively in our diagram. These are insulated from each other. The pieces *l* and *r* are two keys of metal, that are kept pressed against the upper (or *white*) strip by means of a spring. In this,



which is the normal condition of things, there is a complete circuit through A, the upper strip, the line, B, the other upper strip, the earth plate E′, the other earth plate E, and so to A again. The batteries are cut out of the circuit ; since the lower (or *black*) strips, and with them the negative poles of the batteries, are insulated.

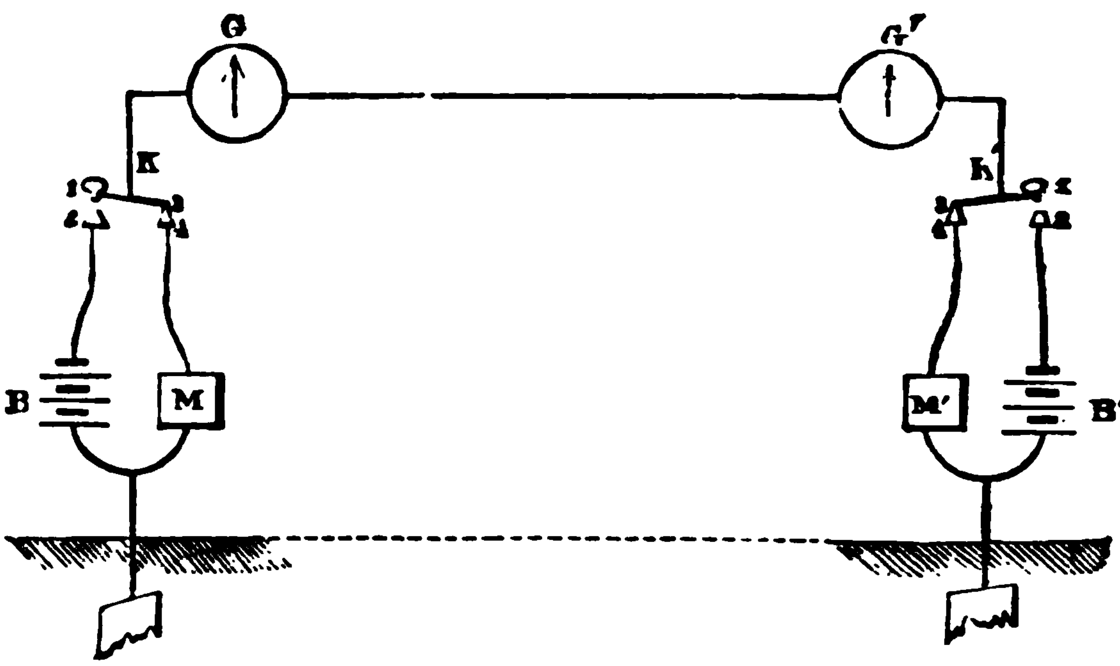
Now let the key *l* be depressed ; thus leaving the upper, and touching the lower, strip. The connections make it evident that this will send a current along the line to B, through E′ and E, and back to the – pole of the battery by A and *l*. If the other key be depressed, the current will pass in the other direction.

If similar keys be depressed at both stations simultaneously the circuit will be broken, and no message can be sent.

§ 5. **The Morse System.**—The *Morse* instrument is in all essentials the *sounder* of § 2, fig. ii. ; but it is modified so as to print *dots* or *dashes* on a strip of paper.

In the figure we show the simplest manner of connecting two stations by means of Morse instruments. The two *Morses* are at M and M′ ; K and K′ are simple keys, not commutators ; G and G′ are galvanometers to indicate to the sender whether or no the current actually passes, or whether the line is stopped ; this being

needed, inasmuch as the current does not pass through the sender's *Morse*. In the position shown, the keys, kept in place by a spring, make contact at 3 and 4, so that each station is ready to receive a message. On either key being depressed a current is sent along



the line and through the other instrument; so that *dots* or *dashes* will there be printed, according to the duration of the contact made by the sending key.

If both keys be depressed at once no message can pass. A preliminary signal is usually sent, so that the receiving clerk may set going the clock-work that moves the paper.

The *Morse* is often worked by sound, the ear soon acquiring the power of reading off the message in this way.

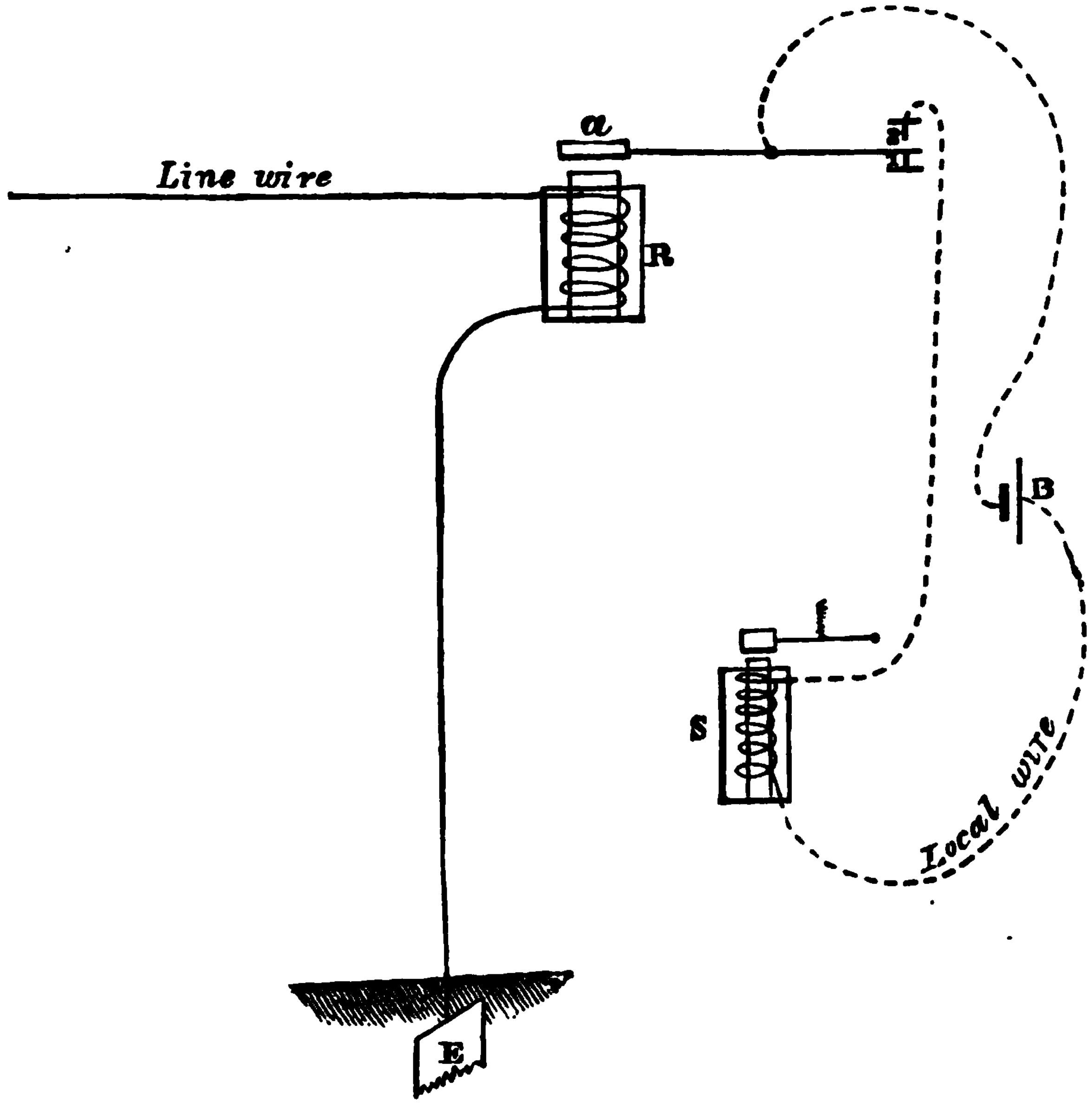
§ 6. **Relays.**—On long lines the current will usually be too feeble to work the printing *Morse*. In such cases a *relay* is employed. This is an arrangement consisting of a local battery in circuit with the *Morse*, and a key that can make or break this circuit; this key being worked by the feeble line current through the intermediency of an electro-magnet.

In the figure on the next page the feeble line current works the electro-magnet R; this acts upon the soft iron piece *a*, and by means of the lever makes contact at 1, 2; the local circuit is thus completed, and the *Morse* S is worked by the strong local current.

§ 7. **Transmission through Cables, under Water.**—When the line connecting two places has to be laid under water, it then consists of one or more copper wires thickly coated with some insulating material; round this again is generally twisted stout iron wire in order to give the strength necessary, and this is coated

with some waterproof material. The whole is called a *cable*; the messages conveyed by it are sometimes called *cablegrams*.

Difficulties occurring in transmission by cable.—It was soon found that it was not possible to send, through a cable of any considerable length, the clearly separated *dots* and *dashes* required by the *Morse* receiver. The sharp tap on the key at the sending end is represented at the receiving end by a feeble current that slowly rises to a maximum and again slowly sinks to zero. With such currents we cannot use any of the instruments described



above. Had not the *Thomson's reflecting galvanometer*, or some similar instrument, been invented, cablegraphy would hardly have been practicable. With this instrument, however, the reading of a message is easy. The principle of the reflected beam of light gives the means of magnifying the motion to any desired extent. The needle is of very small mass and possesses but little moment of inertia about its axis of suspension; and hence its deflexion will follow, with but an inappreciable lag, the varying undulatory current. And the extreme delicacy of the instrument enables very

small currents to be perceived. Finally, the small mass of the needle, its method of suspension, and the command of it that is given by the controlling magnet, renders it possible to use a modified form of this instrument on board ship during the laying of the line.

Reading the messages.—On a land line we are able to give to the needle very rapid deflexion to right and to left of the zero mark. But on a cable the currents last so long that it is not practicable to wait for the dying away of one current before sending another. Hence the currents overlap one another, and allowance has to be made for this in reading the message. The clerk at the receiving end watches the spot of light, and observes, *not* its movements to right or left of the zero mark in the middle of the scale, but its jerks to right or left wherever it happens to be. In fact, a shifting zero mark is used.

Cause of retardation of signals.—The origin of the alteration of the sharp signals sent, into the slow and gradual undulations received, lies mainly in the fact that the cable forms a long condenser of very great capacity.

In the case of an overland wire, the sharp making of contact effected by striking the key is followed by the passage along the wire of what we may call a tide of electricity; this tide having a relatively abrupt front, as has the well-known 'bore' that is sometimes seen on a tidal river. The receiving instrument is affected, almost instantaneously, with the full force of the current. But in the case of the cable matters are very different. The tide of electricity rushes into the cable, but as it proceeds it is to a great extent detained to charge electrostatically the cable; this being a condenser in which the wire and the sea-water form two coatings separated by the insulating di-electric. Thus the rise to a maximum occurs very gradually, as would the rise of tide in a stream out of which ran many side channels that must be filled by the tide before it can advance up the river. (We need not say that the analogies here given are very incomplete.)

Hence at the receiving instrument the current rises very slowly to a maximum.

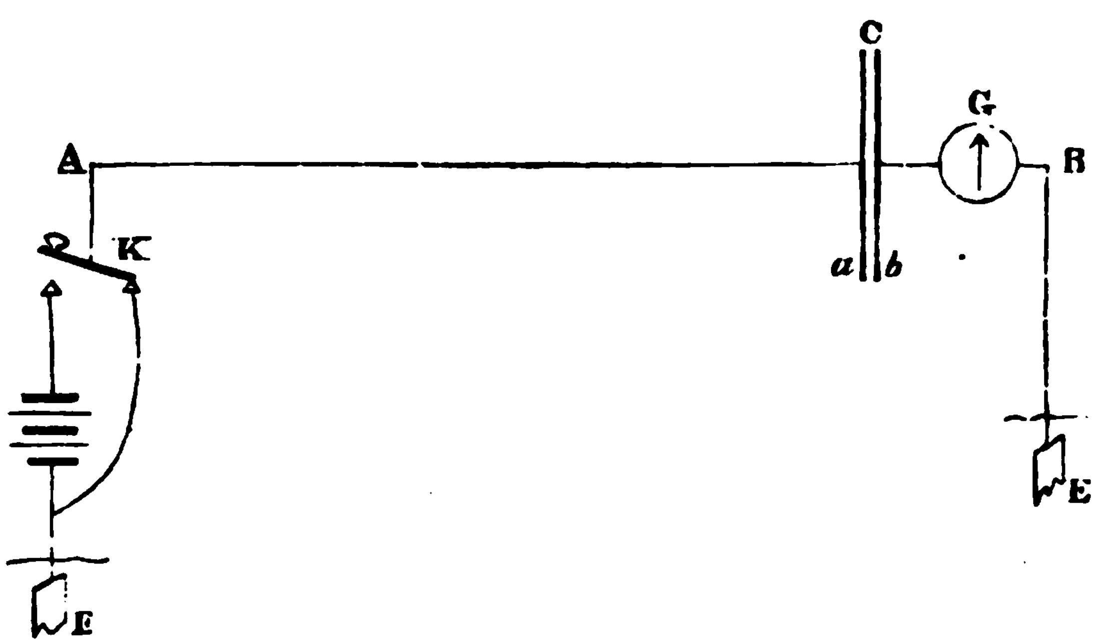
§ 8. **Earth Currents. Condenser System of Working.**—It is found that currents are continually flowing from one point of the earth to another, these currents being due to thermo-electric

or to other unknown causes. When the two earth plates of a telegraphic line happen to be buried where the earth is not at one potential, a current will flow along the line wire. The greater the ΔV of the two places at which the plates are sunk, the stronger will be the E.M.F. driving the current along the line. During those disturbances of the earth's magnetic field which are known as *magnetic storms*, these earth currents become very strong, and may entirely over-ride and nullify the ordinary currents by which the messages are transmitted; such 'storms' occur, for example, during brilliant displays of the aurora borealis, and during any noticeable changes in the surface of the sun.

These earth currents give great trouble on long lines, and especially in long cables. There are two methods of meeting the difficulty.

(i.) *The return wire method.*—Where it is practicable, a return wire may be used instead of an 'earth.' If the clerks find the earth current to be so strong on any particular day as to give trouble, they may temporarily complete the circuit by means of any wire that does not happen to be in use on that day.

(ii.) *The condenser system.*—The diagram here given represents the *principle* of the condenser method. In this system the line is



not continuous, but the line from the sending station is connected with one condenser plate, while from the other plate passes away the line to the receiving station. In the simple arrangement shown in the figure, a depression of the key would send a charge into the condenser plate *a*. This would call up an equal and opposite charge in the plate *b*; and the effect upon the instrument G would

be the same as though the current sent had passed on. This action of course ceases when the condenser is charged, but, as this latter is always of very great capacity, the current due to the electrostatic induction lasts as long as is necessary.

When the key is again released, the plate *a* is discharged to earth; and hence there will be a discharge of *b* also, and therefore a reverse current through the instrument G.

In practice there is a condenser at each end, the cable being totally insulated. There is double induction, but, by the electrostatic principles discussed in Chapter X. and earlier, the instrument at the receiving end will be influenced by currents passing in the same direction as if they had come direct from the sending station.

The *condensers* employed consist usually of many sheets of tin-foil separated by insulating sheets of paraffined paper; the sheets of tin-foil being connected up into two alternate sets, answering to the two plates of a condenser.

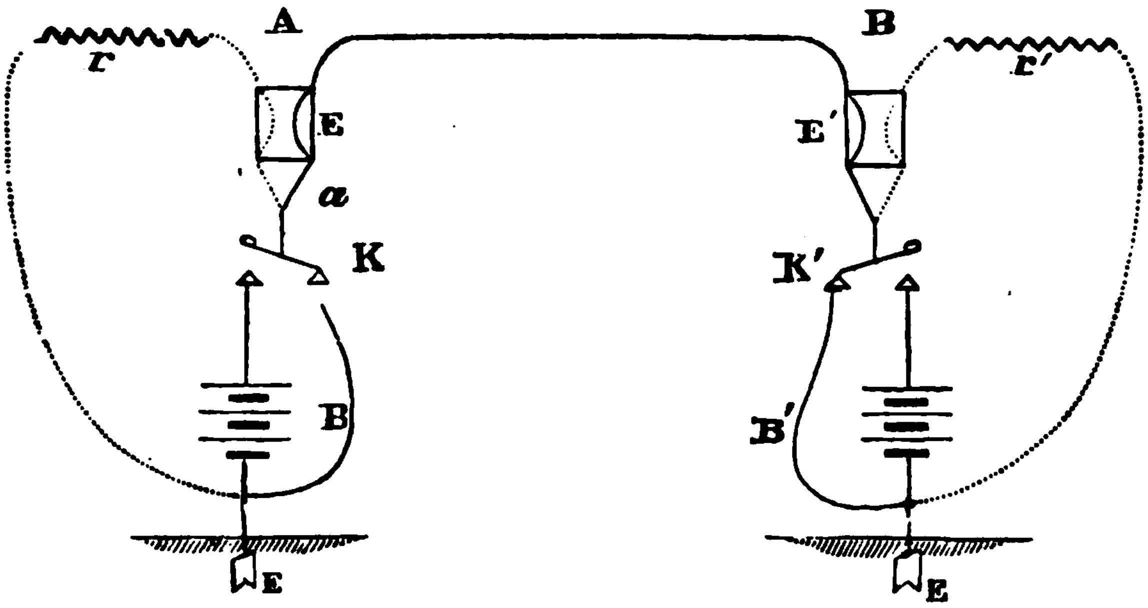
§ 9. **Insulation of Wires.**—If we cut connection with the earth at one end of a line, and send a current into the wire at the other end, a galvanometer in the line will settle down to some steady deflexion. In fact, the 'insulated' line wire leaks to earth along the whole distance; the insulators acting as conductors of very great resistance. The longer the line, the more is the leakage and the lower is the resistance offered by the insulators collectively. It is usual to fix upon some definite resistance (in ohms) as the resistance *per mile* that ought to be offered by any given line of wire. If, without any special cause, such as fogs, rain, or snow, the insulation fall below this standard, then it is suspected that some accidental connection with earth has been made.

§ 10. **Duplex Telegraphy.**—In §§ 4 and 5 we noticed that it was not possible to send a message from each station to the other respectively at one and the same time. Could this be done, it is evident that we could get twice the use out of a line.

We can readily devise a method in which each clerk could transmit his own message through his own instrument; the problem to be solved is much harder, viz. how the message sent from A shall be indicated by the instrument at B only, while that sent from B shall be indicated at A only.

This problem has been solved in two ways, one of which we will briefly describe.

(i.) *The differential method.*—In the figure, E and E′ represent the two *Morse* instruments (or relays), the electro-magnet in each being wound with two equal coils. The line from the key K branches at *a*; one path is through the one coil of E, through the line *l*, through the one coil of the receiving instrument E′, and so to earth; the other path is through the second coil of E,



through an adjustable resistance box *r*, and so direct to the other terminal of the battery. The same may be said with respect to the paths from the other key K′.

In order to work this system it is necessary that the two paths from either key offer equal resistances. In this case, since there are in each instrument two equal and opposite coils, the depression of the key K alone will not affect E, but will affect E′; while the depression of K′ alone will not affect E′, but will affect E. In the diagram the keys are shown in their normal position. This fact enables us to test for the desired equality of resistances; *r* and *r*′ being adjusted until the desired end is attained.

It is to be noticed that in the action on the *Morse* instruments or relays we need not consider the direction of the current.

Let us now suppose that, while A is sending a message to B by depressing the key K, the key K′ is also depressed. This will destroy the balance previously existing. Through the one coil of E there will now flow less current than before, or zero current, or even a reverse current; this depending upon the relative E.M.F.s of the two opposed batteries. So long, then, as K and K′ are both depressed, the instrument E is worked by this difference of

currents in its two coils ; and, if K be released, E will be worked directly by the current from B alone. Hence, under both conditions E will attract its armature, or release it, in obedience to the movements of the key K′ at the station B. The same holds good with respect to the instrument E′.

Thus, by means of a system in which checks to the currents sent are recorded as currents received, the problem of duplex telegraphy was solved.

(ii.) *The Wheatstone's bridge method.*—In another method a somewhat different principle is employed. Here each instrument is placed in a 'bridge,' and is unaffected as long as the extremities of the bridge are at the same potential. When matters are properly adjusted, this state of equilibrium at the one station A is destroyed by the depression of the key at the other station B; and this is the case whether the key at A be worked or no. Hence duplex working is rendered possible.

TELEPHONES.

§ 11. **Telephones. Introductory.**—Telephones are instruments by means of which it is possible to transmit between two stations, more or less remote from each other, musical notes or other sounds, and even articulate speech; an instrument at the one end is spoken to or sung to, and the instrument at the other end gives out the words spoken or the tune sung. [The same result can be obtained for comparatively short distances by means of speaking tubes ; but to such a system, which is merely a case of reflexion of sound, the word telephone is not applied.]

The only telephones much used are those in which the transmission is effected electrically. There are mechanical telephones, sometimes used, in which mechanical impulses are transmitted along the connecting wire ; but in electric telephones it is an undulatory electric current that passes.

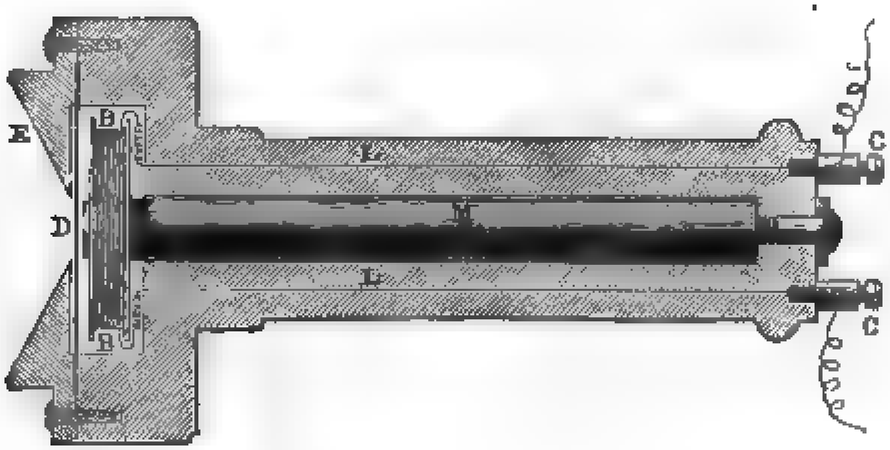
In all electric telephones we have a *transmitter* into which the message is spoken, and a *receiver* which again utters the message: these two instruments may be identical or different in form.

In some telephones the speaking of the messages causes an undulatory current to spring into existence : and this again causes the receiver to utter the message. In such, there is no external

source of current needed. In other telephones there is an external source of current, and the speaking of the messages causes this previously existing current to become undulatory.

§ 12. **The Bell Telephone.**—The most complete form of telephone, for general use, is that invented by Graham Bell, called, after his name, the *Bell telephone.* In this instrument no external source of current is needed, and it can act either as receiver or as transmitter.

In the figure we have a section of the instrument; from this the construction can be explained. E is the mouthpiece into which we speak; M is a bar-magnet of about four inches long; B is a long coil of very fine insulated wire surrounding the pole of the magnet; C C are the terminal screws; and finally, D is a vibrating membrane made of soft iron and very thin When we speak into



the mouthpiece, the soft iron diaphragm vibrates in exact accord with the air-waves impinging upon it. This acts inductively upon the steel magnet, and alters the number of marked lines of force piercing the coil of fine long wire. This gives rise to induced currents in the coil; these will be feeble, but of high E.M.F. Hence, when we speak into the *transmitting* instrument, we cause undulatory currents to pass along the line wire, these undulations answering to the air vibrations, and therefore answering to the words spoken.

These currents arrive at the *receiving* instrument, and there pass round the coil. This alters the magnetism of the magnet's pole, and this again causes the iron diaphragm to vibrate. Finally, this iron diaphragm, vibrating in exact accord with that of the transmitter, will cause the air to vibrate, and, when the ear is held close to the instrument, the words of the message will be heard.

It is to be noticed that the vibrations of the diaphragm of the *transmitter* to and fro will give rise to induced currents in opposite directions respectively; and that in the *receiver* these currents will strengthen or weaken the magnet pole, and therefore attract further or release somewhat the diaphragm of the transmitter respectively. Hence, the to and fro vibrations of the one diaphragm will be reproduced in the other.

Sensitiveness of the telephone.—This telephone is of remarkable sensitiveness. By it we can detect currents too faint to affect any ordinary galvanometer, provided that the currents are undulatory, or at least subject to abrupt variations in strength. Hence, in many cases the telephone can be used as a very delicate galvanoscope.

If a telephone (protected by a bridge when necessary) be in a secondary circuit such as that of a Ruhmkorff's coil, the note that it emits will indicate the number of induced currents passing per second; or the number of makes-and-breaks per second in the primary circuit. It has been used in investigations regarding the stratifications that appear in vacuum tubes; and it has thrown light upon the relation that these bear to the number per second of induced currents, and also to the steadiness of the contact maker in the primary.

In the circuit of a 'Gramme,' the telephone shows plainly, by emitting a note, that the current is really undulatory, though for many purposes it acts as if continuous.

Induction disturbances on telephonic lines.—The great sensitiveness of telephones is a source of difficulty in their practical use. If the return be made through the earth, conversation cannot as a rule be carried on at all. For, what with earth currents and leakage from other circuits, the instrument emits a continuous bubbling or frying sound that drowns the faint 'speech.' Return wires are therefore always used. But even then there is much induction if the wires run anywhere near ordinary telegraph wires; and it requires a very special method of laying the telephone wires to reduce the Babel of sounds, due to these induced currents, to comparative silence.

Details as to the precautions taken will be found in more technical works.

§ 13. **Telephones with External Source of Current.**—The Bell telephone is a wonderful instrument, certainly; it acts as transmitter or receiver, and it requires no battery to work it. But it must be remembered that the currents transmitted by it are very small indeed, and that consequently the message is delivered by the receiving instrument in a very faint way.

To obviate this difficulty, telephones have been invented which make use of strong currents driven by an external source, such as a Leclanché cell.

Edison's carbon transmitter is an example of this class of telephone. This instrument has a mouthpiece and a vibrating metallic diaphragm, but no magnet or coil. At the back of the diaphragm is a button of carbon, this resting against a piece of metal. As the diaphragm vibrates the carbon will make better or worse contact with the pieces of metal touching it on the two sides; better contact when it is pressed harder, worse when it is somewhat released. Now the two pieces of metal and the carbon between them form part of the circuit of a Leclanché or other cell. Hence, as the diaphragm vibrates, the current flowing in this circuit will vary in strength, the variations corresponding exactly to the original sound vibrations. Now this current forms the primary to a small induction coil, the secondary wire of which is in circuit with the line to the distant station. No current will flow in the secondary, and therefore none through the line, so long as the primary is steady. But when some one speaks to the transmitter and thus causes variations to occur in the primary, then will there be currents induced in the secondary, in exact accord with the air vibrations of the speech uttered to the transmitter. These currents traverse the line to the distant station, and the receiver (which we may suppose, *e.g.*, to be a Bell telephone) will reproduce the original speech.

§ 14. **Microphones.**—Professor Hughes found that when in a circuit there is a loose contact, or still better a group of loose contacts, the current is exceedingly sensitive to even very slight mechanical disturbances, such, *e.g.*, as those produced by sound-waves impinging upon the loose contact or upon the stand supporting the same. Undulations are produced in the current that is passing, and these undulations are so exactly in accord with the mechanical vibrations to which they are due, that a telephone

included in the circuit will reproduce with greater or less distinctness the sounds spoken near the loose contact.

An instrument constructed on the principle and for the object indicated above is called a *microphone*.

It is found that loose contacts of *carbon* give the most striking results, and so in the usual form of instrument a carbon rod rests loosely between two other carbon pieces. If a fly walk over the stand of this instrument, the faint jarring due to its tread produces undulations in the current passing, and a telephone in the circuit will give out what we may, not however very exactly, call the 'sound' of the fly's tread.

§ 15. **Properties of Selenium. The Photophone.**—It is well known that in general the electrical resistance of bodies depends upon their temperature, and varies with it either inversely or directly.

There seems no reason to doubt but that whenever radiant energy (whether this consist of rays whose chief action is *heating*, or of rays remarkable chiefly for the impressions of *light* that they produce when they impinge upon the retina) impinges upon a body, the molecules of that body are to a greater or less degree affected; and that consequently the electrical resistance of the body is affected.

Now it is found that the body *selenium*, at least when in one of its several physical conditions, is affected in this way to a very remarkable degree. Even when the longer waves, those whose action is *par excellence* 'heating,' are sifted out of the ray, so that the radiation left is remarkable mainly for its action upon the retina and for its chemical action, still the effect produced upon selenium is very great. Hence one often hears of 'the action of *light* on selenium.' In order to exhibit or use this variability in conductivity, we must construct what is called a *selenium cell*. In one form of this there are two separate spirals of wire wound round a cylinder of hard wood or of other insulating material. These spirals are separated from each other, but run parallel to, and close to, one another through all their length. The one spiral is attached to the one end of the battery circuit, the other spiral to the other. So that the circuit is, as far as we have described it, broken, but the two halves run close to each other for a considerable length and in a compact manner. The space between the two

spirals is now filled up with selenium, which is melted on. We thus have the circuit completed by what is equivalent to a strip of selenium of very small length for the current to traverse, but of very great width. Thus the resistance is small, while the action of the radiant energy upon the selenium ought to produce a relatively great effect upon the current, there being such an extent of selenium to be acted upon.

With the help of such a 'selenium cell' a piece of apparatus, called a *photophone*, has been constructed, by means of which sounds, and even articulate speech, can be transmitted from a distance. It is so contrived that, on speaking to the transmitting instrument, a beam of light reflected from it to the receiving station varies its intensity in exact accordance with the vibrations due to the voice. The beam falls on a selenium cell, which is in a circuit with a battery and a telephone. The resistance of the cell varies with the variations in the beam, and so also does the current. Hence the current at the receiving station varies in accordance with the vibrations due to the voice at the transmitting station, and the telephone thus reproduces the sounds uttered.

ELECTRIC LIGHTING.

§ 16. **General Account of Electric Lighting.**—Illumination by means of electricity is effected in two main ways. We will give a brief preliminary description of each of these, and will then proceed to some detail.

(i.) *Incandescent lighting.*—In the one system a thin conducting filament is heated to whiteness by the passage of a current. The filament is usually or always enclosed in a vacuous glass vessel, and hence is not consumed. Here we have simply a case of Joule's law: if the resistance be R and the current be C, we have given out per second radiant energy measured in *watts* by $C^2 R$. It is necessary to make the filament small and highly resisting, so that the radiant energy may be intense in quality, *i.e.* so that the temperature may be very high.

(ii.) *Arc lighting.*—If we have a battery of high E.M.F. and a circuit of not too great resistance (forty Bunsen's cells of fair size will fulfil these conditions), and the current be passed through two rods of hard carbon which first touch one another, the point of contact will soon glow and attain a dazzling brilliancy; and if now

the carbons be slightly separated, the current will continue to flow across the interval in the intensely dazzling continuous discharge known as the *electric arc*. The illumination seems to be due to the intense ignition of a stream of carbon particles that are continually transported from the + to the − terminal. The *arc* can be deflected by a draught of air, or by being acted upon by a magnetic field. Its length depends upon the current-strength; *i.e.* upon the E.M.F. of the battery and the total resistance in the circuit.

It is found that the arc is the seat of a reverse *E.M.F.* e, against which the current is driven by the battery, and that also it offers resistance R as does a conductor. Hence the radiant energy evolved per second in the arc is expressed in *watts* by the sum of the two *activities* Ce and $C^2 R$. Here then we have *not* a simple case of Joule's law.

§ 17. **The Incandescent 'Lamp.'**—The substance chosen for the 'filament' spoken of in § 16 (i.) is now invariably *carbon*.

This substance possesses the great advantages of being practically infusible and of being a very good radiator.

The main difficulty encountered in its use was that of obtaining a filament that should be at once thin enough, and at the same time tough and not liable to break. *Edison* found the suitable material in the prepared fibres of a certain kind of bamboo. *Swan* and others acted upon cotton thread or fine strips of card with strong sulphuric acid, and then carbonised the tough threads thus obtained.

In the figure is shown a *Swan* lamp. It will be observed that, in order to give a greater amount of light within the same space, the filament is given a twist into one or more spirals. Near the ends the filament is thicker; this is to obviate any great heating near the points of support, as this would lead to the breakage of the filament.

If the globe be filled before exhaustion with a hydrocarbon gas instead of with air, the slight residue that always exists even in

a good 'vacuum' will act rather to strengthen the filament by deposition of carbon than to weaken it by combustion.

It is found that the carbon filament is slowly dispersed and otherwise weakened by continued use. Each lamp has, in fact, a definite 'life' that is usually expressed in 'hours of use with so many *ampères*,' or in 'ampère-hours.'

'Candles per horse-power,' in incandescent lamps.—It is of great interest and importance to know the average relations between the *candle-power* of an incandescent lamp, and the *horse-power* expended upon the lamp.

We know that the activity in *watts* expended upon the lamp is $C^2 R$, where C is the current in *ampères* and R is the resistance of the lamp in *ohms*. Or it is measured by $\frac{C^2 R}{746}$ (approx.) in English *horse-power*. The unit by which we usually measure candle-power is very arbitrary and somewhat uncertain ; it is the illumination given by a wax-candle of a certain form in which 120 grains of wax are consumed per hour. It is fairly evident that the desired relation must be established by experiment, and cannot be calculated.

In 'Nature,' vol. xxiv. p. 270, may be found a series of results obtained by Sir W. Thomson and Mr. J. T. Bottomley. It is found that if we wish to obtain such a degree of ignition only as shall give the lamp a chance of a fairly long life, we must be content with from 200 to 250 candles per horse-power. It must be noticed that we obtain, for the same horse-power expended, better *luminous* effects if we employ high temperatures. The total radiant energy emitted each second from several lamps moderately heated may equal that emitted from one lamp intensely heated ; but the radiation from the former may be useless from the luminous point of view. We should therefore raise the lamp to as high a temperature as is compatible with its safety. From what we have said, then, it is evident that the expression $C^2 R$ does not give us a measure of the light, but only of the total radiant energy, emitted per second by the lamp.

The resistance of a lamp varies.—As the temperature of carbon rises, its resistance decreases. Hence the resistance R of the

lamp is not constant. In order to calculate the *watts* or *horse-power* expended on the lamp when any definite current C is running, it is necessary to measure the difference of potential E_B^A existing between the terminals of the lamp at the time. The expression $C \times E_B^A$ gives us the required number of *watts* without error. We can measure C by means of a galvanometer of low resistance included in the circuit, and E_B^A by means of a potential galvanometer of high resistance (*see* Chapter XIV. § 11) connected up with the terminals of the lamp.

§ 18. **Arrangement of Incandescent Lamps in Parallel.**—The simplest manner of arranging a number of incandescent lamps of equal resistance is to connect them *in parallel* with two stout conductors that are connected with the two terminals of the dynamo respectively. The lamps are of equal resistance, and possess no reverse E.M.F. Hence the current between the two stout conductors (which we may add are called *leads*) is divided equally between the lamps. If one lamp be turned off, the current that now flows is divided between the others.

Referring to the case of '*m* equal branches,' discussed in Chapter XIII. § 9, II., let us suppose that here the resistance of each lamp is R; that of the rest of the circuit, including the dynamo, is negligible; and that the E.M.F. is E. Then, when the m lamps are arranged in parallel, the equivalent resistance ρ will equal $\frac{R}{m}$; and the current will be $C = \frac{E}{\frac{1}{m}R} = \frac{mE}{R}$. Now the current flowing in each will be $\frac{1}{m}C$, or will be $\frac{E}{R}$. Therefore, under the not quite attainable conditions assumed, viz. of zero resistance in circuit and in dynamo, the turning off or on of any lamps would not alter the current flowing through each of the remaining lamps; for, if m lamps were left, we should have for each lamp a current that is $\frac{1}{m}$th of $\frac{mE}{R}$, or should have $\frac{E}{R}$ as before.

We may put the matter somewhat differently. In order to supply each lamp of resistance R with a constant current C, it is necessary that the terminals of each lamp be kept at a constant ΔV measured by E, where $E = CR$. Now the stout conductors

between which the incandescent lamps are slung have, we assume, *zero* resistance, and are therefore each at one potential throughout its length. We shall then attain our end if we keep these copper *leads* at the required ΔV measured by E. But, by Ohm's law, if we have zero resistance in the dynamo and *leads*, it follows that the whole fall of potential in the circuit will occur across the lamps, and none will occur through the rest of the circuit. That is, the *leads* will be maintained at the full ΔV measured by E, and the current through each lamp will be constant.

In practice, however, we are not able to construct an armature, though we can make *leads*, of *zero* resistance. If the armature have a resistance B, while the resistance of m lamps in parallel is $\frac{1}{m}$ R, then the total resistance of the circuit is $B + \frac{1}{m} R$. Hence by Ohm's law there will be through the armature a fall of potential measured by $\dfrac{B}{B + \frac{1}{m} R} \cdot E$, while the ΔV between the *leads* will be measured by the remaining fall $\dfrac{\frac{1}{m} R}{B + \frac{1}{m} R} \cdot E$. Now as m increases this expression is not constant, but decreases. Hence, in practice it will not do to supply a set of lamps in parallel from a machine of constant *E.M.F.* E; for the current in each lamp will decrease as the number of lamps increases. We must have a dynamo in which E increases somewhat when m increases, and conversely. This condition is fulfilled by some dynamo of the *series* nature. For, as m (*i.e.* the number of lamps) increases, the total resistance which is measured by $B + \frac{1}{m} R$ will decrease; hence C increases, and therefore [from the nature of a series-dynamo] E will also increase.

§ 19. **The Economy of Incandescent Lighting.**—Supposing that we have 100 lamps, each of 50 *ohms* resistance when hot, arranged in parallel. This will give an equivalent resistance of but ·5 *ohm*. (We will neglect the resistance of the *leads*.) If we wish to use 90 per cent. of our energy in the lamps, and waste only 10

per cent. in heating the dynamo, then by Joule's law the resistance of the dynamo must bear to that of the lamps the ratio of 1 to 9. That is, the resistance of the dynamo must be but $\cdot 0\dot{5}$ *ohms*.

Hence, for incandescent lighting we must have dynamos in which the current passes only through very thick wire, dynamos of very low resistance. If the current demanded by each lamp be 1·2 *ampères*, then we must maintain the leads at a ΔV measured by 60 *volts*; since by Ohm's law we have . . . . .

$$E = C \cdot R = 1{\cdot}2 \times 50 = 60, \text{ for each lamp.}$$

Again, this ΔV will be $\frac{9}{10}$ of the whole fall of potential given by the dynamo; or the *E.M.F.* demanded of the dynamo must be $\frac{10}{9} \times 60 = 66\frac{2}{3}$ *volts*. We use *activity* measured by $(1{\cdot}2)^2 \times {\cdot}5$ *watts*; and we waste activity measured by $(1{\cdot}2)^2 \times {\cdot}0\dot{5}$ *watts*.

If, as we have done, we neglect all other waste, then we shall require $\frac{(1{\cdot}2)^2 \times ({\cdot}5 + {\cdot}0\dot{5})}{746}$ *horse-power* to do work at the rate here required.

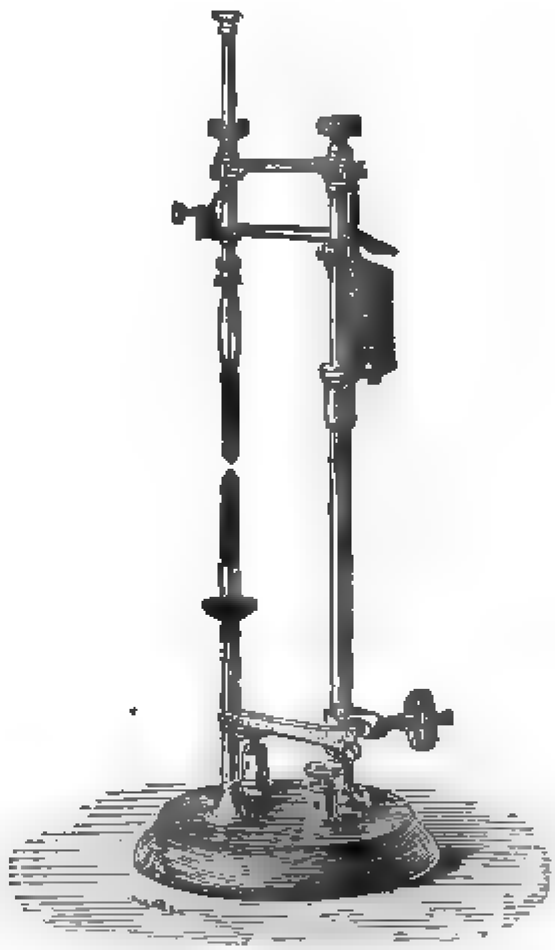
The relation between activity and candle-power is not an exact one, since the latter depends upon the quality of the radiant energy emitted per second as well as upon its quantity. But it will give some idea of the relations between the two if we mention that according to Mr. Swan ('Nature,' vol. xxvi. p. 358) we can obtain at least 200 candle-power for each horse-power of activity evolved in the lamp (*see also* § 17).

Another authority states that, allowing for 10 per cent. waste in heating the armature, we can run nine twenty-candle Swan lamps per horse-power. This estimation agrees practically with the last, since in that no deduction was made for waste in the circuit.

We must further, if we aim at economy, run the lamps at as high a temperature as is consistent with their safety; as was explained in § 17.

§ 20. **The Arc Lamp.**—In the figure is shown a simple form of arc lamp, such as is now used only for lecture-room purposes. Initially the two carbons, made of hard gas coke or of some other very dense form of carbon, are in contact. When, however, the current is well established, by which time the carbons at the point of contact will have been raised to a very intense white heat, a small

electro-magnet that is in the circuit acts upon a lever; and this, by means of a clutch that acts by friction, raises the rod bearing the upper carbon. Thus the carbons are slightly separated and the arc is established. If by the using up of the carbons the space between becomes too great and the current becomes enfeebled, the electro-magnet's hold on the lever and clutch becomes relaxed; and the latter allows the upper carbon rod to slide down nearer to the lower. When the current again rises to its proper value, this slipping movement is checked by means of the electro-magnet. If the arc be entirely broken, it can be restored only by the carbons coming again into actual contact.



Appearance of the arc.—With the naked eye the arc cannot be studied; the extremities of the carbons and the arc itself presenting a dazzling liquid brightness that, with the consequent irradiation effect, baffles analysis. Through dark glass we can, however, study the arc; and we then see that the source of the light lies in the extremities of the carbons, whence the arc springs across, and in the arc itself. This latter presents a flame-like appearance.

Mechanical transport of carbon.—When thus examined, or when a magnified image is thrown upon the screen by means of a lantern, it is seen that one carbon becomes hollow after some time. By actual measurements of weight it seems that it is the + carbon that wastes most, there being a continual transport of particles from it across to the — carbon; and that the hollow appearance of this latter is due to the fact that this stream of transported particles is deposited upon it in a crater-like manner. Of course both carbons are also consumed slowly when the arc is exposed to

the action of the air. The transference from + to − carbon is therefore best shown when the arc is used in *vacuo*.

This mechanical transport is obviated by the use of alternate current machines.

Temperature of the arc.—According to some experiments of M. Rosetti, of Padua, the + carbon attains the higher temperature. These temperatures vary when the current varies. The experimentalist in question believes that we can regard the − carbon to attain 2,500° C., and the + carbon 3,200° C., at least.

Definition of equivalent resistance.—If under any given conditions of driving-*E.M.F.* E, &c., we remove the arc and substitute such a resistance ρ that the current remains unaltered, then ρ is called the equivalent resistance of the arc. The reader will remember (*see* § 16) that the arc offers a reverse E.M.F. as well as a resistance. Hence the expression '*equivalent resistance*' has not the simple meaning of Chapter XIII.; its exact meaning will be discussed in § 22. It must carefully be noticed that we do not here in the least imply that under all conditions we could substitute ρ for the lamp without altering the current; but only that under the given conditions the one is equivalent to the other.

§ 21. **The Reverse E.M.F. of the Arc.**—If by means of some form of quadrant electrometer we examine the two carbons after the arc has been established, we find a large ΔV existing between them, varying in general from 25 to 55 volts.

Now if we consider any two points in a circuit, there will be a certain ΔV between them. If r_a be the resistance between these two points, V_a the ΔV between them, and C the current, and if there be no source of *E.M.F.* between these points, then by Ohm's law we have $V_a = r_a . C$. In such a case we get double of V_a if we choose points such that the resistance between them is double of r_a, *e.g.* if on a uniform wire we take two points separated by double the former distance, C being constant.

But in the case of the arc we find that the ΔV is nothing like doubled if, when C is maintained constant, we double the length of the arc. In fact it is found that the ΔV between the carbons, or V_a, consists of two parts; the one part is a reverse E.M.F. e_a, while the other part follows Ohm's law and is measured by r_a C, where r is the true resistance of the arc. M. Edlund, in his ex-

periments, found that the total Δ V between the carbons was thus composed. We may then write . . . .

$$V_a = e + r\ C,$$

understanding that of the two constituents of V_a by far the larger is e_a.

M. Edlund even detected the reverse E.M.F. e_a as existing for a fraction of a second after the current had ceased to flow.

It is worthy of remark that we have here a somewhat new case of transformation of energy. We expend activity to the amount of $C\,.\,e_a$, against the reverse E.M.F. e_a; and we get an equivalent, not in chemical activity as in an electrolytic cell, nor in mechanical activity as in a motor, but in heat activity.

§ 22. **'Equivalent Resistance' in an Arc Lamp.**—Consider the terminals A and B of an arc lamp, the ΔV between these terminals being V_a.

Now if E be the E.M.F. of the dynamo, the fall V_a occurs in the lamp, and the remaining fall $(E - V_a)$ occurs in the rest of the circuit. If we remove the lamp from between its terminals and insert the *equivalent resistance* (*see* definition in § 20), the ΔV V_a must remain unaltered, otherwise the fall of potential through the rest of the circuit, and therefore the current, would be altered, which would be contrary to the definition of equivalent resistance.

In the first case, therefore, we have . . . .

$$V_a = e_a + r_a C;$$

and in the second case we have . . . .

$$V_a = \rho C.$$

$$\text{Whence } \rho = \frac{V_a}{C} = \frac{e_a + r_a C}{C}.$$

Now it is found that V_a does not vary in proportion to the current C, but that, within certain limits of current, it remains fairly constant. Hence, as we hinted in § 20, ρ is not a constant at all, but it varies inversely as C approximately, within certain limits of C.

Example.—One form of arc lamp used in the Brush system

requires ten *ampères* current. The ΔV between its poles, or V_a is about fifty *volts*. Here, then, we have . . . .

$$\rho = \frac{50}{10} = 5 \textit{ ohms.}$$

Hence, *on the assumption* that a current of ten *ampères* is to be maintained, we may in our calculations substitute five *ohms* resistance for each lamp.

§ 23. **Series Arrangement of Arc Lamps.**—Since the reverse E.M.F. of a lamp varies from *zero*, when the carbons are in contact, to fifty *volts* or so when the lamp is in full power, it would be impossible to arrange arc lamps in parallel. We could not insure any equality of distribution even for a moment, and the system would be eminently unstable.

We must, therefore, arrange the lamps in series, so as to insure the same current passing through each. Even then there are difficulties that have been overcome by means of many very ingenious regulating contrivances.

We must consequently employ machines of very high *E.M.F.*; and, if we desire to avoid the waste that ensues if the armatures have a high resistance, we must obtain this high *E.M.F.* in part at least by exceedingly rapid rotation of the armature (*see* formula, Chapter XXIII. § 2). It is usual to employ alternate current machines, as these can be readily constructed to give high *E.M.Fs*, and as we thereby waste the carbons equally.

As an example let us suppose that we wish to light twenty Brush lamps in series. With the necessary current of ten *ampères* we may count each lamp as equivalent to five *ohms*. Hence, if E be the required E.M.F., we must by Ohm's law have . . . . . .

$$E = C R = 10 \times (5 \times 20 + R') \textit{ volts},$$

where R′ is the resistance offered by the dynamo and the rest of the circuit. If R′ be about 10 per cent. of the total resistance, we must have . . . .

$$E = 10 \times \left(100 + \frac{100}{9}\right) = 1111\frac{1}{9} \textit{ volts.}$$

Dangers from currents of high E.M.F.—This explains the dangerous character of the currents employed in arc lighting as compared with those employed in incandescent lighting. If a person touch with one hand a wire conveying a current at such a

high E.M.F. as the above, he may form a branch circuit in consequence of leakage to earth; and at this high E.M.F. a fatal current may pass through him. The effects of alternate currents on the human frame are far more serious than are those of continuous currents. Of course contact with two hands is even more dangerous. Instant death has occurred in both these ways. There are of course fire risks, due mainly or entirely to the ignition of wires by the passage of successive currents.

For an account of the various means by which arc lamps are regulated, we must refer the reader to more technical works. To such works we must also refer him for descriptions and figures of other forms of electric lamps, as the *Jablochkoff candle*, *Jamin's lamp*, *Werdermann's lamp*, the *Sun lamp*, &c.

§ 24. **Further Remarks on the Use of Arc Lamps.**

For the reason given before, viz. that it is only the radiant energy of high temperature that is useful for illumination, arc lamps are more economical than incandescent lamps; but of course this holds only when the areas to be illuminated are sufficiently great to demand such powerful centres of illumination. Small arc lamps are not economical, nor indeed can they be readily kept steady; hence for houses the incandescent is the only practicable light. On the same principle it is more economical, though often not at all convenient, to expend our horse-power on one large lamp than on several small ones. One obvious consideration is as follows: that as we must have arc lamps in series, and as two smaller lamps in series will give a combined reverse *E.M.F.* that is very much greater than that of one large lamp, the same current that will maintain the latter in full action may be totally unfit to maintain the former.

In order to insure economy in working it is, as a rule, better to construct dynamos specially for the particular work for which they are intended, and not to use the same dynamo for a large and for a small number of lamps in series.

MORE RECENT MEASURING INSTRUMENTS.

§ 25. **Current-meters or Ammeters.**—As was observed at the end of Chapter XVII., practical measuring instruments must be independent of the earth's magnetic field, and must not be affected by such magnetic disturbances as are of usual occurrence.

We require *two* classes of such instruments. *Firstly*, the necessarily more delicate 'standard instruments,' by means of which we can from time to time test and calibrate the ordinary 'working instruments'; and *secondly*, a rougher and more portable type of instrument, whose construction can be varied to suit various requirements.

Standard instruments.—Our standard instrument should be delicate, uninfluenced by external magnetic disturbances, and capable of measuring large or small currents. Moreover its deflexions should be proportional to the currents causing them; and its 'constant' should not change perceptibly in (say) a few months.

All these requirements are met very fairly indeed by a modified form of a permanent-magnet instrument, an 'improved Deprez-d'Arsonvale galvanometer.'

In this instrument a coil of fine wire, wound on a copper frame for damping purposes (*see* § 27), is suspended between the poles of a permanent horse-shoe magnet whose lines of force lie more or less in the plane of the coil. The current is conveyed to and from the coil by means of very fine wires—[Professors Ayrton and Perry use very fine flat wires of phosphor-bronze]—which give practically no torsion-couple for usual deflexions of the coil. Embedded in the coil, and lying along the lines of force when the coil is in its zero position, are a number of small needles of soft iron.

When a current passes through the coil, the field acts on the coil with a *deflecting* couple tending to set it at right angles to its 'zero-position'; and it also acts on the soft iron needles with a couple that tends to *restore* the needles and coil to their zero-position; and the final deflexion, indicated, as shown on p. 284, by lamp, mirror, and scale, is such as gives equilibrium between these two couples. The particular value of getting rid of a *torsional* restoring couple, and substituting for it a *magnetic* restoring couple, is that even considerable changes in the magnetism of the permanent magnet produce no change in the value of the deflexions; since both deflecting and restoring couples are affected alike.

The galvanometer is usually 'shunted' off the main circuit; and, by varying suitably the resistance of the shunt and that in

the galvanometer circuit, we may arrange the instrument so as to measure currents of any strength in the main circuit [*see* Chapter XIII. § 10]. The instrument is calibrated by electro-chemical means; details of such calibration will be found in practical text-books.

It then serves to calibrate other instruments; these instruments being placed in the main circuit, currents of various strengths measured by the [shunted] standard instrument being passed through them, and their readings being noted.

Ammeters in which soft iron is used.—In a very large number of ammeters the field due to the current causes movement in a piece of soft iron, and this causes an index to move. The deflecting moment so caused is balanced usually by a torsional couple, or by the moment of a 'weight' about an axis. In most of such instruments there may be very large errors due to residual magnetism, if currents of very different magnitudes be measured soon after one another. Such errors can best be obviated by having the iron used both very soft and very thin.

Solenoidal Ammeters.—More accurate, though usually less portable and more expensive, are instruments in which current passes through two coils, one of which is movable. Such an instrument was alluded to in Chapter XVII. § 15. Here the action is between the two coils only; there is no soft iron, and therefore no errors due to residual magnetism.

Sir W. Thomson has devised accurate instruments based upon this principle.

'*Deflexional*' *and* '*zero*' *instruments.*—We may for some purposes divide measuring instruments into two classes:

(i.) those in which we observe the *deflexion* of an index,

(ii.) and those in which a measured force or couple brings the index back to *zero*.

Concerning (i.) it may be said that they enable us to see at once the magnitude of the current [or other quantity to be measured], and to observe changes in it. But the range of movement of the index, and therefore the range of measurement, is usually not great; and the divisions on the scale are usually not very proportional to the current [&c.], since the *arm* of the couple acting on the coil or piece of soft iron usually changes owing to its movement in the field.

Concerning (ii.) it may be said that we get a far larger range of measurement by a suitable choice (*e.g.*) of torsion wire; and, as there is no movement in the field, the deflecting couple, and therefore also the restoring couple, is proportional to the current or to the square of the current. But it is inconvenient not to be able to observe at once changes in the current.

By means of a 'magnifying spring' Professors Ayrton and Perry have contrived to make deflexional instruments in which the index has a long range of movement, and yet the piece of soft iron moves but very slightly. Thus these instruments possess in a large measure the two main characteristic advantages of the 'zero type' of instrument.

§ 26. **Voltmeters.**—Most voltmeters are practically simply ammeters in which the resistance is very great as compared with the resistance lying between the points whose Δ V we wish to measure [*see* Chapter XIV. § 11]. It is clear that if the resistance of the voltmeter changes, the Δ V between the terminals that answers to a certain deflexion will change also. This is a source of error in all such voltmeters.

The Cardew Voltmeter.—If we have a wire so shielded that it is not subject to variable currents of air, then its rise in temperature and consequent elongation will depend only upon the magnitude of the current traversing it; that is, upon the Δ V existing between its terminals

On this principle Cardew constructed his 'heated wire voltmeter.' One of its great advantages is that it can be used for alternating currents also.

[It is evident that if the wire is to change in temperature readily, and to heat and cool rapidly, it must be very fine, and therefore be of high resistance. Hence the Cardew instrument, based upon the above principle, is adapted for a *voltmeter*, but not for an *ammeter*].

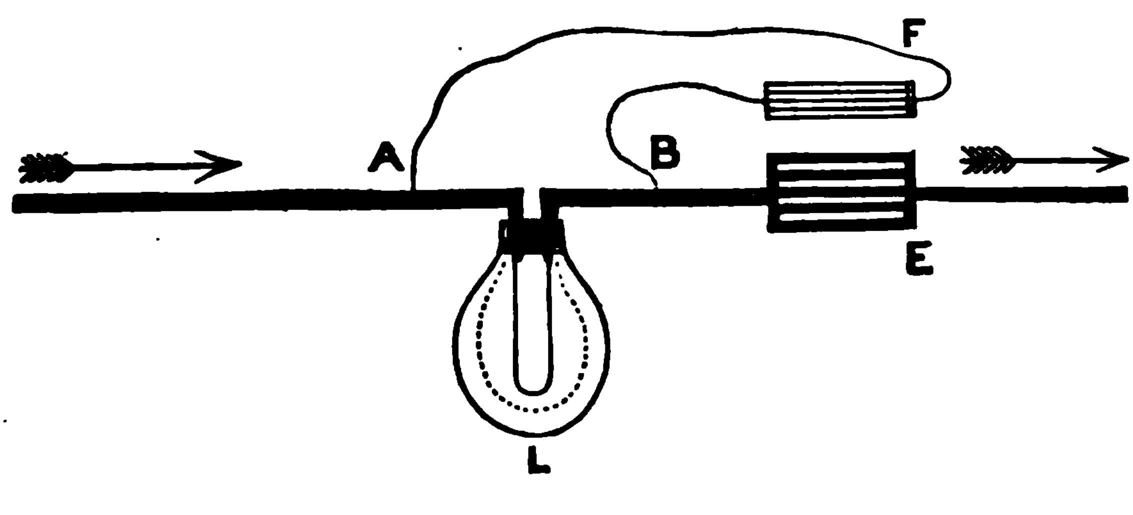
By another ingenious application of their 'magnifying spring' Professors Ayrton and Perry have contrived to produce an instrument of this type that is of much smaller size and greater sensitiveness, and in which the waste of electric energy is much less.

§ 27. **'Damping' in Ammeters and Voltmeters.**—It is usually of considerable importance that the index of the instrument should rapidly come to rest

The method of 'damping' by means of induced eddy-currents [*see* Chapter XXII. § 2] is often employed. Thus ordinary needle-galvanometers usually have copper plates under the needles; in some instruments the needle is of such a shape that it can be entirely enclosed in a copper cylinder; and, in the Deprez-d'Arsonvale instrument, described in § 25 of this chapter, the movable coil is wound on a copper frame that moves with it in the magnetic field. In some of their more recent instruments Professors Ayrton and Perry have made use of the 'viscosity of air'; and, by attaching to the index a vane fringed with fine feathers, have succeeded in getting a very dead-beat action.

§ 28. **Wattmeters.**—It is often desirable to measure the *watts* expended in some particular portion of a conductor, as *e.g.* in an incandescent lamp. We might measure separately the Δ V in *volts* between the extremities of the conductor, and the current in *ampères* flowing through it, by means of voltmeters and ammeters respectively. But instruments have been devised which will give us the *watts* more directly.

As it is necessary to see such instruments in order to understand them thoroughly, we here give merely a representative diagram, illustrating the principle.



Let L represent an incandescent lamp, and A B its terminals; let E_B^A be the Δ V between its terminals, and C the current flowing through it. F is a coil of fine wire of relatively very high resistance; so that the current C′ that flows through it is insignificant as compared with C. Of course C′ is proportional to E_B^A, assuming that the resistance of F does not alter perceptibly.

E is a coil of stout wire, through which passes the whole current (C+C′).

In reality, the coils F and E are part of one and the same instrument; and one of the two, say F, is movable. If the instrument is of the 'zero type' we can bring back F to its zero-position by a measured force or torsion-couple. The electromagnetic force that we thus balance and measure is of course proportional to the product $C' \times (C + C')$; that is, very nearly to $C \times C'$, since C' is relatively small so that $(C')^2$ can be neglected. But C' is proportional to E_B^A. Hence our measured force or torsion-couple is proportional to the *watts*, $E_B^A \times C$, expended in the lamp.

Mr. Swinbourne has devised a means of eliminating the small error mentioned above.

As regards the fine coil F, the wattmeter is subject to error due to change of resistance with changing temperature.

bont § 29. **Integrating Instruments. Coulomb-meters.**—Though it is generally recognised that it is *energy*, or (*quantity*) × (*difference of potential*), that should be measured and paid for, and not *quantity* alone, yet instruments designed to measure the *total quantity* supplied with varying current-strength are worth noticing. We can, however, give but a brief description of one type only. Of course if the Δ V between the 'leads,' where they enter the house, remains constant, then the total quantity supplied will be proportional to the total energy supplied; and this may very well be the case where the house is supplied for incandescent lighting only, since for this the Δ V should be constant.

Dr. Aron's 'Pendulum' Coulomb-meter.—This instrument is in detail somewhat complicated; but the principle of its action, which is all that we can give here, is easily explained. A clock, that under ordinary conditions keeps accurate time, has its bob composed mainly or partly of a permanent magnet of very hard steel. The current passes round a solenoid of thick wire, so placed as to give a vertical magnetic field symmetrical with respect to the pendulum-bob when this latter is in its lowest position. The direction of the current and the polarity of the bob are such that when the current passes there is a force directed vertically upwards, acting on the bob.

When the current passes, then, the *practical* effect is to change the gravitational constant g into some slightly smaller value $(g-k)$. The upward magnetic force acting on the bob must be very small

s compared with its weight; or k must be very small as compared with g. And the field must not be so strong as to alter the strength of the magnet. When these two conditions hold it can be shown that the *increase* of the time T of a complete oscillation [due to the *weakening* of the total downward pull on the bob] is (approximately) in direct proportion to the current-strength.

Hence the amount of 'time lost' by the clock during each minute is also directly proportional to the average current strength during that minute. And so, finally, the total 'time lost' by the clock during the day will be a measure of the total quantity of electricity that has passed during that day. [In infinitesimal notation it will measure $\int C.dt.$]

§ 30. **Integrating Instruments. Energy-meters**—It is, however, a far more important matter to measure total energy supplied. Professors Ayrton and Perry have devised an instrument by means of which this may be done. In general respects it is like the instrument last described; but the bob of the pendulum is a coil of fine wire of relatively very high resistance [*see* Chapter XIV. § 11]. The terminals of this coil are connected with the two 'leads' by which the electrical energy to be measured is being supplied; and thus at each moment the current C′ passing through this fine coil is directly proportional to the Δ V between these 'leads.' The main current C, in comparison with which C′ is negligible owing to the 'relatively very high resistance' of the fine-wire bob, passes through two fixed coils of very thick wire situated one on each side of the bob when this latter is in its lowest position.

Again, as in § 29, we have the practical effect that the gravitational constant g is altered into $(g-k)$. But in the present case the value of k is proportional to the product CC′, and not to C alone; that is, is proportional to the product . . . .

$$(\textit{current}) \times (\Delta\ V \textit{ between the leads});$$

or to the *watts* supplied to the house at each moment. Hence the 'loss of time' during each minute is a measure of the *watts* supplied during that minute; and the total 'loss of time' in the day is a measure of the *total electric energy* supplied during the day. [In infinitesimal notation it will measure $\int W.\,dt$, where W represents the watts at the time t.]

§ 31. **Testing of Coulomb-meters and Energy-meters.**—By means of *ammeters* and *voltmeters* it is not difficult to see how the instruments just described can be tested and their constants found. But a description of the manner in which this is done in practice would be out of place here; indeed, the student should go through a laboratory course if he wishes to understand the details of the use and calibration of measuring instruments.

DISTRIBUTION BY ALTERNATING CURRENTS AND 'TRANSFORMERS.'

§ 32. **General Principle of the System.**—One of the most important problems in electrical engineering is how to supply a district with electrical energy from a central station, without either an undue percentage of waste in the 'leads' from having too thin wires, or undue expense from having these too massive.

Now, in Chapter XXIV. § 8 we saw that *power* could be transmitted from a distance along thin leads with economy and on a sufficiently large scale, provided that we used a high 'driving E.M.F.' in the dynamo, and also a high 'reverse E.M.F.' in the motor. And, again, we could attain the same desired result in the case of incandescent lamps if only their total resistance were very great as compared with the resistance of the leads. As, however, incandescent lamps are worked 'in parallel,' their combined resistance will not be relatively high if the leads be long and thin.

The system named above aims at solving the problem in what we may call a third way; and we will briefly describe the principle of this system, it being now of considerable importance.

Let us suppose that there are upon the same soft iron core two coils of wire; one composed of many turns of very fine wire, and the other of fewer turns of very thick wire; and let an alternating current be sent through the fine-wire coil.

Then we have an instrument that resembles a Ruhmkorff's coil, in which the *long fine coil* is chosen as *primary*, and the *short thick coil* as *secondary*. Instead, therefore, of obtaining small currents of high E.M.F. by means of large currents of low E.M.F., *we conversely obtain large currents of low E.M.F. by means of small currents of high E.M.F.*

When the secondary circuit is closed there will be [by mutual

induction] a reverse *E.M.F.* induced in the primary. Thus if E be the *E.M.F.* of the dynamo, and the reverse *E.M.F.* in the primary be e, while the resistance of the whole primary circuit is R, there will be a current, C, measured by $\frac{E-e}{R}$, and this may be very small. The energy given each second to the secondary circuit will be C e *watts*. Thus we have . . . .

$$\left\{\begin{array}{l}\text{C E } \textit{watts} \text{ supplied by dynamo,}\\ \text{C } (E-e) \textit{ watts} \text{ wasted in heat,}\\ \text{C } e \textit{ watts} \text{ usefully expended in induction on the secondary,}\\ \quad \text{and } \frac{e}{E} \text{ will measure the } \textit{efficiency}.\end{array}\right.$$

Note.—In all the above we have wished only to give the principle of the action. Hence we have used the simple symbols C, E, and e, as if we were dealing with steady currents. To deal properly with alternating currents is a matter for a more advanced book than the present.

It is found that instruments of this kind, called for obvious reasons 'Transformers,' can be so constructed as to give a high *efficiency*, and also [which is a different matter] plenty of current in the secondary circuit. We can thus, for reasons referred to in Chapter XXIV. § 8, use 'leads' of thin wire and small cost without much percentage waste. The central station supplies the whole circuit with an alternating current of high E.M.F. ; and each house can have a transformer, suited to its requirements, from the *secondary* of which it draws its supply.

QUESTIONS AND EXAMPLES.

CHAPTER I.

1. It is sometimes found that two magnetic poles, which at some distance apart repel one another, will attract one another when they are brought nearer. Account for this.

2. Explain the statement that '*repulsion* of the pole of a magnetic needle is the only certain test that the body presented to it is a magnet.'

3. You are required to convert the blade of a pen-knife into a magnet, having the *point* for its *south*-seeking, or −, pole. How would you do this?

4. How could you show experimentally that in the case of action between a magnet and a piece of iron the law that 'action and reaction are equal and opposite' holds good?

5. How would you treat a needle if you wished to give it a *north*-seeking pole at each end, and a *south*-seeking pole at the centre?

6. Give any reason that occurs to you to account for the fact that *white-hot* iron exhibits no magnetic properties.

7. A number of soft iron needles are floating in a vertical position on small and separate bits of cork in a vessel of water; they are very close to one another. A powerful magnetic pole is held above the group. Mention, and explain, any vertical and horizontal movements that will take place.

CHAPTER II.

1. A stone traverses 2 *kilomètres* in $1\frac{1}{2}$ *minutes.* What is its velocity in C.G.S. units?

2. In a certain moving body there is observed a steady increase of velocity; the velocity, added each 3 *minutes*, being a velocity of 30 *mètres* per 20 *seconds.* What is this acceleration in C.G.S. units?

3. A mass of $2\frac{1}{2}$ *kilogrammes* loses, each $2\frac{1}{4}$ *minutes*, a velocity of 300 *mètres* per 2 *hours.* Express in *dynes* the force acting.

4. Two forces, of 30 *dynes* and 40 *dynes* respectively, act on a material particle at right angles to one another. Find the magnitude of their resultant, and the angle that it makes with the direction of the smaller force.

5. A force of 5000 *dynes* is resolved into others acting at right angles to one another. One of these is 3000 *dynes.* Find the other component, and the angle that it makes with the original force.

6. One couple has an arm of 1 *mètre*, and each force is 600 *dynes*. Another couple in an opposite direction has each force 20 *dynes*. What must be its arm in order that it may just balance the former?

7. In a magnetic needle, the pole has a strength of 20 C.G.S. units, and the length between the poles is 12 *cms*. What is the magnetic moment of the needle?

8. In a certain magnetic field a pole of *strength* 13 is urged with 20 *dynes* force. What is the strength of the field?

9. A magnetic field has *strength* 12. In it is placed a needle of *magnetic moment* 10, so as to be at right angles to the lines of force. With what couple is the needle urged? Explain the italicised expressions.

10. A pole of strength 20 is placed in a magnetic field in which another pole of strength 6 was urged with 13 *dynes* force. With what force is the former pole urged? And what is the component of this force in a direction making 30° with the lines of force of the field?

CHAPTER III.

1. In a field of strength 20 is placed a magnetic needle of pole-strength 6, and length 8 *cms.*, lying at right angles to the line of force. What is the magnitude of the couple with which the field acts upon the needle?

2. In the same field is placed a needle of magnetic moment 15, making an angle of 30° with the lines of force. What is the couple in this case?

3. There is a soft iron mast in a certain ship, and the ship's compass is placed between the foot of the mast and the stern of the vessel. Describe in general terms how the presence of the mast will affect the indications of the needle in each of the following cases.

(i.) At the magnetic equator, when the ship lies N and S, E and W, S and N, and N W and S E respectively.

(ii.) In latitudes about 45° north, for the same positions of the ship respectively.

(iii.) At the north magnetic pole.

Hence, make a comprehensive general statement as to the general nature of the disturbances due to such a vertical iron mast.

4. Discuss in a similar manner the action, on the needle, of a horizontal beam of soft iron running right across the vessel between the compass and the bows.

5. Explain, with formulæ also, how to use the torsion balance so as to compare two magnetic fields; assuming the magnetic moment of the needle to remain constant. Point out clearly the importance of this assumption.

6. How would you use the torsion balance to compare the magnetic moments of two needles; assuming that each can be in turn attached to the thread of the same torsion balance? Use formulæ also.

7. What considerations justify the statement that it is a more complicated matter to compare the magnetic moments of two needles by the method of oscillations than by the method of (6)?

8. A small magnetic needle, placed at a certain distance from the pole of a magnet, is found to make 12 oscillations per *minute*; the lines of force due to the magnet having, in the neighbourhood of the small needle, the same direction as those due to the earth. When in the same position with respect to the pole of another magnet that gives just twice as strong a field as the former, it makes 15 oscillations per *minute*. How many oscillations per minute would it make in the earth's field alone?

9 A needle makes 9 oscillations per *minute* in a certain magnetic field. How many will it make when re-magnetised so that its magnetic moment is half as great again as before?

10. Assume that in a certain room the earth's field has been neutralised, and that there a small needle makes 20 oscillations per *minute* when 4 *inches* distant from the pole of a magnet. How many will it make when 3 *inches* distant? [Neglect here the other pole of the magnet.]

11. Two poles of strength + 10 and − 8 respectively are placed 20 *centimètres* apart. Find in *dynes* the stress between them; interpreting the sign.

12. If it be desired for the purpose of experiment to neutralise as far as possible the earth's field about a needle, how would you do it? And how would you test whether or no the desired result had been attained?

13. How would a truly-balanced dip-needle behave under the following conditions? Its plane of swing is in the magnetic meridian; and a bar-magnet is placed in a horizontal position symmetrically on one side of it, with the *north*-seeking pole pointing north and the *south*-seeking pole pointing south.

14. A needle, oscillating at two different places, makes 80 oscillations in 3 *minutes*, and 100 oscillations in 4 *minutes*, respectively, when acted upon by the earth's horizontal field only. Compare the horizontal fields at the two places respectively. What assumption with respect to the needle do we here make?

15. Given that the earth's horizontal field has a strength measured by ·5, and that the dip is 30°, at a certain place; required, (i.) the total field-strength, (ii.) the vertical component, at that place.

16. A needle vibrates in a horizontal plane under the earth's horizontal field, and makes 16 oscillations per *minute*. How many oscillations per *minute* would it make where the horizontal field is stronger in the ratio of 9 to 4?

CHAPTER IV.

1. A person is seen to bring a rod near to a magnetic needle, and this latter is thereby deflected. What further experiment would you direct him to perform in order to determine whether the observed action is magnetic or electrical?

2. Some mercury contained in a dry glass vessel is connected by a wire with a gold-leaf electroscope. The end of a dry glass rod (unexcited) is gently dipped into the mercury and is then suddenly withdrawn. Describe and explain what happens in the two cases respectively.

3. A soap-bubble is blown at the end of a narrow insulated tube, and the

other end of this tube is then closed. Explain what occurs when the whole is charged with electricity.

4. Discuss the statement that 'repulsion is the only certain sign of electrification.'

5. Suggest any experiments tending to prove that an electric charge resides on the surface only of an electrified conductor.

6. Given two charged bodies, and that it is required to determine whether or no their charges are equal and of opposite signs. Explain why it is an accurate test to drop them both into a hollow conducting vessel placed upon the plate of the gold-leaf electroscope, and not an accurate test to lay the two bodies on the plate.

7. A small insulated conducting sphere charged with 4 units of + electricity is at a distance of 2 *cms.* from an identical sphere charged with 2 units of – electricity. What is the stress between them, in *dynes*? And what will be the stress if they are made to touch one another and are then removed to the same distance apart?

8. How would you show that a charge + Q induces on surrounding earth-connected surfaces an equal and opposite – Q?

9. A sheet of tin-foil is rolled upon an ebonite rod and is electrified so that a pair of pith-balls, suspended from the foil by cotton threads, diverge widely. On unrolling the foil it is observed that the balls diverge less widely. Explain this action by reference to definite electrical principles.

10. A charged conductor is placed, insulated, inside a hollow conductor also insulated. The latter is put to earth for a moment and is then re-insulated When the charged conductor is removed, in what condition is the hollow conductor left? Suggest any further experiment needed to prove the truth of your statement.

11. You are given a charged and insulated body A; a hollow insulated conducting vessel B; and a larger hollow insulated conductor C. The vessel B can be placed entirely inside C without touching it, and A can be placed inside B without touching it. Show how it is possible to give to C any multiple of the charge on A, and this without bringing A into contact with anything.

(*Note.*—The methods of § 16 are to be followed.)

CHAPTER V.

1. How is the distribution on an insulated conductor affected by . . . .

(i.) The shape of the conductor?

(ii.) The position of surrounding bodies?

Is the *potential* of the conductor affected by these conditions; and, if so, in what way?

2. A sphere of gun-cotton and a metal sphere of much larger radius, both uncharged and insulated, are rubbed together and then separated. What will be the condition of the two respectively, both as regards quantity of charge and as regards potential?

3. (*Note.*—In the following it will be well to draw first a figure of the arrangement described.)

Two thin gold leaves are suspended within an ordinary glass flask A, by a wire which is fitted air-tight in the neck of the flask and is provided with a metal knob outside. The flask is placed in a larger metal vessel B which is connected with earth; the vessel B reaches up to the level of the top of the flask, but the knob projects above B. A wire-gauze cylinder C, closed at one end, is inverted over the flask, resting on its shoulders; this gauze cylinder is insulated from B and from the knob, and the gold leaves in A hang down below the level of the lower edge of C. State and explain what will take place . . . .

(i.) when a + electrified glass rod is held over the system;

(ii.) when water is then poured into B until it finally touches the lower edge of C.

4. Explain, using a diagram, under what circumstances a conductor may be said to be 'isolated.'

5. Explain, with a diagram, why the 'capacity' of such a conductor [considered alone] increases when it is brought near to the walls of a room.

CHAPTER VI.

1. A thin sheet of ebonite, coated on its upper side with tin-foil extending to within some distance from the edges, is laid upon a conducting table. The tin-foil is struck with dry flannel, and some pith-balls are dropped upon it. A person then lifts the ebonite from the table, holding it by its edges only. Explain what happens at each stage in this experiment.

2. A person holds a charged Leyden jar by its outer coating, and brings its knob into contact with the knob of a similar uncharged jar which another person holds by its outer coating. Describe and explain what happens.

3. A jar is charged +ly while its outer coating is to earth. It is then placed upon an insulating stand, and its outer coating is charged +. Discuss, in a general manner, the final condition of the jar.

CHAPTER X.

1. Two conductors, A and B, are insulated and are placed near to one another.

(i.) A is charged to a potential + V. What will be the potential condition of B?

(ii.) If A be charged to a sufficiently high potential it is observed that at last a spark will pass between A and B. What does this fact prove with respect to the rise of potential of B during the charging of A?

(*Note.*—We here assume that the passage of a spark indicates the attainment of a certain difference of potential between A and B.)

2. Two small pith-balls are each of mass 1 *gramme*. They are suspended from the same point by silk fibres each 10 *centimètres* long, and of negligible

mass. They are then electrified so that each string is inclined 30° to the vertical. How many units of electricity must each ball possess?

(*Note.*—Take moments about the point of suspension, equating the electrical moment and the gravitational moment, g being assumed to be 981.)

3. Two small balls, each of mass 1 *gramme*, are suspended from the same point by silk fibres, each of which is $490\frac{1}{2}$ *centimètres* in length. Taking g to be 981, show that when each ball is charged with a unit of electricity they will diverge to a distance of 1 *centimètre* from each other.

(*Note.*—This example is taken from Professor Garnett's lectures.)

4. Three insulated conducting spheres, A, B, and C, whose radii are respectively 1, 2, and 3 *centimètres*, are so charged that their respective potentials are 3, 2, and 1 units of potential. They are then connected by wires of negligible capacity. What is the total quantity of charge, what is the total capacity, and what the final common potential of the system?

5. A small insulated charged sphere is placed near the knob of an electroscope, this latter being previously uncharged. Explain why the gold leaves diverge. What will happen if we interpose . . . .

(i.) an ebonite plate;

(ii.) an insulated and uncharged brass plate;

(iii.) an un-insulated brass plate, ...

respectively, between the charged sphere and the electroscope?

6. Find the capacity of an insulated sphere that is 10 *centimètres* in diameter, it being surrounded by a concentric sphere 10·2 *centimètres* in internal diameter.

7. Find in absolute electrostatic units the capacities of. . . . . . .

(i.) an isolated sphere of 10 *centimètres* radius;

(ii.) the same when the 'large room,' in which the sphere is placed, is filled with shellac (*specific inductive capacity* σ);

(iii.) a spherical condenser in which the two radii are 10 *centimètres* and 11 *centimètres* respectively, air being the dielectric;

(iv.) a cylindrical Leyden jar in which the radius is 5 *centimètres*, the height of the tin-foil 20 *centimètres*, the thickness of glass ·1 *centimètre*, and the *specific inductive capacity* of the glass is 1·76. (The jar is not coated with tin-foil at the bottom; only round the side.)

8. A Leyden jar has a coated surface of 100 *square centimètres*, the *specific inductive capacity* of the glass is 1·9, and the thickness is ·2 *centimètres*. What would be the diameter of an isolated metal sphere of the same capacity as this jar?

9. A condenser consists of two parallel plates. When there is air between them, the one plate A is put to earth, and the other B is charged to a potential V. Next the plate A is also insulated, and a sheet of ebonite is introduced between the two plates. How is the potential of each plate affected?

10. A sphere is charged, and is placed in the middle of a 'large' room. The lines of force and equipotential surfaces are marked out in the usual manner. The room is now filled with a medium whose specific inductive capacity is greater than that of the air or other medium that previously filled

the room. Discuss and explain any change in the system of lines of force and equipotential surfaces; these being marked out on the same principle as before.

11. Two small conductors are charged; and, when placed a certain distance apart in air, they repel each other with a certain force. They are then immersed in anhydrous paraffin oil, having their charges and distance apart unaltered. Explain in a general way what alteration will be observed in the stress between them.

12. The capacity of a conductor is K, and it is charged with Q units of electricity. How many *ergs* work were done in charging it?

13. An insulated conducting sphere of given radius receives a given charge. Another sphere of equal radius, uncharged, is made to touch the former, and is then removed. Show, in symbols, how to find the energy of discharge attending the contact between them.

14. Discuss the case of the last section when the radii of the spheres are unequal.

15. What main alterations must be made in the wording of §§ 9–16 in order that the results there given may apply also to the magnetic fields?

16. What will be the magnetic potential of a point situate at 10 *centimètres* from the + pole, and 15 *centimètres* from the – pole, of a bar-magnet, the pole-strength of this magnet being 40 in absolute measure?

CHAPTER XI.

1. A zinc vessel having a hole at the bottom is filled with finely powdered copper. Underneath this vessel is an insulated copper cylinder open at both ends, but provided internally with a metal tongue that reaches across from side to side near the centre of the cylinder. The former vessel is put to earth, and the powdered copper drops from the opening, strikes the metal tongue of the cylinder, and then falls clear to the ground. What will be the condition of the insulated cylinder when the action has proceeded for some time? [Assume the view of § 4.]

2. A plate of *pure* zinc is immersed in dilute sulphuric acid. A little copper-sulphate is then mixed with and dissolved in the dilute acid. It is observed that copper is instantly deposited upon the zinc, and that after this hydrogen is given off freely. Of what nature is this latter action? Explain; comparing with any similar action with which you are acquainted.

3. Into a platinum dish is poured some acidified solution of an antimony salt, and then a piece of zinc is dropped in. The platinum is found to acquire a coating of metallic antimony. Explain this action.

4. In Question 2, what will be the action if more copper-sulphate solution be poured in?

5. In Question 3, what will be observed if there be plenty of acid, but very little antimony salt, in the solution?

6. Explain, with a figure, how you would arrange matters if you wished to

try whether an electric current could be obtained from two given liquids and a single given metal.

7. Under what circumstances does a 'constant' battery become polarised and fall off in power?

CHAPTER XII.

1. A copper dish contains an unacidulated solution of copper-sulphate, and in it is suspended a plate of lead so as not to touch the copper. The dish is connected with the – pole of a battery, and the lead with its + pole.

(i.) What changes take place?

(ii.) What happens when the battery is removed, and the copper dish is connected by a wire with the lead plate?

2. The poles of a single Daniell's cell are connected with two platinum plates immersed in dilute sulphuric acid, and a galvanometer is included in the circuit. What is observed (i.) the moment that the circuit is completed; (ii.) and after a time? If we remove the Daniell's cell and complete the circuit without it, what is observed?

3. In the last question what difference does it make if we employ *two* Daniell's cells arranged in series?

4. If a 'storage battery' be charged by a dynamo running at full speed, and if then the speed of the dynamo (and therefore its electro-motive force) fall off, the connections remaining unaltered, what may happen?

5. How would you prepare a secondary battery to give an E.M.F. greater than that of the primary battery employed in the charging?

6. How would you construct a secondary battery if you required a current at a moderate E.M.F. but to last a long time?

7. Compare the action and theory of a secondary battery and of a Leyden jar respectively.

8. Given a number of uncharged secondary cells capable of giving an E.M.F. of 1 *volt*, a large primary cell of E.M.F. of 1·5 *volt*, and an electrolytic cell whose reverse E.M.F. exceeds 1·6 *volt*, show how you can decompose the electrolyte in this cell.

9. Criticise the common expression 'storage of electricity' as applied (i.) to the Leyden jar, (ii.) to the secondary cell, respectively. Give any analogies that may occur to you, applicable to the two cases respectively.

10. Discuss whether there is or is not a true distinction to be made between primary and secondary batteries.

CHAPTER XIII.

Preliminary notes.—(i.) In what follows we shall occasionally use the following units, of which a further account will be given in Chapter XIV.; *viz.* the *ohm* for resistance, the *volt* for E.M.F., and the *ampère* for current. They are connected with one another (by Ohm's law) in the following very simple manner.

$$\textit{Current in ampères} = \frac{\textit{E.M.F. in volts}}{\textit{resistance in ohms}}.$$

/

(ii.) In all graphical examples we may, without causing any error in the results obtained, consider the E.M.F. of a cell to be represented by an abrupt rise in potential occurring at one place in the cell.

If a *volt* and an *ohm* are represented by one and the same length marked off on the axes of ordinates and of abscissæ respectively, then the value of the current in *ampères* is given by *tan* α, where α is the angle made by the line representing the fall of potential with the axis of abscissæ.

For most purposes, unless the contrary is requested, the student may represent the total E.M.F. in the circuit by a single abrupt rise of potential occurring at one place in the circuit.

1. A cell has a tangent galvanometer of zero resistance, and nothing else, in its circuit; the deviation is 30°. When 9 more similar cells are added in series, the deflexion remains unaltered. Show how this experiment tends to confirm Ohm's law.

2. Two cells, of equal E.M.F. but of unequal resistances, are joined up in series, but with opposed poles, and a galvanometer is included in the circuit. Explain in words and in symbols your reasons for stating that a current will or will not pass.

3. A circuit includes a battery, some wire, and an electrolytic cell consisting of dilute acid in which are immersed platinum electrodes. Discuss the application of Ohm's law to such a case.

4. What will happen if . . . .
(i.) one point only in a battery circuit,
(ii.) two points simultaneously . . . .
be put to earth respectively? Discuss whether or no two points could be found which might be put to earth simultaneously without the current being affected.

(*Note.*—This question is perhaps answered most easily by the help of the graphical method.)

5. A cell of E.M.F. e_1 gives a deflexion θ_1; and gives a (smaller) deflexion θ_2 when a resistance r_1 is added to the circuit. Another cell of E.M.F. e_2 is substituted for the first, and the resistance is so adjusted that we have the deflexion θ_1 again; it is then found necessary to add a resistance r_2 in order to reduce the deflexion to θ_2. Show that $e_1 : e_2 = r_1 : r_2$.

6. Given 12 voltaic cells, each of an internal resistance 15 *ohms*, explain in how many different ways they may all be (conveniently) arranged. Which arrangement will give the greatest current when the external resistance is 20 *ohms*?

7. A single Bunsen's cell was found to give the same current through a galvanometer as did two Daniell's cells joined in series. If now the Bunsen's cell and the Daniell's battery be included in the same circuit, but with their *E.M.F.s* opposed, will there necessarily be no current? Explain in words and in symbols.

8. There is a circular ring of uniform wire. Two points in it, 135° apart, are connected with the opposite poles of a battery. Compare the currents in the two segments respectively.

9. In the last case, if the resistance of the whole ring be taken as unity, what is the 'equivalent resistance' between the two points A and B?

✓ 10. What is the best arrangement of 6 cells, each of $\frac{3}{4}$ *ohm* resistance, when the external resistance is 2 *ohms*?

11. In what sort of cases can *two* different arrangements of a given number of cells give the greatest current possible through a fixed external resistance?

✓ 12. Compare the two currents obtained in the following cases respectively.

(i.) Two cells, each of 3 *ohms* resistance, are joined in series, and the circuit is completed through a wire having 20 *ohms* resistance.

(ii.) The same two cells are joined in parallel, and the circuit is completed through the same wire, but this time doubled on itself.

13. If you wished to have a bichromate battery that would have a small internal resistance, and would yield a moderate current for a relatively long time, what instructions would you give to the makers?

14. It is required to drive a current through a fixed external resistance r, and there are at your disposal an indefinite number of Leclanché cells, each of *E.M.F.* E and resistance B. What is the limiting value of the current that you can drive through r by means of any number of these cells all arranged *in series*?

15. If in the last question you may arrange the cells in any way you please, is there a limit to the magnitude of the current? Explain fully.

16. Given an external resistance r that is very small as compared with the resistance B of a single cell, and a number of cells. Would you, in order to obtain the greatest possible current, arrange all your cells *in parallel*, however many cells there may be?

17. In the last case find the limiting value of the current obtainable by coupling an unlimited number of the cells *in parallel*.

18. There are two cells A and B, of E.M.F.s E_A and E_B respectively, where E_A is greater than E_B. They are placed in circuit with a tangent galvanometer (*see* Chapter XVII. § 5). When they are opposed, they give a deflexion $\alpha_2{}^{\circ}$; and when they drive the current in the same direction they give a deflexion $\alpha_1{}^{\circ}$. Find the ratio $E_A : E_B$.

(*Note.*—For the following 'graphical' exercises, *see* the note at the beginning of these questions on Chapter XIII.)

19. There is a cell of E.M.F. 2 *volts*, and internal resistance 1 *ohm*. If the external resistance be 2 *ohms*, exhibit graphically the condition of the circuit when closed.

20. There is a circuit of 12 *ohms* total resistance, this including a battery of 6 *volts* E.M.F. The resistance between the points A and B is 2 *ohms*. Find graphically the difference of potential between A and B.

21. In the last case the points A and B are joined also by a second wire of ·6 *ohm* resistance. Find algebraically . . . .

(i.) the equivalent resistance between A and B;

(ii.) the alteration in total current produced by the introduction of the second wire;

(iii.) the difference of potential between A and B.

22. There are arranged in series six cells each of E.M.F. 2 *volts* and of internal resistance 1 *ohm*. Represent graphically the condition of the circuit—

(i.) considering the whole E.M.F. to occur abruptly at one point;

(ii.) considering the E.M.F. of each cell to occur abruptly at the point where the copper terminal is soldered to the zinc plate of each cell.

23. A battery has a total E.M.F. of 20 *volts*, and an internal resistance of 12 *ohms*. It is in circuit with a storage battery of reverse E.M.F. of 4 *volts*, and resistance 2 *ohms*. The resistance of the rest of the circuit is 3 *ohms*. Considering the total E.M.F., *i.e.* the algebraic sum of the E.M.F.s, to occur abruptly at one point, represent graphically the condition of the closed circuit.

CHAPTER XIV.

1. If the conductivity of copper is to that of aluminium as 2·3 : 1, and their specific gravities are as 3·3 : 1, show what would be the ratio between the masses of two wires made of these two metals respectively, which for the same length offered the same resistance.

2. To measure the resistance of sulphuric acid a Wheatstone's bridge is sometimes employed; but the galvanometer is then replaced by a telephone (*see* Chapter XXV.), and the battery is replaced by an induction coil or other arrangement giving rapidly alternating currents. Explain the object aimed at in these modifications, and state how you would conduct the experiment indicated.

3. A Smee's cell is joined up with a tangent galvanometer (*see* Chapter XVII. § 5) of negligible resistance; the rest of the external circuit having also negligible resistance. A deflexion of θ° is observed where $\tan\theta = 2{\cdot}328$. When resistances of ·775 *ohm* and of 2·325 *ohms* are successively (not simultaneously) introduced into the circuit, the corresponding angles of deflexion had tangents equal to ·966 and ·445 respectively. From these data calculate *two* values for the internal resistance B of the cell, to four places of decimals; showing that these two results agree approximately.

4. The density of silver is 10·5, and the resistance of a silver wire 1 *mètre* in length and of 1 *gramme* mass is ·1689 *ohm*. Find the specific resistance of silver in *ohms*.

(*Note.*—This means the resistance in *ohms* of a piece of silver 1 *centimètre* long and 1 *square centimètre* cross section; the current passing between two opposite faces.)

5. Two separate pounds of copper of the same quality are drawn out into uniform wires, the one twice the length of the other. Find the ratio between their resistances.

6. A current of not less than ·016 *ampère* is to be sent through an external resistance of 360 *ohms*. What is the smallest number of Leclanché cells, each with E.M.F. 1·4 *volts* and resistance 15 *ohms*, by which this can be effected? What would be the maximum strength of current obtainable if double this number of cells were used?

7. The zinc pole of a Daniell's cell being joined to the platinum pole of a Grove's, the other poles are connected up with a tangent galvanometer, and the current is found to be ·5665 *ampère*. The zinc pole of the former is now connected with the zinc pole of the latter, the two positive poles are connected with the two terminals of the galvanometer, and the current is found to be ·0875 *ampère*. The resistances and E.M.F.s being supposed constant, find the ratio between the E.M.F.s of the two elements.

8. An iron wire ·24 *centimètre* in diameter has a resistance of 7·8 *ohms* per mile. What is the diameter of another wire of the same material which for the same length has a resistance of 12·87 *ohms*?

9. How would you use Wheatstone's bridge to ascertain the nature of the effect produced by rise of temperature on the specific resistance of any material?

10. How could you further ascertain the percentage change in resistance for rises in temperature from 0° to 10° C., 10° to 20° C., &c.?

11. The internal resistance of a cell is ·5 *ohm*. When a tangent-galvanometer (*see* Chapter XVII. § 5) of 1 *ohm* resistance is coupled with it by wires of negligible resistance, the deflexion is 15°. What will be the deflexion if the connecting wires have 1·5 *ohms* resistance?

(*Note.*-- A table of 'tangents' must be used.)

CHAPTER XV.

1. I observe some zinc dissolving in dilute acid, and I notice that the amount of heat calculated as due is not evolved. Explain how this may very well be the case, and yet the law of 'Conservation of energy' be unbroken.

2. From noticing the heat-energy evolved when a current passes through a fine wire, should you be justified in speaking of 'electricity' as a 'source of energy'? Explain clearly.

3. A current of 130 *ampères* flows through an armature whose resistance is ·5 ohm. How many *horse-power* are wasted in the armature?

4. Assuming that one *horse-power* is equivalent to 746 *volt-ampères*, and also to 33,000 *foot-pounds per minute*; and assuming that 1390 *foot-pounds* are equivalent to a *pound-degree-Centigrade* unit of heat; find the number of pounds of water that can be heated through 1° C. by the heat emitted in 1 *hour* by a lamp through which a current of 1 *ampère* flows, the difference of potential between the terminals of the lamp being 60 *volts*.

5. Two cells have internal resistances each equal to 3 *ohms*. In one case they are connected in series by a wire of 3 *ohms* resistance. In another case they are joined in parallel by the same wire. Compare the total heat evolved per *second* in the two cases respectively, and show in each case its distribution between cells and wire.

6. A given wire, forming the whole external circuit of a battery, has resistance of 3 *ohms*. You are provided with 8 cells, each of 1 *ohm* resistance and E.M.F. 2 *volts*. How will you arrange the cells so as to make the wire as hot as possible? How many calories will then be evolved per *second* in the

wire? And what *work per second* (in *ergs per second*) will be the equivalent of this?

7. If you wish to heat a platinum wire, at a distance from the battery, to as high a temperature as possible, what sort of connecting wires will you use, and why? And what arrangement of battery-cells will you adopt?

8. If in the last case the insulation of the conducting wires was very imperfect, show whether it would be better to increase the number of cells arranged in series, or the number arranged in parallel; supposing that you have some additional cells at your disposal.

9. A silver wire is joined end to end with an iron wire which is of the same length, but of double the diameter and of six times the specific resistance of the former. The whole is put into circuit with a battery, and when the current has passed for a certain time it is found that the total heat evolved in the two wires is 45 *calories*. How is this shared between the two wires?

10. The difference of potential between two points in a circuit is 50 *volts*, the resistance between them is 1 *ohm*, and the current is 10 *ampères*. Show that there must be a reverse E.M.F. between the two points, and find its value.

11. In the last example, (i.) what is the *work per second* (in *watts*) done against this reverse E.M.F.; and (ii.) what is the *heat per second* (in *calories per second*) evolved between the two points?

12. In a certain circuit there is a battery of E.M.F. 20 *volts*, and resistance 2 *ohms*; a wire of resistance 1 *ohm*; and an electrolytic cell of resistance 1 *ohm*, and reverse E.M.F. 2 *volts*. Find . . . .

(i.) the *watts* expended by the battery;

(ii.) the *calories per second* evolved in the battery, in the wire, and in the electrolytic cell, respectively;

(iii.) the *work per second* (in *watts*), of a chemical nature, spent on the electrolytic cell.

CHAPTER XVII.

1. If you have a sine-galvanometer unprovided with shunts, and a tangent galvanometer, which would you use for the measurement of very feeble currents, and which for the measurement of very strong currents? Explain your answer.

2. What if the sine-galvanometer be provided with shunts and compensating resistances?

3. Given a delicate galvanometer and a current of unknown strength, how would you so proceed in your measurement of this current as to obviate any risk of injury to the galvanometer due to too great strength of current passing through the galvanometer?

4. You are given a set of constant cells; a galvanometer whose law of deflexion is complex and unknown; a Wheatstone's rheostat; and a tangent galvanometer. Show how you could graduate the first-named galvanometer so as to make it afterwards available for measuring, and not merely indicating, currents.

5. Suppose that you have thus calibrated your galvanometer up to a considerable angle of deflexion, and that you desire to use it for larger currents. Show how you can do this if you are provided with a shunt whose resistance bears a known ratio to the resistance of the galvanometer.

6. If you do not know this ratio, show how you can find it by the use of the above apparatus.

7. How would you determine the 'compensating resistance' to be inserted when the shunt is used?

8. In the above series of experiments (4–7), does it matter whether the E.M.F. of the battery is constant or no? Explain your answer.

(*Note.*—Questions 4–8 were suggested by, or taken from, the electrical course described in Worthington's 'Physical Laboratory Practice.')

CHAPTER XVIII.

1. A single turn of wire in the form of a circle of 10 *centimètres* radius is placed in the plane of the magnetic meridian, and two turns of wire of 20 *centimètres* radius are placed with their common plane and centre coinciding with those of the first circle. A small magnet is suspended at the common centre in the plane of the coils. The *same* current is now passed in one direction through the single wire, and in the other direction through the double wire, respectively. What effect is produced on the needle? Explain in words, and prove your answer in symbols.

2. A current is passed along a permanent magnet of hard steel, magnetised in the direction of its length. Give reasons for arguing that the current will take a spiral course along the bar, and show whether the spiral will form a right-handed or a left-handed screw.

3. A current is passed along a bar of perfectly soft iron. Give reasons for arguing that the molecules will now form closed magnetic circuits, which are circles having their planes perpendicular to the direction of the current. And show in what direction the *north*-seeking and *south*-seeking poles of each molecule will point.

4. A circular coil of wire is suspended from some height, with its plane vertical, so that it can both swing and rotate, and a current is passed through it. Discuss the action of the pole of a magnet on the coil in various cases.

5. If it were true that the earth's magnetism is due to currents traversing the earth's surface, show what would be their general direction.

6. An elastic spiral of wire hangs so that its lower end *just* dips into a vessel of mercury. Describe and explain what happens when the top of the spiral is connected with the one pole of a battery, and the mercury is connected with the other pole of the same.

7. A coil of 50 turns of wire has a mean radius of 20 *centimètres*, and a current of 6 *ampères* traverses it. Find, in absolute C.G.S. units, the strength of the magnetic field produced at the centre of the coil.

8. If, in the last example, the coil be placed in the plane of the magnetic

meridian, and if the earth's horizontal field be of strength ·2 in absolute measure, find the tangent of the angle of deflexion of a small needle balanced (so as to move horizontally) at the centre of the circle.

9. There is a circle of *one* turn of wire. A current of C absolute units traverses it. What is the '*strength*' $\rho\, t$ of the equivalent magnetic shell?

10. A circular coil consists of n turns of wire, and is 1 *centimètre* thick, and the current is C. If the equivalent magnetic shell were also 1 *centimètre* thick, what would be the surface density ρ of its magnetism?

11. In a similar case the thickness of the coil is not given; but you are told that the wire is coiled at the rate of n turns per 1 *centimètre* of thickness. If the thickness of the equivalent magnetic shell be the same as that of the coil, what must be its surface density ρ of magnetism?

CHAPTER XX.

1. What are the best materials respectively for . . . .

(i.) magnetic needles?

(ii.) the cores of electro-magnets?

(iii.) the wire forming the coils of an electro-magnet? Explain each answer.

2. If a solenoid be made of wire carrying a current C in absolute measure, there being n turns of wire per 1 *centimètre* of length, what would be the surface density ρ of magnetism on the faces of the equivalent bar-magnet? (*See* Chapter XVIII. Question 11 *above.*)

3. It is generally assumed, as sufficiently accurate when the helix is long as compared with its diameter, that the field-strength throughout the interior of the helix is constant and is measured by $4\, \pi\, n$ C. What will be the surface-density ρ of magnetism at the ends of an iron core inserted in the helix, the 'coefficient of magnetisation' being k?

4. If the helix and core have a common cross-section of A *square centimètres*, what is . . . .

(i.) The pole-strength of the helix alone?

(ii.) That of helix and core together?

5. Applying the results of Chapter X. §§ 14, 15, &c., to the case of magnetism, and assuming that *all* the resultant 'marked lines of force' proceed *outwards* from the face of a magnet, show what will be the number of resultant 'marked lines' due . . . .

(i.) To the pole of the helix alone;

(ii.) To the pole of the iron core alone.

CHAPTER XXI.

1. If two wires, attached to the terminals of a Grove's battery of three or four cells, be held in the hands, and be alternately placed in the hand and separated, nothing is felt. But if a large electro-magnet be in the circuit, then a shock is felt when contact is broken. Account for this.

2. A bar of perfectly soft iron is thrust into the interior of a coil of wire whose terminals are connected through a galvanometer. An induced current is observed. Explain how this may be.

3. Could the coil and bar be placed in such a position that the above action might nearly or entirely disappear? Explain fully.

4. A plane rectangular iron frame is placed vertically so that it faces due 'magnetic north.' It then is overthrown, falling northward into a horizontal position. Discuss, in a general manner, what occurs.

5. In the last example assume some exact values, in absolute C.G.S. units, for . . . .

(i.) the area of the frame;

(ii.) the dip and strength of the earth's field;

(iii.) the time occupied in the fall;

and from these data deduce the average induced E.M.F.

6. A current flows through a wire, and a magnetic pole is consequently urged into rotation round the wire. If the battery be now removed and be replaced by a delicate galvanometer, and if the pole be moved by hand in the same direction as before, what will be observed? Of what laws is this an example?

CHAPTER XXII.

1. In constructing an induction coil, what special contrivances must be introduced in order to obtain long sparks?

2. An insulated wire is coiled round a bar of copper, and a current is momentarily passed through the wire. What effects are produced?

3. In the last case it is found that, while the current is flowing, there is some resistance felt to the withdrawal of the copper bar from within the coil. To what general law may this phenomenon be referred?

4. A wire is coiled round a bar-magnet, and a bar of soft iron is brought near to one pole of the magnet. Explain whether or no there will be a current induced in the coil, and, if there be such a current, show in what direction it will run.

5. In passing a current from an induction coil through a 'vacuum-tube' the current passes in one direction only. How is this? And how can you determine what is this direction of current? (Give two methods, at least.)

6. The number of oscillations made by a given needle in one and the same place, in a given time, is affected by placing a copper plate under it. Explain this, and point out what practical use can be made of the fact.

7. The poles of a voltaic battery are connected with two mercury-cups. These cups are connected successively by . . . .

(i.) a long straight wire;

(ii.) the same wire arranged in a close spiral, the wire being covered with some insulating material;

(iii.) the same wire coiled round a soft iron core.

Describe and discuss what happens in each case when the circuit is broken.

8. In the automatic 'make-and-break' spiral, described in Question 6 of Chapter XVIII. p. 458, discuss the effect of introducing a copper core that does not touch the spiral.

9. In the same case discuss the effect of a similar soft iron core.

10. In the same case discuss the effect of a similar core consisting of hard steel magnetised longitudinally, with one or the other pole respectively at the lower end.

CHAPTERS XXIII., XXIV., and XXV.

1. How should a dynamo, intended for electro-plating work, be constructed? Explain your answer.

2. Explain in what cases it would be dangerous for a person . . . . .

(i.) to touch with *one* hand only the wire in circuit with a dynamo;

(ii.) to touch the same at two different points with the *two* hands simultaneously.

3. The heat supplied per *second* to a boiler is equivalent to 750 *kilogram-mètres* of work. *One-tenth* of this is usefully employed in driving a dynamo, the efficiency of which is ·75. The current drives a motor that is also of efficiency ·75, and 10 *per cent.* of the energy of the dynamo's current is lost in the form of heat. Find the *work per second* of the motor in *kilogram-mètres per second.*

4. A dynamo has a constant E.M.F. of 200 *volts*, and is in circuit with a motor, the total resistance of the whole circuit being 6 *ohms*. Find. . . .

(i.) the current when the motor is at rest;

(ii.) the current, activity used in the motor, total activity, and activity wasted in heat, when the motor is run at such a rate that there is 'maximum work' per *second* done;

(iii.) the current, total activity, and activity used in the motor, when this is run at such a rate that the efficiency is 90 *per cent.*

5. It is desired to measure the resistance of a certain incandescent lamp when the carbon is rendered white-hot by a current of 1·3 *ampères*. It is found that then there is a difference of potential of 65 *volts* between the terminals of the lamp. What is the required resistance?

6. A cable a mile long, whose ends are A and B, has a total resistance of 3·59 *ohms*. It is put into water, and a small hole is pierced to the core at a certain point. When the end B is insulated and the resistance is tested from the end A, this is found to be 2·81 *ohms*. When A is insulated, the resistance tested from the end B is found to be 2·76 *ohms*. Find the distance in yards of the puncture from the end A, and its resistance in *ohms*.

ANSWERS TO QUESTIONS.

CHAPTER I.

(4) Float the two on a piece of cork, on water.

CHAPTER II.

(1) $2222\frac{2}{9}$. (2) $\frac{5}{6}$. (3) $69\frac{4}{9}$.
(4) 50 *dynes*; $\theta = \tan^{-1}\frac{4}{3}$. (5) 4000 *dynes*; $\theta = \tan^{-1}\frac{3}{4}$.
(6) 30 *mètres*. (7) 240. (8) $1\frac{7}{13}$.
(9) 120. (10) $43\frac{1}{3}$ *dynes*; $21\frac{2}{3}\sqrt{3}$ *dynes*.

CHAPTER III.

(1) 960. (2) 150.
(6) Deflect each to same angle with lines of earth's 'horizontal field.'
(7) The moments of inertia of the needles enter into the formulæ of calculation.
(8) $\sqrt{63} = 8$, nearly. (9) 11, nearly. (10) 26·6.
(11) ·2 *dynes*; there is *attraction*.
(12) By means of a large magnet suitably placed.
(14) Ratio is 256 : 225; magnetic moment of needle assumed constant.
(15) $\frac{1}{\sqrt{3}}$; $\frac{1}{2\sqrt{3}}$. (16) 24.

CHAPTER IV.

(7) Attraction, 2 *dynes*; repulsion, ·25 *dyne*.

CHAPTER V.

(3) (i.) A and C are at a + potential, B at a zero potential, and the leaves diverge with +. (ii.) B C all at zero potential, and the leaves hang together.

CHAPTER VI.

(3) Both outside and inside are raised in potential, their difference of potential remaining (practically) constant.

CHAPTER X.

(1) B is at a + potential, but lower than A's. That, as we charge A, the potential of B rises more slowly than does that of A.

(2) $Q = \frac{98,100}{\sqrt{3}}$.

(4) Q = 10; total K = 6; common potential = $1 \cdot \dot{6}$.

(5) Divergence increases; divergence decreases; leaves collapse.

(6) K = 255. (7) 10; 10 σ; 110; 880.

(8) Diameter = $\frac{475}{\pi}$ *centimètres.*

(9) That of A rises, that of B falls, somewhat.

(10) The force on a + unit at a given distance will be less; hence the 'marked lines' will be fewer, and the equipotential surfaces further apart.

(11) It will decrease. (12) $\frac{Q^2}{2K}$ *ergs.*

(15) Merely substitute magnetic unit pole for electric unit quantity, and take infinity, not 'earth,' as region of zero potential.

(16) $1\frac{1}{3}$.

CHAPTER XI.

(1) Its potential will have fallen.

(2) Copper is deposited by chemical substitution, and then the whole acts as a closed cell.

(3) We have a closed cell, and the nascent hydrogen sets free antimony on the platinum plate.

(4) More copper is set free by the nascent hydrogen.

(5) Hydrogen will be set free from the platinum plate as in an ordinary closed cell.

(7) If the current be too 'dense,' *i.e.* too great per unit area of the plates.

CHAPTER XII.

(1) (i.) Copper deposited on copper dish, lead dioxide on the lead plate. (ii.) A back current, and a return (at least *partial*) to the initial condition.

(2) A deflexion, and then a return to zero. A deflexion the other way, and a return to zero.

(3) A continued deflexion.

(5) By suitable changes in method of coupling.

(8) Same method as in Question 5.

CHAPTER XIII.

(1) Easily proved by formula.

(5) Use formula for current in each case.

(6) In *six* main ways. 4 in *series* and 3 in *parallel.*

(7) Depends on the respective internal resistances, as well as on E.M.F.s.

(8) Ratio is 3 : 5. (9) $\frac{15}{64}$.

(10) Either 6 in *series*, or 3 in *series* and 2 in parallel.

(12) $\frac{E}{13}$, and $\frac{2E}{13}$, *ampères* respectively. (14) Limit is $C = \frac{E}{B}$.

(15) There is no limit. Prove by formula.

(16) No. Prove by general formula. (17) Limit is $\frac{E}{r}$.

(18) $\frac{E_A}{E_B} = \frac{\tan\alpha_1 + \tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2}$. (20) 1 *volt*.

(21) (i.) ·5 *ohm*; (ii.) from ·5 *ampère* to $\frac{4}{7}$ *ampère*; (iii.) $\frac{2}{7}$ *volt*.

CHAPTER XIV.

(1) $\frac{\text{Mass of copper}}{\text{Mass of aluminium}} = \frac{3{\cdot}3}{2{\cdot}3}$.

(3) The two answers are B = ·5496, and B = ·5494, *ohms* respectively.

(4) $1{\cdot}608 \div 10^6$, nearly. (5) Ratio is 4 : 1.

(6) 5 cells in series; ·027 *ampères*, nearly.

(7) E.M.F.s have ratio of 479 : 654. (8) ·187 *centimètre*, nearly.

(11) 7° 37′.

CHAPTER XV.

(3) 11·32 h.p., nearly. (4) 115 *lbs.* nearly.

(5) (i.) Heat in first case : Heat in second case = $\frac{\cdot 32}{3}E^2 : \frac{\cdot 16}{3}E^2 = 2 : 1$.

(ii.) In the first case $\frac{2}{3}$ of the heat is internal and $\frac{1}{3}$ external, while in the second case $\frac{1}{3}$ is internal and $\frac{2}{3}$ external.

(6) 4 in *series*, 2 in *parallel*; 1·8432 *calories per second*; 77,414,400 *ergs per second*.

(8) In parallel. (9) $\frac{\text{Heat in silver wire}}{\text{Heat in iron wire}} = \frac{2}{3}$.

(10) Reverse E.M.F. = 40 *volts*.

(11) 400 *watts*; 24 *calories per second*.

(12) (i.) 90 *watts*; (ii.) 9·72, 4·86, and 4·86 *calories per second* respectively; (iii.) 9 *watts*.

CHAPTER XVIII.

(1) The needle is unaffected. (4) Consider the equivalent magnetic shell.

(5) From E to W.

(6) It will vibrate vertically, making and breaking contact alternately.

(7) 9·425, nearly. (8) $\tan\theta = 47{\cdot}124$, nearly. (9) Strength = ρt = C.

(10) $\rho \times 1 = n\,C$, or $\rho = n\,C$.

(11) Let t be the common thickness, expressed in *centimètres*, of coil and shell. Then $\rho\,t = t\,n\,C$, or $\rho = n\,C$.

CHAPTER XX.

(2) $+\,n\,C$ and $-\,n\,C$ at the two ends respectively.

(3) $\rho = 4\pi n\,C \times k$. (4) (i.) $n\,C\,A$; (ii.) $n\,C\,A + 4\pi n\,C\,k\,A$.

(5) (i.) $4\pi n\,C\,A$; (ii.) $4\pi \times$ *pole-strength* $= (4\pi)^2 \times n\,C\,k\,A$.

CHAPTER XXI.

(2) Consider the earth's field and the magnetisation of the soft iron due to this.

(6) This is an example of Lenz's law.

CHAPTER XXII.

(2) Eddy currents arise. (3) Induction; Lenz's law.

(4) By magnetic induction the field due to the magnet is altered.

(6) Eddy currents arise, and Lenz's law holds.

(7) (i.) Hardly any extra current; (ii.) decided extra current; (iii.) still greater extra current.

(8) Slower vibration. Compare with Question 6.

(9) Still slower vibration.

(10) Observe that with suitable pole downwards the vibrations may even be stopped altogether.

CHAPTERS XXIII., XXIV., and XXV.

(1) Small internal resistance, low E.M.F.

(2) Much depends on whether or no the dynamo can 'leak' to earth.

(3) 37·96 *kilogram-mètres per second.*

(4) (i.) $33\frac{1}{3}$ *ampères*; (ii.) $16\frac{2}{3}$ *ampères*, $1666\frac{2}{3}$ *watts*, $3333\frac{1}{3}$ *watts*, $1666\frac{2}{3}$ *watts*; (iii.) $3\frac{1}{3}$ *ampères*, $666\frac{2}{3}$ *watts*, 600 *watts*.

(5) 50 *ohms*. (6) 892 *yards*, nearly; ·99 *ohm*.

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PRINTED BY
SPOTTISWOODE AND CO., NEW-STREET SQUARE
LONDON

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